FLUID PRESSURE AND ITS MEASUREMENT

FLUID PRESSURE

Definition of 'Pressure' or 'intensity of pressure'

It is defined as the force exerted on a unit area.

Let F be the total force that is uniformly distributed over an area A. Then, the pressure at any point on the area is given by $p = (F/A)$. But, if the total force F acting on the area A is not uniformly distributed, the expression $p = (F/A)$ gives only the average value of pressure on the area A. or in other words, when the force F is not uniformly distributed, the pressure varies from point to point on the area A. in such a case, the magnitude of pressure at any point on the area A can be expressed (obtained) by the following expression

$$p = \frac{dF}{dA} \qquad \text{......(1)}$$

where dF = force acting on an infinitesimal area dA

SI units of pressure

Pressure is expressed in N/m^2 (or Pa) in SI units.

It should be noted that 1 N/m^2 = 1 Pa

We know that a fluid is a substance that is capable of flowing. When a certain mass of fluid is held at rest (static equilibrium) by keeping it in a vessel or a container, then the fluid exerts forces against the solid boundary surfaces. What is the direction in which the fluid at rest exerts force on the surface of the vessel in which it comes in contact? The force is exerted by the fluid always in a direction normal (perpendicular) to the surface with which it comes in contact. *Why?* This is because a fluid at rest (static equilibrium) cannot sustain (carry on) shear stress which acts parallel to the surface of contact. Hence, the forces do not have any tangential components. The fluid pressure can now be defined as the force exerted by a fluid per unit area of the surface with which it comes in contact in a direction normal to the surface. It should be noted that even if an imaginary surface is considered within a fluid body, the fluid pressure and pressure force (force due to the fluid pressure) on the imaginary surface are the same as those acting on any real surface.

VARIATION OF PRESSURE IN A FLUID

Consider a small fluid element of size $\delta x \times \delta y \times \delta z$ at any point in a static (standing) mass of fluid (fluid at rest) as shown in Figure 1 below.

