COASTAL ENGINEERING

Dr.N .Ashok Kumar
Associate Professor
Department of Civil Engineering

Harbour

- Harbour can be defined as a basin or haven of navigable waters well protected naturally or artificially from the action of winds and waves. They are situated at sea-shore or river estuary or lake or canal connected to the sea.

Classification of Harbour

- There are many Classification of Harbours. let us discuss briefly about each one of them.
- **<u>CLASSIFICATION OF HABOUR DEPENDING UPON THE PROTECTION NEEDED</u>**

➢ Natural Harbours or Natural Roadsteads

Inlet protected from storms and waves by natural configuration of the land A deep navigable channel with a protective natural bank or shoal to seaward is a good example of a natural roadstead

- **Semi-natural Harbour**
- ➢ Protected on sides by headlands and requires man-made protection only at the entrance
- Eg: Vishakapatanam
- **Artificial Harbours Or Artificial Roadsteads**
- ➢ No natural facilities are available for the harbour,also known as man-made harbours.
- ➢ An area protected from the effect of waves either by breakwaters.
- Eg: Madras harbour
- ➢ Artificial roadsteads – created by constructing a breakwater or wall parallel to the coast or curvilinear from the coast

CLASSIFICATION OF HARBOUR DEPENDING UPON THE UTILITY

- **Harbours of Refuge**
  - All naval crafts small or big require refuge in an emergency
  - Modern big ships will require a lot of elbow room for turning and manoeuvring
  - Should provide commodious accommodation
- **Requirements**
  - Ready accessibility from the high seas
  - Safe and convenient anchorage against the sea.
  - Facilities for obtaining supplies and repairs

Continued

- **Commercial Harbours**
  - Not situated on coasts of big rivers or even on island river coast
  - Do not have any emergency demand
- **Requirements**
  - Spacious accommodation of mercantile marine
  - Ample quays space and facilities for transporting, loading and unloading cargo
  - Storage sheds for cargo
  - Good and quick repair facilities to avoid delay
  - More sheltered conditions as loading and unloading could be done with advantage in calmer waters

- **Fishery Harbours**

- ➢ Constantly open for departure and arrival of fishing ships.
- ➢ Loading and unloading facilities & quick despatch facilities for the perishable fish catch.
- ➢ Refrigerated stores with ample storing space for preserving the catch.

- **Military Harbour**

➢ Should accommodate the naval vessels.

➢ They serve as supply depots also.

➢ Bombay and Cochin harbours have naval bases.

- **Marina Harbours**
- *Marina is a harbour providing* facilities for fuel, food, showers, telephones etc. for small boat owners having temporary or permanent berths
- **Classification**
- Large marinas – have 200 or more berths.
- Small marinas – have less than 100 berths.
- **Facilities Provided**
- Resort Facilities
- Yacht Club
- Sport, Fishing Facilities
- Marina Pubs

CLASSIFICATION OF HARBOUR DEPENDING UPON THE LOCATION

- ***(1) Canal Harbour***
  - ➢ Harbour located along canals for sea navigations.
  - ➢ Dredging is negligible
- ***(2) Lake Harbour***
  - ➢ Harbour constructed along the shore of the lake.
  - ➢ No tidal action
- ***(3) River Harbour***
  - ➢ Also known as Estuary harbour
  - ➢ Harbour constructed along the banks of the river.
- ***(4) Sea Or Ocean Harbour***
  - ➢ Harbour located on the coast of the sea or an ocean.

Wind And Waves

- The ocean is never still. Whether observing from the beach or a boat, we expect to see waves on the horizon.
- Waves are created by energy passing through water, causing it to move in a circular motion. However, water does not actually travel in waves.
- Waves transmit energy, not water, across the ocean and if not obstructed by anything, they have the potential to travel across an entire ocean basin.

- Waves are most commonly caused by wind.
- **Wind-driven waves**, or **surface waves**, are created by the friction between wind and surface water. As wind blows across the surface of the ocean or a lake, the continual disturbance creates a wave crest. These types of waves are found globally across the open ocean and along the coast.

- More potentially hazardous waves can be caused by severe weather, like a hurricane. The strong winds and pressure from this type of severe storm causes storm surge, a series of long waves that are created far from shore in deeper water and intensify as they move closer to land

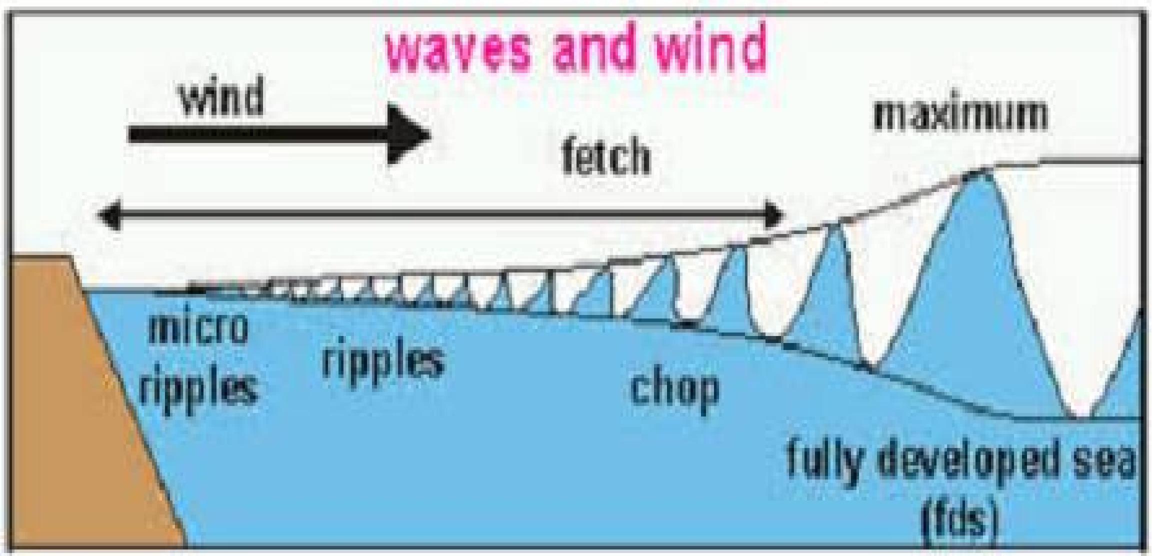
- Other hazardous waves can be caused by underwater disturbances that displace large amounts of water quickly such as earthquakes, landslides, or volcanic eruptions. These very long waves are called tsunamis.
- Storm surge and tsunamis are not the types of waves you imagine crashing down on the shore. These waves roll upon the shore like a massive sea level rise and can reach far distances inland.

Tidal Waves

- The gravitational pull of the sun and moon on the earth also causes waves. These waves are tides or, in other words, **tidal waves**. It is a common misconception that a tidal wave is also a tsunami. The causes of tsunamis are not related to tide information at all but can occur in any tidal state.

Three different types of wind waves develop over time

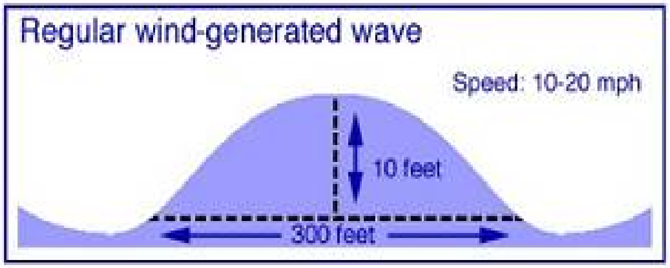
- Capillary **waves**, or ripples, dominated by surface tension effects.
- Gravity **waves**, dominated by gravitational and inertial forces. Seas, raised locally by the **wind**.
- Swell



waves and wind
wind
maximum
fetch
micro ripples
ripples
chop
fully developed sea (fds)

Types of Wind Waves

1. Capillary
2. Breakers:
   1. Surging
   2. Spilling
   3. Plunging
3. Swells



- **Sea**' is a description of the wind waves raised by the wind in the immediate neighbourhood of the place of observation at the time of observation.
- '**Swell**' is a description of the **ocean** waves that are not raised by the local wind.