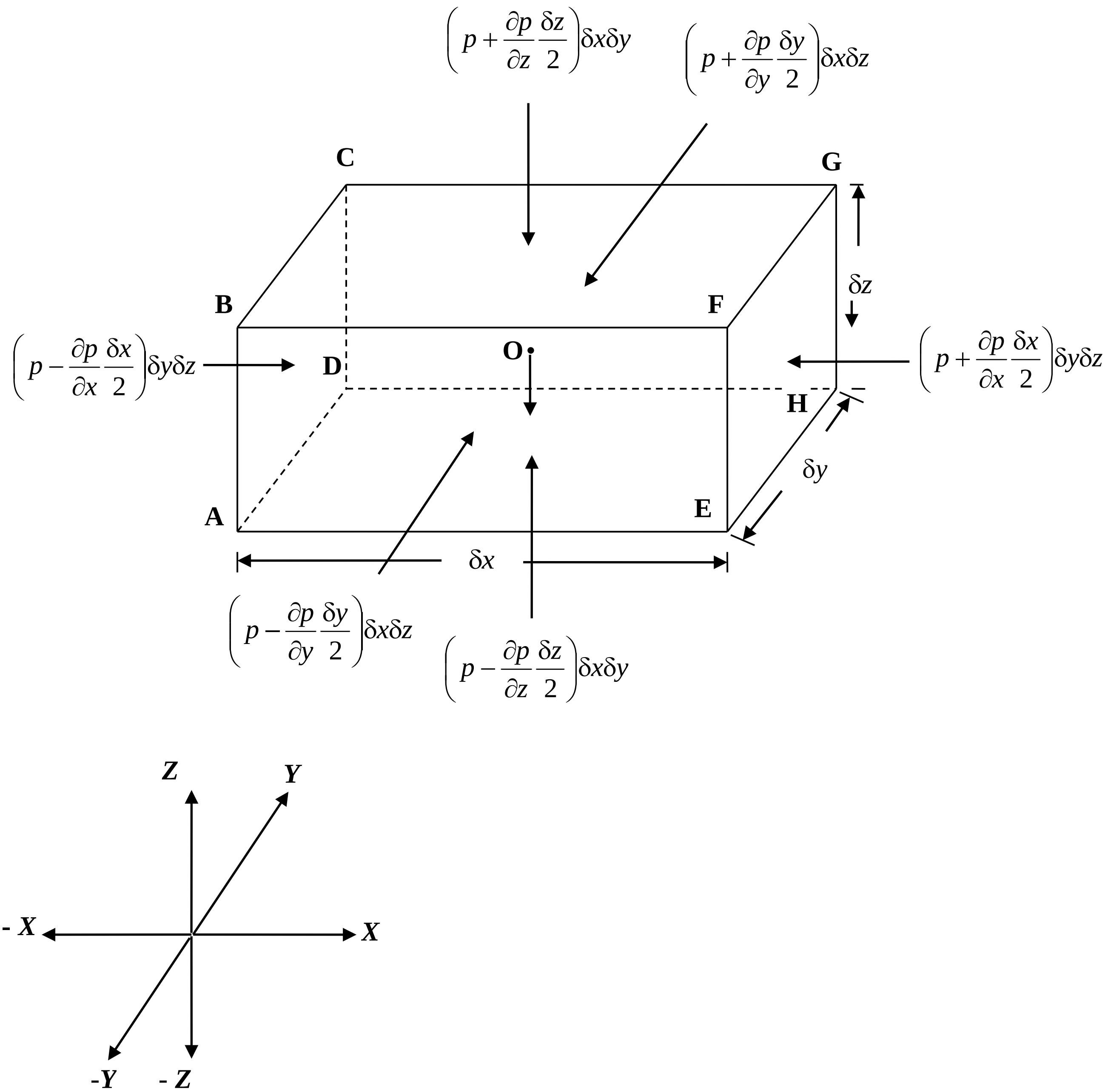

Figure 1 - Fluid element with forces acting on it in a static mass of fluid

As the fluid is at rest, the fluid element should also be at rest (in static equilibrium) under the various forces acting on it.

Forces that act on the fluid element under consideration are:

(i) Pressure forces on the six faces of the fluid element
(ii) Self-weight of the fluid element

Let *O* be the mid-point (centre) of the fluid element. Let the fluid pressure intensity at point *O* be *p*. Let us assume that the pressure intensity varies from point to point in the fluid element, that is, the pressure intensity varies in all the three mutually perpendicular reference directions, *x*, *y* and *z*. Let the mid-point *O* be considered as the origin through which the three mutually perpendicular reference axes pass through.

Let us evolve the expression for the pressure intensity on the left hand face *ABCD* of the fluid element. The face *ABCD* is normal to the *X* – direction. The pressure intensity over the face *ABCD* is along the *X* – direction. Hence, for computing the magnitude of pressure intensity on face *ABCD*, we have to consider the variation of pressure intensity in *X* – direction. As per the sign convention, to the left of *O*, the pressure intensity is considered to decrease, and to the right of *O*, the pressure intensity is considered to increase. Let the rate of variation of pressure intensity in *X* – direction be represented as $\left(\frac{\partial p}{\partial x}\right)$. The length of the fluid element in to the left of mid-point *O* is $\left(\frac{\delta x}{2}\right)$. Therefore, the total variation in pressure intensity over a length $\left(\frac{\delta x}{2}\right)$ to the left of point *O* is the product of rate of variation in pressure intensity in *X* – direction, $\left(\frac{\partial p}{\partial x}\right)$ and the length of the fluid element $\left(\frac{\delta x}{2}\right)$. Now, the pressure intensity on face ABCD in *X* – direction is given by $\left[p-\left(\frac{\partial p}{\partial x}\right)\left(\frac{\delta x}{2}\right)\right]$. Similarly, the pressure intensity on the right hand face *EFGH* of the fluid element can be computed. Like face *ABCD*, face *EFGH* is perpendicular the *X* – direction. Face *EFGH* is located to the right of the mid-point *O* of the fluid element. Hence, pressure intensity is considered to increase to the right of *O* in *X* – direction. Therefore, the pressure intensity on face *EFGH* in negative *X* – direction is given by $\left[p+\left(\frac{\partial p}{\partial x}\right)\left(\frac{\delta x}{2}\right)\right]$.

The corresponding pressure force on the left hand face *ABCD* of the fluid element is the product of pressure intensity on the face *ABCD* and the area of the face *ABCD*. That is,

Pressure force on left hand face *ABCD* = (pressure intensity on left hand face *ABCD*) x (area of face *ABCD*)

$$= \left[p - \left(\frac{\partial p}{\partial x}\right)\left(\frac{\delta x}{2}\right)\right](\delta y.\delta z)$$

Similarly, pressure force on the right hand face *EFGH* = $\left[p + \left(\frac{\partial p}{\partial x}\right)\left(\frac{\delta x}{2}\right)\right](\delta y.\delta z)$

Pressure intensity on the front face ABFE and Pressure intensity on the rear face DCGH of the fluid element

Face *ABFE* is normal to *Y* – axis. Face *ABFE* is in the negative *Y* – direction. The pressure intensity is considered to decrease in the negative *Y* – direction. Let the rate of variation of pressure intensity in *Y* – direction be $\left(\frac{\partial p}{\partial y}\right)$. The length of the fluid element in the negative *Y*- direction from the mid-point *O* is $\left(\frac{\delta y}{2}\right)$. The face *ABFE* is located at a distance $\left(\frac{\delta y}{2}\right)$ from point *O*. Now, the pressure intensity on face *ABFE* normal to it in *Y* – direction is given by $\left[\left(p - \frac{\partial p}{\partial y}\frac{\delta x}{2}\right)\right]$. Similarly, the pressure intensity on the rear face *DCGH* is given by $\left[\left(p + \frac{\partial p}{\partial y}\frac{\delta x}{2}\right)\right]$. It should be noted that the face *DCGH* is normal to *Y* – axis and in the *Y* – direction; it is considered that the pressure intensity increases in *Y* – direction.

The corresponding pressure force on the front face *ABFE* of the fluid element is the product of pressure intensity on the face *ABFE* and the area of the face *ABFE*. That is,

Pressure force on left hand face *ABFE* = (pressure intensity on left hand face *ABFE*) x (area of face *ABFE*)

$$= \left[\left(p - \frac{\partial p}{\partial y}\frac{\delta x}{2}\right)\right](\delta x.\delta z)$$

Similarly, pressure force on the rear face *DCGH* = $\left[\left(p + \frac{\partial p}{\partial y}\frac{\delta x}{2}\right)\right](\delta x.\delta z)$

Pressure intensity on the top face BCGF and Pressure intensity on the bottom face ADHE of the fluid element

Top Face *BCGF* of the fluid element is normal to the *Z* – axis (Vertical axis). Face *BCGF* is above the mid-point *O* of the fluid element. As per the sign convention, pressure intensity is considered to increase in *Z* – direction (vertical upward direction) and decrease in negative *Z* – direction (vertical downward direction). Let the rate of variation of pressure intensity in *Z* – direction be represented by $\left(\frac{\partial p}{\partial z}\right)$. The length of the fluid element in *Z* – direction from *O* is $\left(\frac{\delta z}{2}\right)$. Therefore, the total variation in pressure intensity over a length $\left(\frac{\delta z}{2}\right)$ above point *O* is the product of rate of variation in pressure intensity in *Z* – direction, $\left(\frac{\partial p}{\partial z}\right)$ and the length of the fluid element $\left(\frac{\delta z}{2}\right)$. Now, the pressure intensity on top face *BCGF* (acting normal to face *BCGF*) in the vertical downward direction (negative *Z* – direction) is given by $\left[p+\left(\frac{\partial p}{\partial z}\right)\left(\frac{\delta z}{2}\right)\right]$. Similarly, the pressure intensity on bottom face *ADHE* (acting normal to face *ADHE*) in the vertical upward direction (positive *Z* – direction) is given by $\left[p-\left(\frac{\partial p}{\partial z}\right)\left(\frac{\delta z}{2}\right)\right]$, since the pressure intensity is considered to decrease in the negative *Z* –direction, from point *O* towards the bottom face *ADHE* normal to *Z* - axis.