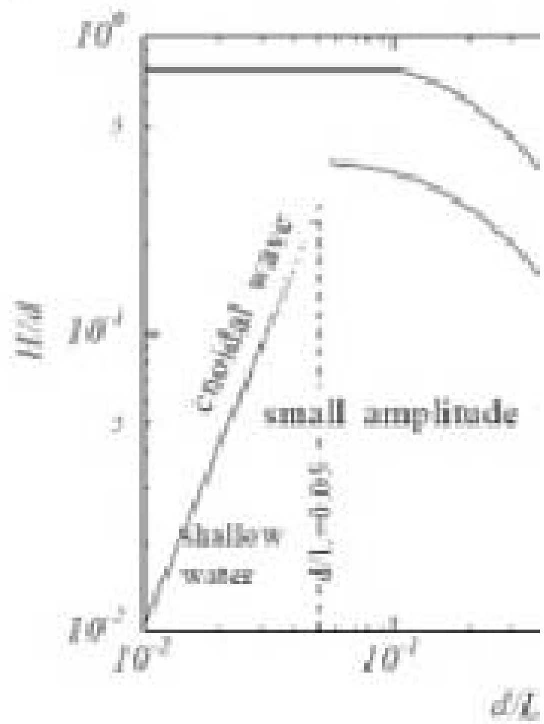
Causes of Swell

- Waves located on the ocean's surface are commonly **caused** by wind transferring its energy to the water, and big waves, or **swells**, can travel over long distances. A wave's size depends on wind speed, wind duration, and the area over which the wind is blowing (the fetch).

WAVE THEORIES

Wave theories are approximations to reality.

1. **Linear wave theories**
   - Small amplitude wave theory
2. **Non-linear wave theories**
   - Stokes finite amplitude wave theory
3. **Other theories**
   - Nonlinear shallow water eave theories
   - KdV and Boussinesq Eqs
   - Cnoidal wave theory
   - Stream function theory
   - Fourier approximation



WAVE THEORIES

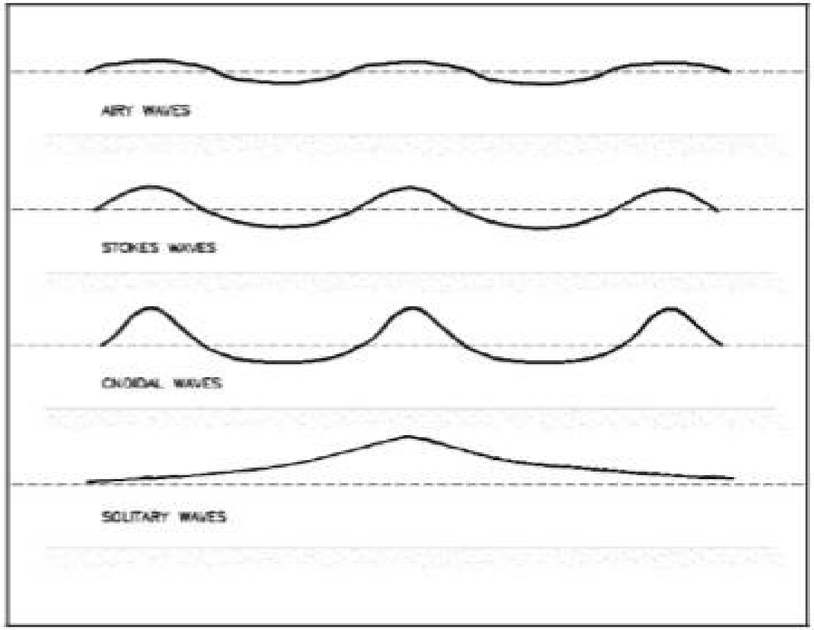


Figure: Wave profile of different progressive gravity waves

SMALL AMPLITUDE WAVE THEORY

The earliest mathematical description of periodic progressive waves is that attributed to Airy in 1845.

Airy wave theory is strictly only applicable to conditions in which the wave height is small compared to the wavelength and the water depth.

It is commonly referred to as **linear or first order wave theory**, because of the simplifying assumptions made in its derivation.

SMALL AMPLITUDE WAVE THEORY

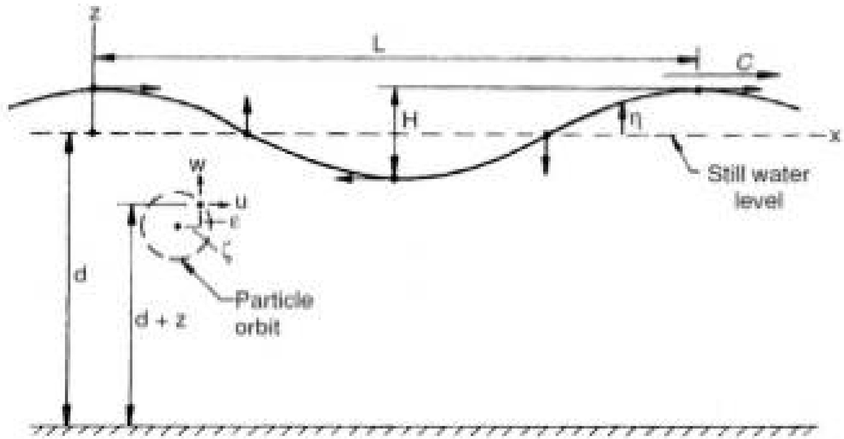


Figure: Definition of progressive surface wave parameters

L= wave length
T=wave period
σ=angular frequency=2π/T
C=wave speed (celerity)
H=wave height
a= amplitude of wave (H/2)=horizontal excursion of water particle
η=instantaneous surface water elevation
d= local water depth

u and w = The horizontal and vertical components of the water particle velocity at any instant.
ς and ε = The horizontal and vertical coordinates of a water particle at any instant.

SMALL AMPLITUDE WAVE THEORY

Assumptions:

The basic assumptions for linear wave theory are as follows:

1. Water is homogeneous and incompressible, wave lengths are greater than 3m so that capillary effects may be ignored

2. Flow is irrotational, i.e. no shear stress present anywhere. Thus the velocity potential, Φ, must satisfy the Laplace equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad (1)$$

Laplace equation is actually continuity equation:

$$\underbrace{\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}}_{\text{continuity}} = 0 = \underbrace{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2}}_{\text{Laplace}}$$

Where,

$$u = \partial \phi / \partial x, \; w = \partial \phi / \partial z$$

SMALL AMPLITUDE WAVE THEORY

Assumptions:

3. The bottom is not moving and is impermeable and horizontal, i.e. no energy transfer through the bed. This lead to:

$$w = \left.\frac{\partial \phi}{\partial z}\right|_{z=-d} = 0$$

4. The pressure along the air-sea interface is constant, i.e. no effect of weather related pressure differences

$$\frac{\partial \phi}{\partial t} + gz + \frac{p}{\rho} + \frac{1}{2}\left(u^2 + w^2\right) = C(t) \quad \text{at } z = \eta \quad \xrightarrow[\substack{u^2 + w^2 = 0 \\ p/\rho = 0}]{\text{Assuming}} \quad \eta = -\frac{1}{g}\left.\frac{\partial \phi}{\partial t}\right|_{z=\eta}$$

5. The wave amplitude is small compared to the wave length and water depth.

$$\eta = -\frac{1}{g}\left.\frac{\partial \phi}{\partial t}\right|_{z=\eta} \quad \xrightarrow{\eta \text{ is very small}} \quad \eta = -\frac{1}{g}\left.\frac{\partial \phi}{\partial t}\right|_{z=0}$$

SMALL AMPLITUDE WAVE THEORY

Basic Equations:

The water surface profile is given as:

$$\eta = (H/2)\cos(kx - \sigma t)$$

Where, k is wave number (=2 π /L) and

σ is angular frequency(=2 π /T)

Since the velocity potential, Φ, should be cyclic with horizontal position and time, and it should vary with depth, we can assume that:

$$\phi = f(z)\sin(kx - \sigma t) \qquad (2)$$

Substituting Eq.(2) into (1), one can get:

$$\frac{\partial^2 f}{\partial z^2} - k^2 f = 0$$

SMALL AMPLITUDE WAVE THEORY

The general solution to this partial differential equation is:

$$f(z) = A\exp(kz) + B\exp(-kz)$$

Where A an B are arbitrary constants. Putting this value of the functi in Eq.(2), we get:

$$\phi = \{A\exp(kz) + B\exp(-kz)\}\sin(kx - \sigma t)$$

Two boundary conditions are required to find A and B.

SMALL AMPLITUDE WAVE THEORY

(a) The vertical velocity, *w*, at the bottom must be zero (Assumption 3):

$$w = \left.\frac{\partial \phi}{\partial z}\right|_{z=-d} = 0$$

From Eq. (3)

$$\frac{\partial \phi}{\partial z} = k\{A\exp(kz) - B\exp(-kz)\}\sin(kx - \sigma t)$$

At $z=-d$

$$\left.\frac{\partial \phi}{\partial z}\right|_{z=-d} = k\{A\exp(-kd) - B\exp(kd)\}\sin(kx - \sigma t) = 0$$

Since k and $\sin(kx-\sigma t)$ can not be equal to zero (if they are zero, there is no wave!), we shall have always:

$$A\exp(-kd) - B\exp(kd) = 0$$

Or

$$A = B\frac{\exp(kd)}{\exp(-kd)}$$

Substituting this value of A in Eq.(3) we get:

$$\phi = B\exp(kd)[\exp\{k(d+z)\} + \exp\{-k(d+z)\}]\sin(kx - \sigma t)$$

Since, $[\exp\{k(d+z)\} + \exp\{-k(d+z)\}] = 2\cosh k(d+z)$, we get:

$$\phi = 2B\exp(kd)\cosh k(d+z)\sin(kx - \sigma t) \qquad (4)$$

SMALL AMPLITUDE WAVE THEORY

(b) Second boundary condition (on the surface) may be derived from Bernoulli's equation for time-varying flow in two dimensions, i.e.

$$\frac{\partial \phi}{\partial t} + gz + \frac{p}{\rho} + \frac{1}{2}\left(u^2 + w^2\right) = 0$$

On the surface atmospheric pressure is there, i.e. $p/\rho = 0$. Assuming particle velocities to be small, $u^2 + w^2 = 0$ at the surface, $z = \eta$.