The corresponding pressure force on the top face *BCGF* of the fluid element is the product of pressure intensity on the face *BCGF* and the area of the face *BCGF*. That is,

Pressure force on top face *BCGF* = (pressure intensity on left hand face *BCGF*) x (area of face *BCGF*)

$$= \left[p+\left(\frac{\partial p}{\partial z}\right)\left(\frac{\delta z}{2}\right)\right](\delta x.\delta y)$$

Similarly, pressure force on the bottom face *ADHE* = $\left[p-\left(\frac{\partial p}{\partial z}\right)\left(\frac{\delta z}{2}\right)\right](\delta x.\delta y)$

Self – weight of the fluid element

This is evaluated as the product of the specific weight of the fluid and the volume of the fluid element. Let the specific weight of the fluid element be γ. The volume of the fluid element is $\delta x.\delta y.\delta z$

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Hence, self-weight of the fluid element = $\gamma\ (\delta x \delta y \delta z)$

As the fluid is at rest, the fluid element is in static equilibrium under the influence of the above forces. That is, the algebraic sum of forces acting on the fluid element in any direction must be zero. Thus, considering the forces acting on the fluid element along *X*, *Y* and *Z* – directions, we have,

Algebraic sum of forces along *X* – direction = 0, that is, $\sum F_x = \mathbf{0}$

Forces along *X* – axis are the pressure forces acting normal to the left face *ABCD* and the right face *EFGH*. Pressure force acting normal to face *ABCD* is along *X* – direction and pressure force acting normal to face *EFGH* is along negative *X* – direction. Therefore,

$$\left[p - \left(\frac{\partial p}{\partial x}\right)\left(\frac{\delta x}{2}\right)\right](\delta y.\delta z)\ \text{-}\ \left[p + \left(\frac{\partial p}{\partial x}\right)\left(\frac{\delta x}{2}\right)\right](\delta y.\delta z) = 0$$

$$\Rightarrow -\ 2\left(\frac{\partial p}{\partial x}\right)\left(\frac{\delta x}{2}\right)(\delta y.\delta z) = 0$$

$$\Rightarrow -2\left(\frac{\delta x}{2}\right)(\delta y.\delta z)\left(\frac{\partial p}{\partial x}\right) = 0$$

$$\Rightarrow -\ (\delta x \delta y \delta z)\left(\frac{\partial p}{\partial x}\right) = 0$$

$$\Rightarrow -\left(\frac{\partial p}{\partial x}\right) = 0$$ (since, $(\delta x \delta y \delta z)$ represents the volume of the fluid element)

$$\Rightarrow \left(\frac{\partial p}{\partial x}\right) = 0 \qquad \ldots\ldots (2)$$

This shows that the pressure intensity does not vary in *X* – direction.

Algebraic sum of forces along *Y* – direction = 0, that is, $\sum F_y = \mathbf{0}$

Forces along *Y* – axis are the pressure forces acting normal to the front face *ABFE* and the rear face *DCGH*. Pressure force acting normal to face *ABFE* is along *Y* – direction and pressure force acting normal to face *DCGH* is along negative *Y* – direction. Therefore,

$$\left[p - \left(\frac{\partial p}{\partial y}\right)\left(\frac{\delta y}{2}\right)\right](\delta x.\delta z)\ \text{-}\ \left[p + \left(\frac{\partial p}{\partial y}\right)\left(\frac{\delta y}{2}\right)\right](\delta x.\delta z) = 0$$

$$\Rightarrow -\ 2\left(\frac{\partial p}{\partial y}\right)\left(\frac{\delta y}{2}\right)(\delta x.\delta z) = 0$$

$$\Rightarrow -2\left(\frac{\delta y}{2}\right)(\delta x.\delta z)\left(\frac{\partial p}{\partial y}\right) = 0$$

$$\Rightarrow -\ (\delta x \delta y \delta z)\left(\frac{\partial p}{\partial y}\right) = 0$$

$$\Rightarrow -\left(\frac{\partial p}{\partial y}\right) = 0 \qquad \text{(since, } (\delta x \delta y \delta z) \text{ represents the volume of the fluid element)}$$

$$\Rightarrow \left(\frac{\partial p}{\partial y}\right) = 0 \qquad \ldots\ldots (2)$$

This shows that the pressure intensity does not vary in *Y* – direction.

Algebraic sum of forces along *Z* – direction = 0, that is, $\sum F_Z = 0$

Forces along *Z* – axis are the pressure forces acting normal to the top face *BCGF* and the bottom face *ADHE* and the self-weight of the fluid element acting vertically downward at the mid-point *O*. Pressure force acting normal to top face *BCGF* is vertically downward (negative *Z* – direction) and pressure force acting normal to bottom face *ADHE* is vertically upward (*Z* – direction). The self-weight of fluid element passing through mid-point *O* is acting vertically downward (negative *Z* – direction). Therefore,

$$\left[p - \left(\frac{\partial p}{\partial z}\right)\left(\frac{\delta z}{2}\right)\right](\delta x . \delta y) - \left[p + \left(\frac{\partial p}{\partial z}\right)\left(\frac{\delta z}{2}\right)\right](\delta x . \delta y) - \gamma\,(\delta x \delta y \delta z) = 0$$

$$\Rightarrow -\ 2\left(\frac{\partial p}{\partial z}\right)\left(\frac{\delta z}{2}\right)(\delta x . \delta y) - \gamma\,(\delta x \delta y \delta z) = 0$$

$$\Rightarrow -2\left(\frac{\delta z}{2}\right)(\delta x . \delta y)\left(\frac{\partial p}{\partial z}\right) - \gamma\,(\delta x \delta y \delta z) = 0$$

$$\Rightarrow -\ (\delta x \delta y \delta z)\left(\frac{\partial p}{\partial z}\right) - \gamma\,(\delta x \delta y \delta z) = 0$$

$$\Rightarrow\ -\left(\frac{\partial p}{\partial z}\right) = \frac{\gamma(\delta x \delta y \delta z)}{(\delta x \delta y \delta z)}$$

$$\Rightarrow \left(\frac{\partial p}{\partial z}\right) = -\ \gamma \qquad \ldots\ldots (4)$$

From the previous discussion, it can be concluded that the pressure intensity *p* at any point in a static mass of fluid does not vary in *X* and *Y* directions and it varies only in *Z* – direction (vertical direction).

Hence, the expression $\left(\frac{\partial p}{\partial z}\right) = -\ \gamma$ can be written as

$$\left(\frac{dp}{dz}\right) = -\ \gamma \qquad \ldots\ldots (5)$$

That is, the partial derivative $\left(\frac{\partial p}{\partial z}\right)$ is replaced by the total derivative $\left(\frac{dp}{dz}\right)$.

The minus sign in equation (5) indicates that the pressure intensity p decreases in Z – direction (vertical upward direction).

Equation (5) is the basic differential equation representing variation of pressure intensity (or simply 'pressure') in a fluid at rest. This equation holds good for both compressible and incompressible fluids.

The pressure intensity decreases at the rate of specific weight of fluid with increase in elevation z. Or in other words, the pressure intensity increases at the rate of specific weight of fluid in the vertical downward direction.

In equation (5), if $dz = 0$, then $dp = 0$. *What do we mean by this?* This shows that the pressure intensity remains constant over any horizontal plane in a fluid.

What will we get when we integrate equation (5)?

$$\int \frac{dp}{dz} = -\int \gamma$$
$$\Rightarrow \int dp = -\int \gamma dz$$
$$\Rightarrow p = -\gamma \int dz$$
$$\Rightarrow p = -\gamma z + C \qquad \ldots\ldots (6)$$

where, p is the pressure at any point at an elevation z in the static mass of fluid. C is a constant of integration. So, integration of equation (5) has resulted in the pressure at any point in a fluid at rest. Let us now evaluate the value of C. Let us consider a liquid of specific weight γ contained in an open vessel as shown in figure below. The free surface of liquid is exposed to atmosphere. Let the atmospheric pressure acting normal to the free surface of liquid be denoted as p_a. Let the height of free surface of liquid in the vessel measured from the bottom of vessel be H. Let the bottom of vessel is located at a height z_o from the datum level. Hence, the height of free surface of liquid from the datum level be z. Now, $z = (z_o + H)$. using equation (6), the atmospheric pressure at the free surface of liquid can be written as

$$p_a = -\gamma (z_o + H) + C$$
$$\Rightarrow C = p_a + \gamma (z_o + H) \qquad \ldots\ldots (7)$$