Hence, $\partial\phi/\partial t + g\eta = 0$, at the surface.

Or
$$\eta = -\frac{1}{g}\frac{\partial \phi}{\partial t}\bigg|_{z=\eta}$$

Considering η to be very small, we can say:

$$\eta = -\frac{1}{g}\frac{\partial \phi}{\partial t}\bigg|_{z=0}$$

$$\eta = (H/2)\cos(kx - \sigma t)$$

At t=0, x=0 and z=0, η=H/2

Or
$$\frac{gH}{2} = -\frac{\partial \phi}{\partial t}\bigg|_{z=0}$$

SMALL AMPLITUDE WAVE THEORY

From Eq.(4)

$$\left.\frac{\partial \phi}{\partial t}\right|_{z=0} = -2\sigma B \exp(kd) \cosh k(d+z)\big|_{z=0} \cos(kx - \sigma t)$$

At t=0, x=0 (crest of the wave), $\cos(kx - \sigma t) = 1$

So,

$$2B\exp(kd) = \frac{Hg}{2\sigma \cosh kd}$$

Eq.(4) becomes:

$$\phi = \frac{H}{2} \frac{g \cosh k(d+z)}{\sigma \cosh kd} \sin(kx - \sigma t) \qquad (5)$$

The velocity potential given by Eq.(5) can be used to find the velocities in x and z directions as will be shown later.

SMALL AMPLITUDE WAVE THEORY

Another useful relationship may be derived by considering the vertical component of velocity of a particle on the water surface, w, which is given as:

$w = \frac{\partial \eta}{\partial t}$ but at the water surface, $\eta = -\frac{1}{g}\frac{\partial \phi}{\partial t}\Big|_{z=0}$ so,

$$w = -\frac{1}{g}\frac{\partial^2 \phi}{\partial t^2}\Big|_{z=0}$$

Also, $w = \frac{\partial \phi}{\partial z}$, we can write:

$$\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial z} = 0$$

Inserting ϕ from Eq.(5) and solving, we get the so-called ***dispersion relation***:

$$\sigma^2 = gk\tanh(kd) \qquad (6)$$

Dispersion *of water waves generally refers to frequency dispersion, which means that waves of different wavelengths travel at different phase speeds*

SMALL AMPLITUDE WAVE THEORY

Since, the wave celerity C, is defined as: $C=L/T=\sigma/k$
From Eq.(6) we get:

$$C = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)} \tag{7}$$

Or

$$C = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{L}\right) \tag{8}$$

And

$$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right) \tag{9}$$

Eq.(9) is an implicit relationship by virtue of the wave length L and therefore, it can be solved by trials only. Fenton (1990) has given an approximate explicit relationship from which the wave length can be directly calculated from the available knowledge of wave period and depth of water as follows:

$$L = \frac{gT^2}{2\pi}\left\{\tanh\left(\frac{\sigma^2 d}{g}\right)^{3/4}\right\}^{2/3} \tag{10}$$

SMALL AMPLITUDE WAVE THEORY

Summary:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2}$$

continuity Laplace

$$\eta = (H/2)\cos(kx - \sigma t)$$

$k = (2\pi/L)$ & $\sigma = (2\pi/T)$

$$\phi = \frac{H}{2}\frac{g\cosh k(d+z)}{\sigma\cosh kd}\sin(kx - \sigma t)$$

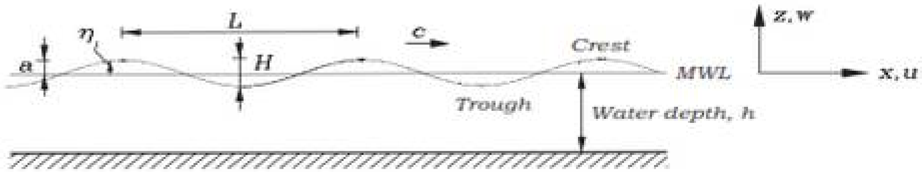
$$\sigma^2 = gk\tanh(kd)$$

$$C = L/T = \sigma/k$$

$$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$$

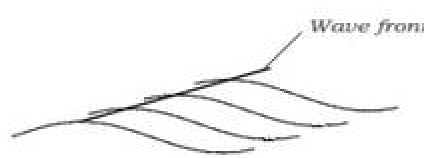
$$C = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{L}\right) \quad \& \quad C = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)}$$

1.3 Definitions and Symbols



H	wave height
a	wave amplitude
η	water surface elevations from MWL (posituve upwards)
L	wave length
$s = \dfrac{H}{L}$	wave steepness
$c = \dfrac{L}{T}$	phase velocity of wave
T	wave period, time between two crests passage of same vertical se
u	horizontal particle velocity
w	vertical particle velocity
$k = \dfrac{2\pi}{L}$	wave number
$\omega = \dfrac{2\pi}{T}$	cyclic frequency, angular frequency
h	water depth

Wave fronts



Wave orthogonals

What causes swell at sea?

- Waves located on the ocean's surface are commonly **caused** by wind transferring its energy to the water, and big waves, or **swells**, can travel over long distances. A wave's size depends on wind speed, wind duration, and the area over which the wind is blowing (the fetch).

How is sea swell measured?

- It's **measured** from the trough (very lowest point) to peak (very highest point) of each wave. Generally speaking the larger the **swell** the larger the waves it'll create.

What's the **difference between waves**, seas and **swell**?

- **Waves** are generated by wind moving over water; they indicate the speed of the wind in that area. **Swell** are **waves** (usually with smooth tops) that have moved beyond the area where they were generated.

What is a good swell period?

- **Swell period** is a measure of that acquired momentum and it determines how far a **swell** will be able to travel in the open ocean. Short-**period swell**, (11 seconds or less) will usually decay within a few hundred miles, while long-**period swell**, (above 14 seconds), is capable of far greater journeys.

What is a big swell?

- **Swells** are collections of waves produced by storm winds raging hundreds of miles out to sea, rather than the product of local winds along beaches. They are formed by a combination of factors and are coveted by surfers looking to catch a **big** wave

What is a good swell?

- **Swell** period is the time passed between two waves. ... A wave period above 8 seconds is average, above 11 seconds is **good**, above 14 seconds is **great** (at this point the waves has definitely traveled long enough to not be affected at all by the storm that created them, they are pure ground **swell**).

What are two types of waves?

- Waves come in two kinds, longitudinal and transverse. Transverse waves are like those on **water**, with the surface going up and down, and longitudinal waves are like of those of sound, consisting of alternating compressions and rarefactions in a medium.

What are the types of waves?

- Usually, waves are around us, it can be sound waves, **radio waves**, water waves, sine waves, cosine waves, string waves, slinky waves etc. These are created through disturbance.

What do you call a group of waves?

- *Often, when several **waves** are travelling along a medium as shown above, the continuous g**roup of waves** is **called** a **wave** train*.

What are 3 causes of waves?

- Waves are dependent on three major factors – **wind speed**, **wind** time and **wind** distance.

- What is a very large wave called?
- tsunami. noun. a **very large wave** or series of **waves** caused when something such as an earthquake moves a **large** quantity of water in the sea.

1.1 Wave Classification

Various types of waves can be observed at the sea that generally can be divided into different groups depending on their frequency and the generation method.

Phenomenon	Origin	Period
Surges	Atmospheric pressure and wind	1 – 30 days
Tides	Gravity forces from the moon and the sun	app. 12 and 24 h
Barometric wave	Air pressure variations	1 – 20 h
Tsunami	Earthquake, submarine land slide or submerged volcano	5 – 60 min.
Seiches (water level fluctuations in bays and harbour basins)	Resonance of long period wave components	1 – 30 min.
Surf beat, mean water level fluctuations at the coast	Wave groups	0.5 – 5 min.
Swells	Waves generated by a storm some distance away	$<$ 40 sec.
Wind generated waves	Wind shear on the water surface	$<$ 25 sec.

- Waves generated locally by wind are known as ***sea. It consists of waves of many*** different wave heights and periods as shown in the time series in Fig. 2.3. These waves propagate more or less in the wind direction.
- Waves are formed by a crossing pattern of two wave trains propagating at a small angle away from the wind direction as shown in Fig. 2.4. Local peaks in the water level occur where the two wave trains add and lower water levels exist where they subtract, resulting in the irregular wave pattern of Fig. 2.3 at any particular location.

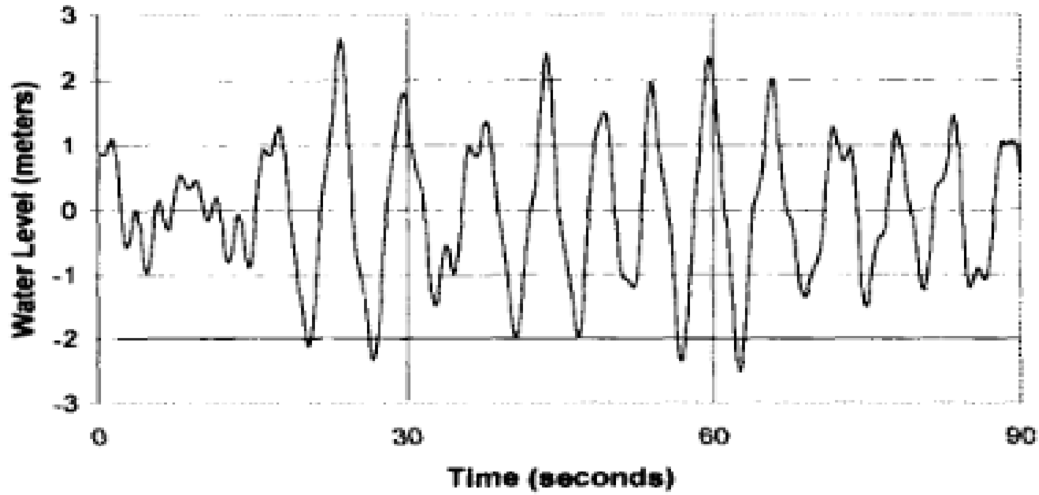


Figure 2.3 Record of Locally Generated Sea

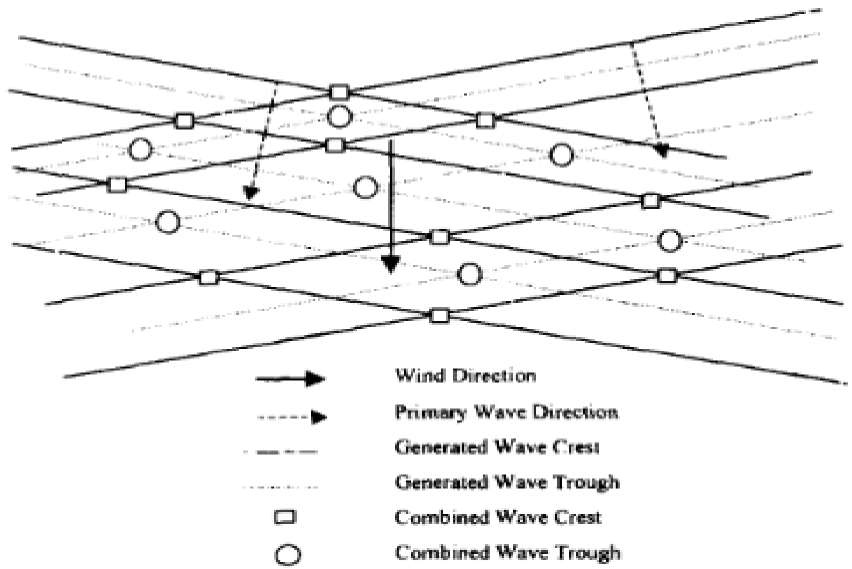


Figure 2.4 Crossing Pattern of Waves

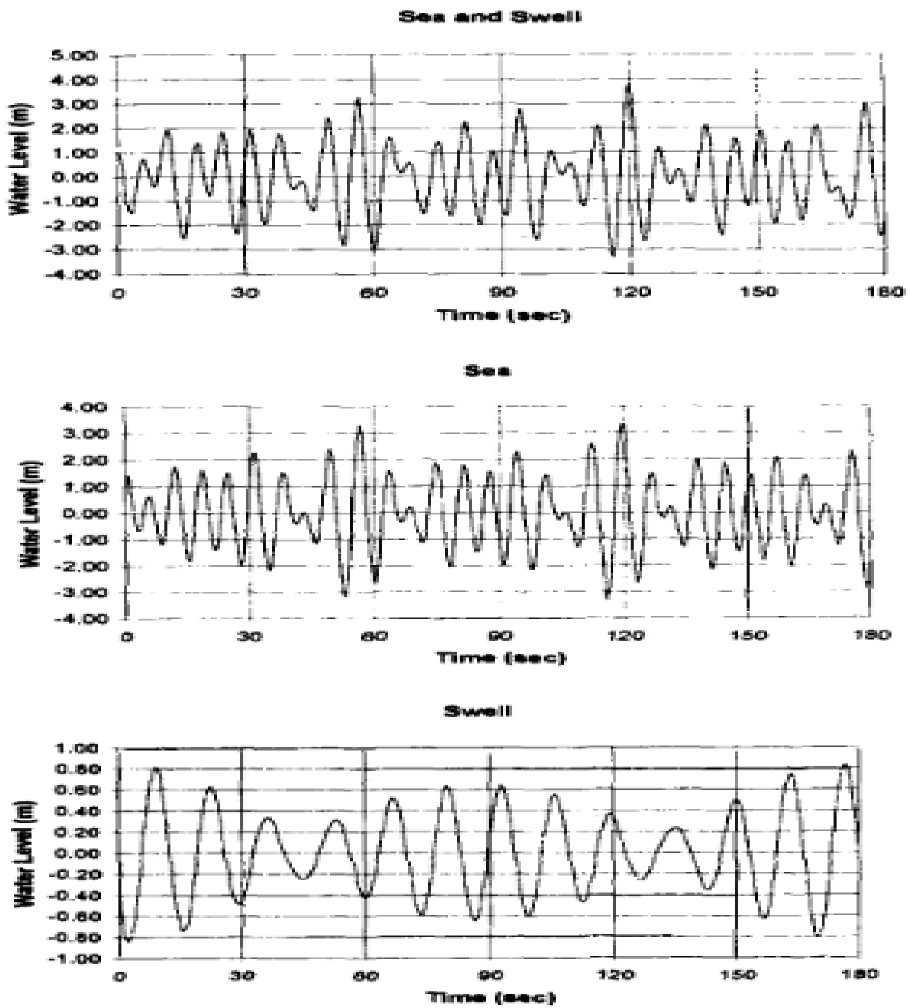


Figure 2.5 Sea and Swell Separated

- Waves, generated some distance away are called ***swell. The difference between sea*** and swell is shown in Fig. 2.5.

Wave Table

Table 2.2: Common Expressions For Progressive Waves

		Deep Water (d/L>0.5)	Shallow Water (d/L>0.5)
1. Water Surface [m]	$\eta = \frac{H}{2}\cos(kx-\omega t)$		
2. Velocity of Propagation [m/s] (Dispersion Equation)	$C = \frac{L}{T} = \frac{\omega}{k} = \frac{gT}{2\pi}\tanh kd$ $= \sqrt{\frac{gL}{2\pi}\tanh kd}$	$C_o = \frac{gT}{2\pi}$	$C = \sqrt{gd}$
3. Wave Length [m]	$L = CT = \frac{gT^2}{2\pi}\tanh kd$	$L_o = \frac{gT^2}{2\pi}$	
4. Horizontal Component of Orbital Velocity [m/s]	$u = \frac{\pi H}{T}\frac{\cosh k(z+d)}{\sinh kd}\cos(kx-\omega t)$	$u_o = \frac{\pi H_o}{T}e^{k_o z}\cos(k_o x-\omega t)$	
5. Vertical Component of Orbital Velocity [m/s]	$w = \frac{\pi H}{T}\frac{\sinh k(z+d)}{\sinh kd}\sin(kx-\omega t)$	$w_o = \frac{\pi H_o}{T}e^{k_o z}\sin(k_o x-\omega t)$	
6. Horizontal Orbital Semi-Axis [m]	$A = \frac{H}{2}\frac{\cosh k(z+d)}{\sinh kd}$	$A_o = \frac{H_o}{2}e^{k_o z}$	
7. Vertical Orbital Semi-Axis [m]	$B = \frac{H}{2}\frac{\sinh k(z+d)}{\sinh kd}$	$B_o = A_o$	