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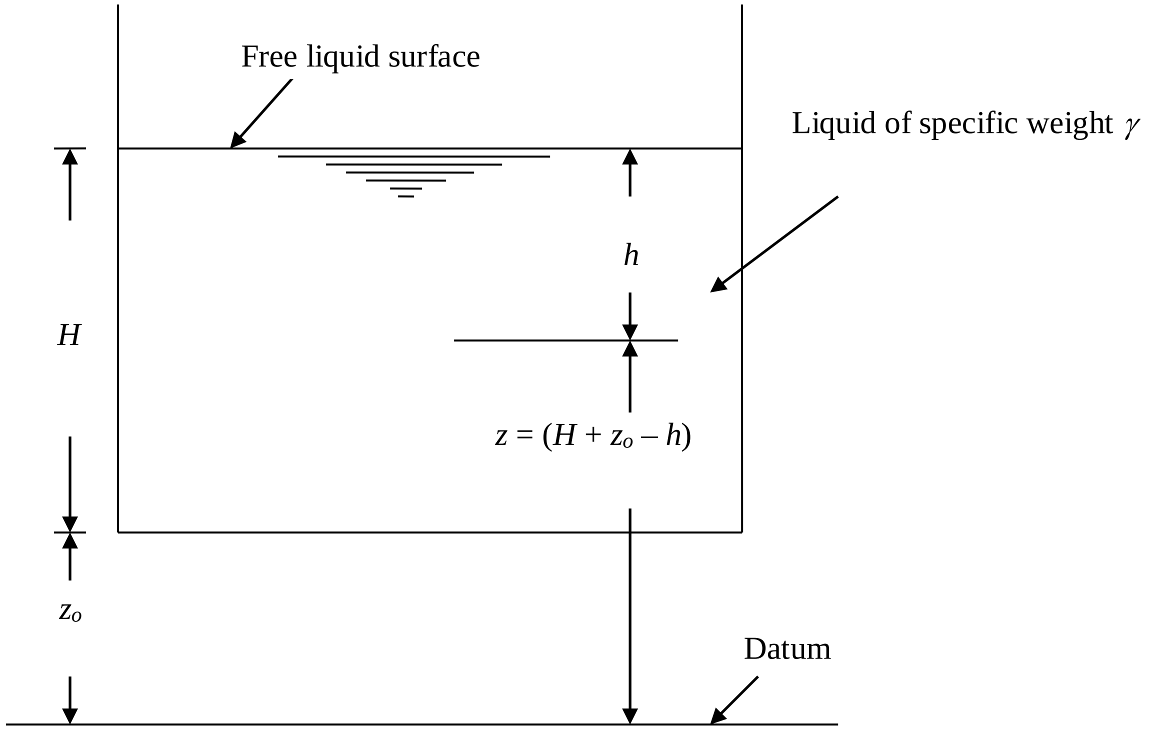

Figure 2 - Pressure at a point in a liquid

Putting the value of C from equation (7) in (6), we have,

$$p = -\gamma z + [p_a + \gamma(z_o + H)] \qquad \ldots\ldots (8)$$

if we consider a point in the fluid mass located at depth h from the free liquid surface in the vessel, we have, the height of the point from the datum level, $z = (H + z_o - h)$. Substituting this value of z in equation (8), we have,

$$p = -\gamma(H + z_o - h) + [p_a + \gamma(z_o + H)]$$
$$\Rightarrow p = -\gamma H - \gamma z_o + \gamma h + p_a + \gamma z_o + \gamma H$$
$$\Rightarrow p = p_a + \gamma h \qquad \ldots\ldots (9)$$

Equation (9) shows that the pressure at any point in a static mass of liquid depends only upon the vertical depth, h, of the point below the free surface and the specific weight of the liquid. It does not depend upon the shape and size of the container in which it is kept. It does not depend upon the weight of the fluid present in the vessel. This is called ***hydrostatic paradox***.

Figure below shows four vessels, each having the same base area A and each containing liquid of the same specific weight γ to the same height h.