Wave Table

		Deep Water (d/L>0.5)	Shallow Water (d/L>0.5)
8. Pressure [m of water]	$\frac{p}{\rho g} = -z + K_p \eta$		
9. Pressure Response Factor	$K_p = \frac{\cosh k(z+d)}{\cosh kd}$	$K_p = e^{k_o z}$	
10. Energy Density [j/m²]	$E = \frac{1}{8} \rho g H^2; \quad KE = PE = \frac{E}{2}$		
11. Wave Power [w/m]	$P = EC_G$	$P_o = \frac{EC_o}{2}$	$P = EC$
12. Group Velocity [m/s]	$C_G = nC$	$(C_G)_o = \frac{C_o}{2}$	$C_G = C$
13. Group Velocity Parameter	$n = \frac{1}{2}\left[1 + \frac{2kd}{\sinh 2kd}\right]$	$n_o = \frac{1}{2}$	$n = 1$
14. Mass Transport at Bottom [m/s]	$U_B = \frac{5}{4} \frac{a^2 k \omega}{\sinh^2 kd}$		
15. Wave Breaking Criterion	$\left(\frac{H}{L}\right)_{max} = 0.142 \tanh kd$	$\left(\frac{H}{L}\right)_{max} = 0.142$	$\left(\frac{H}{d}\right)_{max} = 0.78$
16. MWL – SWL [m]	$\Delta = \frac{H^2 k}{8} \coth kd$	$\Delta = \frac{H^2 k}{8}$	

2.3.2 *Small Amplitude Expressions*

Waves propagate with velocity C, but the individual water particles do not propagate; they move in particle orbits as shown in Fig. 2.9. For small amplitude wave theory, such particle orbits are elliptical and if the water is 'deep', they become circular. Their size decreases with depth. Horizontal and vertical orbital velocity components, u and w, and the horizontal and vertical orbital amplitudes, A and B, are given in Eqs. [4] to [7].

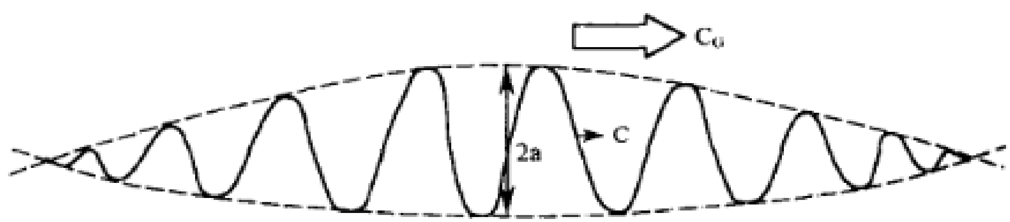
The pressure fluctuations at any point below the water surface are related to the water level fluctuations at the surface. If the wave were infinitely long, the water level would be horizontal at any time, there would be no particle motion and the pressure fluctuations would be hydrostatic and equal to ρgH, where ρ is the fluid density and g is the gravitational acceleration. For waves of limited length the pressure fluctuations are smaller than (ρgH). The ratio of the actual pressure fluctuations to (ρgH), is called the pressure response factor, K_p, and it is a function of wave length (or wave period) and depth below the surface. For longer waves or close to the water surface, the pressure response factor approaches 1. For shorter waves or far below the water surface, the pressure response factor approaches 0. Equations. [8] and [9] quantify the pressure response.

Equation [10] expresses wave energy per unit surface area, or energy density E, in joules/m*. It is made up of half potential energy and half kinetic energy.

Equation 11 gives wave power, P, arriving at any location. Its units are wattslm of wavecrest.

Equation. [2] indicates that longer period waves travel faster than shorter period waves. A real wave train, as in Fig. 2.3, contains many different wave periods, and therefore it would stretch out (disperse) as it traveled. The longest waves would lead and run further and further ahead with time and distance, while the shortest waves would lag further behind. Hence Eq. [2] is called the ***dispersion equation.***

Equation **[2] also means that away from their immediate, generating area, waves of** roughly the same period tend to travel together. From basic physics we know that waves of almost the same period interfere to form beats or wave groups. The theoretical expression for the interference pattern of waves of almost the same period ***is shown in Fig. 2.10.***



$$\eta_t = 2a\cos\left[\frac{k_1+k_2}{2}x+\frac{\omega_1+\omega_2}{2}t\right]\cos\left[\frac{k_1-k_2}{2}x+\frac{\omega_1-\omega_2}{2}t\right]$$

Figure 2.10 Wave Group

The resulting wave consists of two waves that are superimposed – one related to the average values of k and ω of the two interfering waves and another, much longer wave, called the wave group, related to the differences in k and ω. There are two wave speeds involved (ω/k) - one for the short waves $C=(\omega_1+\omega_2)/(k_1+k_2)$ and another for the wave group $G_G=(\omega_1-\omega_2)/(k_1-k_2)$. The speed of the wave group is related to C by the factor n, as given in Eq. [13]. In deep water $n \rightarrow \frac{1}{2}$ and in shallow water $n \rightarrow 1$. Thus $C_G<C$, but in very shallow water $C_G \rightarrow C$. Figure 2.10 shows that the wave group consists of a series of individual waves that increase in size and then decrease.

This gives rise to the adage "every 7th wave is a big one". Because $C > C_G$, the individual waves travel through the group. At the back of the group they are small. Then they increase in size as they travel through the group, decrease in size past the centre of the group and eventually disappear at the front of the group.

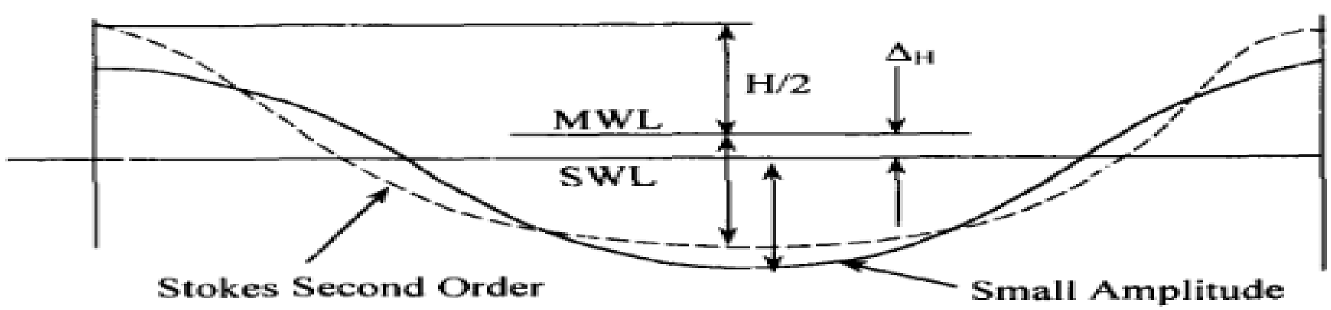


Figure 2.11 Wave Shape and Mean Wave Level

Eqs. [14], **[I51 and [I61 are expressions derived from other theories, often used as** simple extensions of small amplitude theory. According to higher order theory, the particle orbits of Fig. 2.9 are not closed. There is a small net movement of the water

particles in the direction of wave propagation called Mass Transport. The mass transport velocity at the bottom, **UB, which is important for calculating sediment** transport, is given in Eq. **[14]. Three wave breaking criteria are given in Eq. [15].** There is a limit to the steepness of the wave. In shallow water, this reduces to a limit of the ratio (H/d), known as the breaking index. Solitary wave theory defines this limit as 0.78, a value supported by experimental observation. Finally, although the wave crest and trough are equidistant (H/2) from the still water level in small amplitude wave theory, higher order wave theory estimates that the wave crests are higher and the wave troughs become shallower than in small amplitude theory (Fig. 2.11**). This creates a difference between mean wave level midway between crest** and trough and still water level, as given by Eq [**16].**

Wave table

To solve Eq. [2] and all the other equations in Table 2.2, it is necessary to know the wave length, L, which may be calculated using Eq. [3]. However, Eq. [3] is implicit and can only be solved numerically. Tables of solutions have been prepared that yield L as well as other important wave characteristics. Such tables are known as wave tables and have been published in CERC (1984) and Wiegel (1964). An abbreviated set of wave tables is presented in Table 2.3. To use the wave tables, we first calculate the deep water approximation of wave length as given by Eq. [3].

$$L_0 = \frac{gT^2}{2\pi} \qquad (2.6)$$

Then we use the depth of water d to calculate d/L_0 and from there we can evaluate all the remaining wave parameters from Table 2.3. An example is given at the end of the next section.

Table 2.3 Wave Table

d/L_o	tanh kd	d/L	kd	sinh kd	cosh kd	n	d/L_o	tanh kd	d/L	kd	sinh kd	cosh kd	n
0.000	0.000	0.0000	0.000	0.000	1.00	1.000	0.22	0.909	0.242	1.52	2.18	2.40	0.646
0.002	0.112	0.0179	0.112	0.113	1.01	0.996	0.23	0.918	0.251	1.57	2.31	2.52	0.635
0.004	0.158	0.0253	0.159	0.160	1.01	0.992	0.24	0.926	0.259	1.63	2.45	2.65	0.626
0.006	0.193	0.0311	0.195	0.197	1.02	0.998	0.25	0.933	0.268	1.68	2.60	2.78	0.616
0.008	0.222	0.0360	0.226	0.228	1.03	0.983	0.26	0.940	0.277	1.74	2.75	2.93	0.608
0.010	0.248	0.0403	0.253	0.256	1.03	0.979	0.27	0.956	0.285	1.79	2.92	3.09	0.599
0.015	0.302	0.0496	0.312	0.317	1.05	0.969	0.28	0.952	0.294	1.85	3.10	3.25	0.592
0.020	0.347	0.0576	0.362	0.370	1.07	0.959	0.29	0.957	0.303	1.90	3.28	3.43	0.585
0.025	0.386	0.0648	0.407	0.418	1.08	0.949	0.30	0.961	0.312	1.96	3.48	3.62	0.578
0.030	0.420	0.0713	0.448	0.463	1.10	0.939	0.31	0.965	0.321	2.02	3.69	3.83	0.571
0.035	0.452	0.0775	0.487	0.506	1.12	0.929	0.32	0.969	0.330	2.08	3.92	4.05	0.566
0.040	0.480	0.0833	0.523	0.548	1.14	0.919	0.33	0.972	0.339	2.13	4.16	4.28	0.560
0.045	0.507	0.0888	0.558	0.588	1.16	0.910	0.34	0.975	0.349	2.19	4.41	4.53	0.555
0.050	0.531	0.0942	0.592	0.627	1.18	0.900	0.35	0.978	0.358	2.25	4.68	4.79	0.550
0.055	0.554	0.0993	0.624	0.665	1.20	0.891	0.36	0.980	0.367	2.31	4.97	5.07	0.546
0.060	0.575	0.104	0.655	0.703	1.22	0.880	0.37	0.983	0.377	2.37	5.28	5.37	0.542
0.065	0.595	0.109	0.686	0.741	1.24	0.872	0.38	0.984	0.386	2.43	5.61	5.70	0.538
0.070	0.614	0.114	0.716	0.779	1.26	0.863	0.39	0.986	0.395	2.48	5.96	6.04	0.535
0.075	0.632	0.119	0.745	0.816	1.29	0.853	0.40	0.988	0.405	2.54	6.33	6.41	0.531
0.080	0.649	0.123	0.774	0.854	1.31	0.845	0.41	0.989	0.415	2.60	6.72	6.80	0.529
0.085	0.665	0.128	0.803	0.892	1.34	0.836	0.42	0.990	0.424	2.66	7.15	7.22	0.526
0.090	0.681	0.132	0.831	0.929	1.37	0.827	0.43	0.991	0.434	2.73	7.60	7.66	0.523
0.095	0.695	0.137	0.858	0.968	1.39	0.819	0.44	0.992	0.443	2.79	8.07	8.14	0.521
0.10	0.709	0.141	0.886	1.01	1.42	0.810	0.45	0.993	0.453	2.85	8.59	8.64	0.519
0.11	0.735	0.150	0.940	1.08	1.48	0.794	0.46	0.994	0.463	2.91	9.13	9.18	0.517
0.12	0.759	0.158	0.994	1.17	1.54	0.778	0.47	0.995	0.472	2.97	9.71	9.76	0.516
0.13	0.780	0.167	1.05	1.25	1.60	0.762	0.48	0.995	0.482	3.03	10.3	10.4	0.514
0.14	0.800	0.175	1.10	1.33	1.67	0.747	0.49	0.996	0.492	3.09	11.0	11.0	0.513
0.15	0.818	0.183	1.15	1.42	1.74	0.733	0.50	0.996	0.502	3.15	11.7	11.7	0.512
0.16	0.835	0.192	1.20	1.52	1.82	0.718	0.75	1.000	0.746	4.69	54.5	54.5	0.501
0.17	0.850	0.200	1.26	1.61	1.90	0.705	1.0	1.000	0.981	6.16	269.5	269.5	0.500
0.18	0.864	0.208	1.31	1.72	1.99	0.692							
0.20	0.888	0.225	1.41	1.94	2.18	0.668							
0.21	0.899	0.234	1.47	2.05	2.28	0.656							

Example 2.1 Use of the Wave Table

We will now calculate the wave characteristics for a wave of period T = 8 sec and a wave height H = 1.5 m in a depth of water d = 6 m. We use small amplitude wave theory (Table 2.2) and the wave table (Table 2.3).

It is first necessary to calculate the deep water wave length and relative depth

$$L_o = \frac{gT^2}{2\pi} = 1.56T^2 = 1.56(64) = 100m; \quad \frac{d}{L_o} = \frac{6}{100} = 0.060 \tag{2.7}$$

The wave table (Table 2.3) now yields the following

$$\begin{aligned} &\frac{d}{L} = 0.104; \quad \tanh kd = 0.575; \\ &\sinh kd = 0.703; \quad \cosh kd = 1.22; \quad n = 0.881 \end{aligned} \tag{2.8}$$

From the value of $\frac{d}{L}$, the wave length in 6 m of water and wave number, k, may now be calculated

$$L = \frac{d}{0.104} = 57.5m; \quad k = \frac{2\pi}{L} = 0.109\, m^{-1} \tag{2.9}$$

From these, the following parameters may be computed; ρ is assumed to be 1035 kg/m^3 for sea water.

$$C = \frac{L}{T} = 7.2\ m/s; \quad C_G = nC = 0.881(7.2) = 6.35\ \ m/s;$$
$$E = \frac{\rho g H^2}{8} = 2854\ \ j/m^2; \quad P = EC_G = 18{,}124\ \ w/m\ \text{of wave crest} \tag{2.10}$$

At the bottom:

$$z = -d; \quad k(z+d) = 0; \quad \sinh k(z+d) = 0; \quad \cosh k(z+d) = 1.0 \tag{2.11}$$

and the horizontal component of orbital velocity is

$$\begin{aligned} u_B &= \frac{\pi H}{T}\frac{1}{\sinh kd}\cos(kx - \omega t) \\ &= \frac{\pi(1.5)}{8}\frac{1}{.703}\cos(kx - \omega t) = 0.84\cos(kx - \omega t) \end{aligned} \tag{2.12}$$

Thus, at the bottom, u_B has a maximum value $\hat{u}_B$ =0.84 m/s and the vertical velocity component of orbital motion at the bottom is zero. The amplitude of the orbital motion at the bottom is

$$A_B = \frac{H}{2\sinh kd} = \frac{1.5}{2\,(0.703)} = 1.07\ m. \tag{2.13}$$

and the orbital diameter is $2A_B$ = 2.14 m. The pressure response factor K_p at the bottom is

$$(K_p)_B = \frac{1}{\cosh kd} = \frac{1}{1.221} = 0.82 \tag{2.14}$$

which means that the maximum pressure fluctuation is

$$K_p H = 0.82\,(1.5) = 1.23\,(\text{m of water}) \tag{2.15}$$

or for sea water

$$\rho g(K_p H) = 1035\,(9.81)(0.82)(1.5) = 12{,}788\ Pa = 12.8\ kPa \tag{2.16}$$

At a distance of 4 m below the water surface: z = -4m and k(z+d) = 0.218. This gives sinh k(z+d) = 0.220 and cosh k(z+d) = 1.024, $\hat{u}$ = 0.84(1.024) = 0.86 m/s, $\hat{w}$ = 0.84(0.220)=0.18 m/s, A = 1.07(1.024) = 1.10 m/s, B = 1.07(.220) = 0.24 m/s. Finally, K_p = 0.82(1.024) = 0.84.

Calculation by Computer

The above calculation using the wave table is fine if you have only a few calculations to make. What about if you need to make many calculations and use a computer? In that case, L or C may be calculated using a root finding technique such as Newton-Raphson, but such a technique requires iteration. To speed up such computations, approximations may be used, such as the one proposed by Hunt (1979)

$$\frac{C^2}{gd} = \left[y_o + (1 + 0.6522y_o + 0.4622y_o^2 + 0.0864y_o^4 + 0.0675y_o^5)^{-1}\right]^{-1} \quad (2.17)$$

where

$$y_o = \frac{2\pi d}{L_o} \quad (2.18)$$

Fig. 2.12 presents a spreadsheet calculation for Example 2.1, using Eq. 2.17.

Beaufort Wind Force	Wind Speed (knots)[2]	Description of Wind	Description of Sea	Approx H_s (m)	Approx T (sec)
0	0-1	Calm	Sea is like a mirror.	0	1
1	1-3	Light airs	Ripples are formed.	0.025	2
2	4-6	Light breeze	Small wavelets. Still short but more pronounced; crests have a glassy appearance, but do not break	0.1	3
3	7-10	Gentle breeze	Large wavelets. Crests begin to break. Perhaps scattered white caps.	0.4	4
4	11-17	Moderate breeze	Small waves, becoming larger. Fairly frequent white capping.	1	5
5	17-21	Fresh breeze	Moderate waves, taking a more pronounced long form. Many white caps are formed (chance of some spray).	2	6
6	22-27	Strong breeze	Large waves begin to form. The white foam crests are more extensive everywhere (probably some spray).	4	8
7	28-33	Moderate gale	Sea heaps up and white foam from breaking waves begins to be blown in streaks along the direction of the wind (spindrift).	7	10
8	34-40	Fresh gale	Moderately high waves of greater length. Edges of crests break into spindrift. The foam is blown in well-marked streaks along the direction of the wind. Spray affects visibility.	11	13
9	41-47	Strong gale	High waves. Dense streaks of foam along the direction of the wind. Sea begins to roll. Visibility is affected.	18	16
10	48-55	Whole gale[3]	Very high waves with long overhanging crests. The resulting foam is in great patches and is blown in dense white streaks along the direction of the wind. On the whole, the surface of the sea takes a white appearance. The rolling of the sea becomes heavy and shocklike. Visibility is affected.	25	18
11	56-63	Storm[3]	Exceptionally high waves (small and medium sized ships might for a long time be lost to view behind the waves). The sea is completely covered with long white patches of foam lying along the direction of the wind. Visibility is affected.	35[4]	20[4]
12	64-71	Hurricane[3]	Air filled with foam and spray. Sea completely white with driving spray; visibility very seriously affected.	40[4]	22[4]

Example 2.3-1

A wave in water 100 m deep has a period of 10 s and a height of 2 m. Determine the wave celerity, length, and steepness. What is the water particle speed at the wave crest?

Solution:

Assume that this is a deep water wave. Then, from Eq. (2.17)

$$L_o = \frac{9.81(10)^2}{2\pi} = 156\,\text{m}$$

Since the depth is greater than half of the calculated wave length, the wave is in deep water and the wave length is 156 m. [Otherwise, Eq. (2.14) would have to be used to calculate the wave length.] The wave celerity is from Eq. (2.2)

$$C_o = \frac{156}{10} = 15.6\,m/s$$

and the steepness is

$$\frac{H_o}{L_o} = \frac{2}{156} = 0.013$$

For deep water the particle orbits are circular having a diameter at the surface equal to the wave height. Since a particle completes one orbit in one wave period, the particle speed at the crest would be the orbit circumference divided by the period or

$$u_c = \frac{\pi H_o}{T} = \frac{3.14(2)}{10} = 0.63\,m/s$$

Note that this is much less than C_o.

When the relative depth is less than 0.5 the waves interact with the bottom. Wave characteristics depend on both the water depth and the wave period, and continually change as the depth decreases. The full dispersion equations must be used to calculate wave celerity or length for any given water depth and wave period. Dividing Eq. (2.13) by Eq. (2.16) or Eq. (2.14) by Eq. (2.17) yields

$$\frac{C}{C_o} = \frac{L}{L_o} = \tanh\frac{2\pi d}{L} \tag{2.18}$$

which is a useful relationship that will be employed in a later chapter. Waves propagating in the range of relative depths from 0.5 to 0.05 are called intermediate or transitional water waves.

When the relative depth is less than approximately 0.05, tanh $(2\pi d/L)$ approximately equals $2\pi d/L$ and the dispersion equation yields

$$C = \sqrt{gd} \tag{2.19}$$

or

$$L = \sqrt{gd}\,T \tag{2.20}$$

Waves in this region of relative depths are called shallow water waves. In shallow water the small-amplitude wave theory gives a wave celerity that is independent of wave period and dependent only on the water depth (i.e., the waves are not period dispersive). The finite-amplitude wave theories presented in the next chapter show that the shallow water wave celerity is a function of the water depth and the wave height so that in shallow water waves are amplitude dispersive. Remember that it is the relative depth, not the actual depth alone, that defines deep, intermediate, and shallow water conditions. For example, the tide is a very long wave that behaves as a shallow water wave in the deepest parts of the ocean.

Example 2.3-2

Consider the wave from Example 2.3-1 when it has propagated in to a nearshore depth of 2.3 m. Calculate the wave celerity and length.

Solution:

Assuming this is a shallow water wave, Eq. (2.19) yields

$$C = \sqrt{9.81(2.3)} = 4.75\,\mathrm{m/s}$$

and Eq. (2.2) yields

$$L = 4.75(10) = 47.5\,\mathrm{m}$$

So $d/L = 2.3/47.5 = 0.048 < 0.05$ and the assumption of shallow water was correct. Compare these values to the results from Example 2.3-1

Wave Measurement

- In order to understand the coastal environment, probably the most important parameter to determine is the wave climate - the waves that are present at a location over the long term (years) and over the short term (storms and individual waves). The measurement of such waves is the topic of this section.

Wave measurements can be made with different types of recorders kept either at the sea surface or over and below it. The airborne devices include the satellite based sensing of the surface using a radar altimeter. The floating recorders could be either of electrical resistance gauges, ship borne pressure sensors or wave rider buoys. The submerged category involves the pressure gauges and the echo sounders. Out of all above types the wave rider buoy (Fig. 1.9) is most commonly employed in routine wave data collection.

Fig 1.9 Wave Rider Buoy **(www.niot.res.in)**

- It is in the form of a spherical buoy that is kept floating on the sea surface. It undergoes accelerations in accordance with the wave motion. The vertical accelerations are continuously recorded by an accelerometer located inside the buoy. These are further integrated twice electronically to obtain records of the sea surface elevations which in turn are sent to a shore based receiving station. Commonly, a 20-minuterecord collected once in every 3 hours, as a true statistical sample during the period, is practiced to optimize the data collection.

- Forecasting of waves for operational or design purpose needs to be made by measuring and analyzing the actual wave observations at a given location.
- But considering the difficulties and costs involved in getting large scale wave data, many times, the readily available wind information is gathered and then converted into corresponding wave information although this procedure is less accurate than the actual wave analysis.

- The wind information required to forecast the waves can be obtained by making direct observations at the specific ocean site or at a nearby land site. The latter observations require projection to the actual location by applying some overland observation corrections. Wind speed and its direction can be observed at regular intervals and hourly wind vectors can be recorded.

- Alternatively use of synoptic surface weather maps can also be made to extract the wind information. These maps may give Geostrophic or free air speed, which is defined as the one undisturbed by effects of the boundary layer prevalent at the interface of air and sea. Instead of this speed, which may exist at a very large height from the sea surface, the wind prediction formulae incorporate the wind speed value at a standard height of 10 m above the mean sea level (U10) which can be obtained by multiplying the geostrophic speed by a varying correction factor.

This value of U10 so deduced needs further corrections as below before it can be used as input in the wave prediction formulae.

(i) **Correction for overland observations:** This is necessary when wind is observed overland and not over water in which case the roughness of the sea surface is different. If wind speed overland (U_L) is greater than 1.85 m/sec, i.e., 41.5 mph, the correction factor $R_L = U_{10}/U_L$ may be taken as 0.9. If $U_L \sim 15$ m/sec, R_L =1.0. If U_L<15 m/sec, R_L=1.25.

- (ii) **Correction for the difference in air and sea temperature:**

  This difference affects the boundary layer. The correction factor can be substantial - varying from 1.21 for a temperature difference of -20 degrees to about 0.78 for the temperature difference of +20 degrees.

- (iii) **Correction for shortness of observations duration: Since the wind is observed for a very**

  short duration of say 2 minutes at a time, its stable value over duration of an hour or so is required to be calculated. Empirical curves are available to obtain the corrections (i), (ii) and (iii) above. (SPM, 1984).

- (iv) **Correction to account for the non-linear relation between the measured wind speed**
- **and its stress on the seawater:** This correction is given by,

$$U_{corrected} = (0.71)\, U_{10}^{\;1.23}$$

If the wind speed in a given region does not change by about ± 2.5 m/sec with corresponding direction changes of about ± 15 degrees then such a region can be regarded as fetch region. Its horizontal dimension expressed in distance scale, called Fetch, is required as another input in the wave prediction formulae. For coastal sites the upwind distance along the wind direction would give the required fetch value. Alignment (curvature or spreading) of the isobars in weather maps also yields the wind fetch.

Constant wind duration forms an additional input in the formulae of wave prediction. This is obtained by counting the time after allowing deviations of 5 percent in speed and 15 degrees in the directions.

The problem of wave forecasting aims at arriving at the values of the significant wave height (H_s) and the significant wave period (T_s) from given wind speed, duration and fetch distances over which the speed remains constant.

- If we have a collection of pairs of individual wave heights and wave periods (or zero cross periods, meaning thereby that the crests should necessarily cross the mean zeroth line. (Fig 1.10), then an average height of the highest one third of all the waves (like H1, H2, H3...of Fig 1.10) would give the significant height. (Hs) while an average of all wave periods (like T1, T2, T3...of Fig 1.10) would yield significant wave periods (Ts). These definitions are empirical in origin.

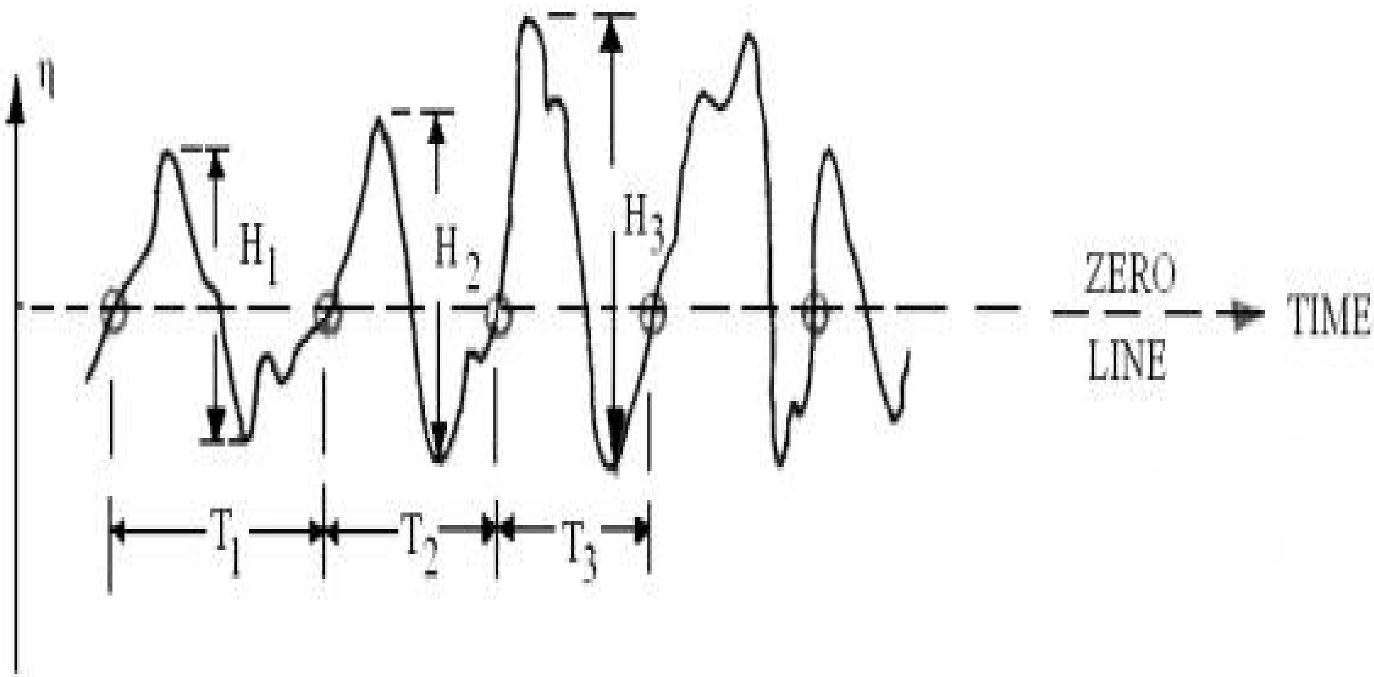


Fig 1.10 Individual Waves

Wave Direction

- If a single sensor is placed to measure waves, the record resembles Fig. 2.3. It gives information of water level fluctuations with respect to time, but it gives no idea of the direction in which the waves were traveling when the recording was made. Such a wave record is simply the sum total of all wave components arriving from all directions. If the direction of approach of the various components must be known, more sensors need to be placed and the record of each sensor needs **to be related to** the others.

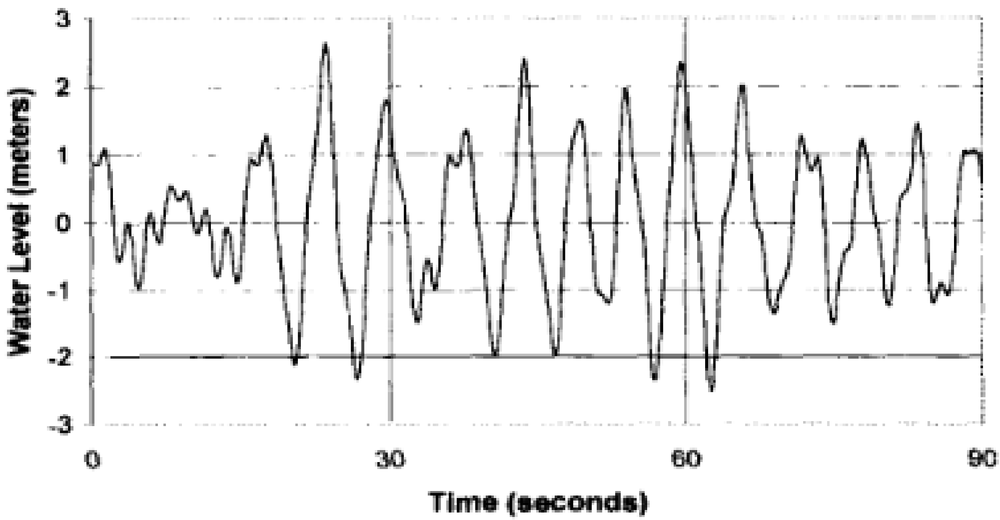


Figure 2.3 Record of Locally Generated Sea

- It is easy to visualize that for the array of sensors in Fig. 2.15, long crested waves traveling ***from the North, (Direction N) would give identical records on all sensors.***
- The records of C and D would be exactly in phase while the records of A and B would lead C and D in time. Conversely, long crested waves traveling from the west, (Direction W) would again give identical records on all four sensors. Those on A and B would be exactly in phase while C would lead and D would lag behind

A and B. Thus from inspection of the records and comparison of phase differences between them, the direction of wave travel may be obtained.

- Even if the direction of travel were such that none of the records occur at the same time, a little arithmetic can determine wave direction. The intuitive method of comparison of separate wave recordings, as described above, may be generalized using cross-correlation techniques to produce estimates of wave direction.

- Since waves are normally not long-crested, the wave recordings should be taken as closely together as possible to ensure that all measurements are of the same short-crested wave. One common technique actually uses one pressure sensor and two orthogonally mounted current meters, placed at the same location and close enough to the surface to obtain strong signals.

- Using equations such as in Table 2.2, q can be computed from p, **u and v,** where v is the velocity measured in the y direction. Cross-correlation of the three signals then makes it possible to calculate wave direction from these three simultaneous measurements.

- Remote sensing with radar or air and satellite photography can also be used to determine wave direction. When wave direction is not measured, it is generally inferred **from the wind direction, as described in Ch.** ***5, in combination with*** refraction, diffraction and reflection patterns of the waves, described in Ch. **7. This** is a rather inaccurate business, but often it is the only source of directional information.

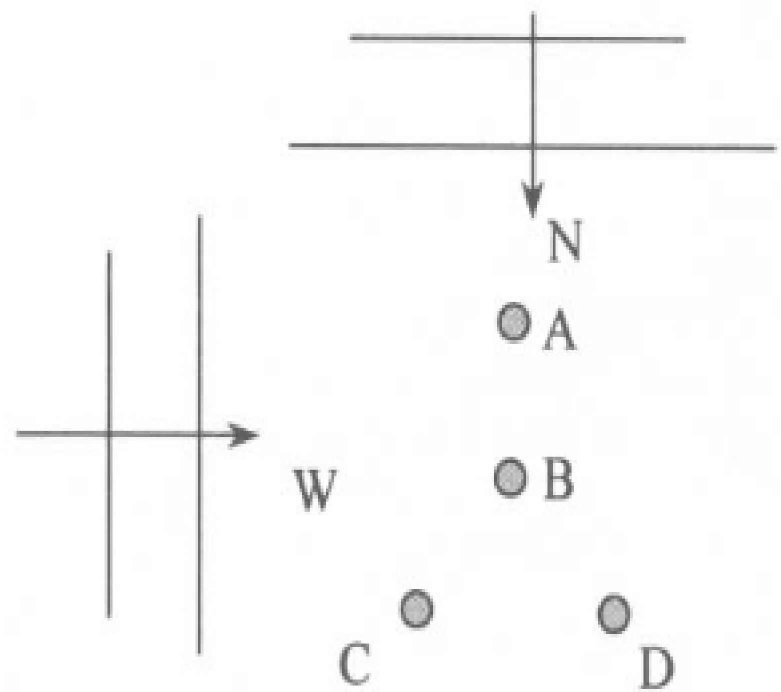


Figure 2. 15 Wave Direction and Array of Four Wave Gauges