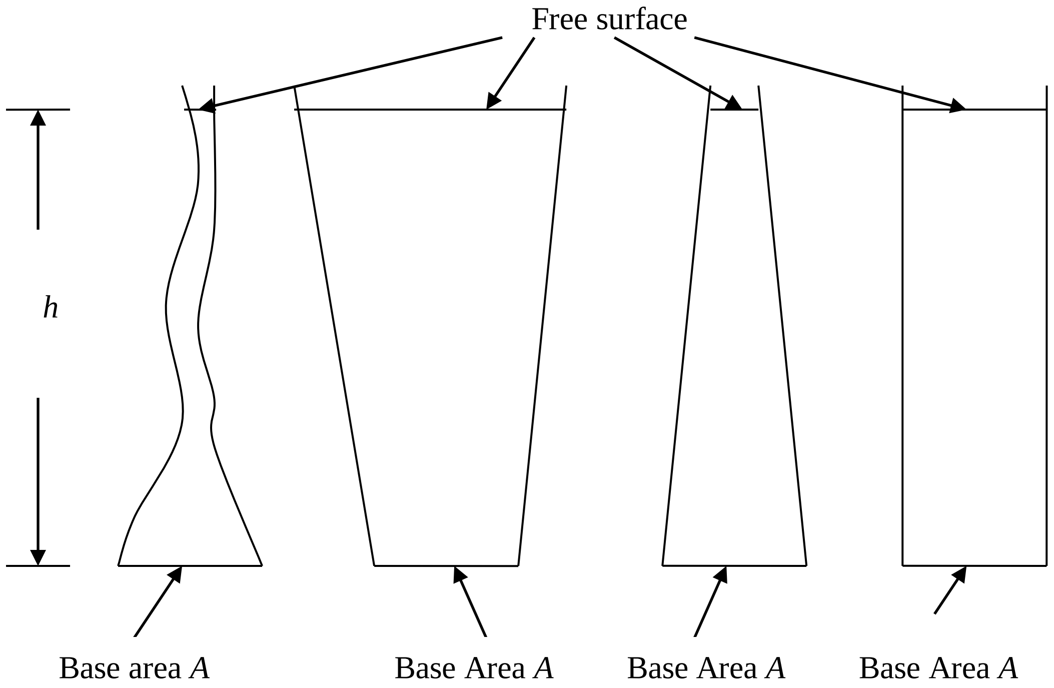

Figure 3 – Hydrostatic Paradox

When the liquid contained in a vessel is exposed to atmosphere, that is, the liquid has a distinct free surface, then, the pressure at any point in the liquid is expressed in gauge pressure, i.e., pressure in excess of atmospheric pressure (atmospheric pressure, p_a, is ignored). That is, equation (9) can be expressed as

$$p = \gamma h \qquad \ldots\ldots (9a)$$

In each of the four vessels shown in figure above, the pressure at the base of the vessel is given by equation (9a). The pressure force acting on the base area of each vessel is given by

Pressure force on bottom of vessel = (pressure at base of vessel) x (area of base)

$$= pA = (\gamma h)A \qquad \ldots\ldots (9b)$$

As the shapes and sizes of the four vessels are different from one another, even as each vessel has the same base area A and same liquid of specific weight γ to the same height h, the weight of the liquid is different in the four vessels. But, the pressure force exerted on the base area of each vessel is the same given by equation (9b).

PRESSURE HEAD

Referring to Figure 2, let us consider the point in the liquid mass in the vessel at a vertical depth h from the free surface of liquid. The vertical height, h, of the free surface of liquid above the point under consideration in the liquid at rest is known as the pressure head. From equation (9a), the pressure head may be expressed as

$$h = \frac{p}{\gamma} \qquad \ldots\ldots (10)$$

The pressure at any point in a liquid depends upon the height of the free surface of liquid above the point. Hence, it is convenient to express fluid pressure at a point in terms of pressure head.

Pressure head is expressed in terms of meters of liquid column.

Let it be required to exert a pressure of $p\ N/m^2$ at a point. Let us consider two different liquids of specific weights γ_1 and γ_2. To exert a pressure of $p\ N/m^2$ at the point, let the pressure head (height of column of liquid) required by the two liquids be h_1 and h_2. Therefore, using equation (9a), we have,

$$p = \gamma_1 h_1 = \gamma_2\, h_2 \qquad \ldots\ldots (11)$$

Let S_1 and S_2 be the specific gravities of the two liquids. Then, we have, $\gamma_1 = S_1\gamma_w$ and $\gamma_2 = S_2\gamma_w$. Therefore, from equation (11), we have,

$$p = (S_1\gamma_w)h_1 = (S_2\gamma_w)h_2$$
$$\Rightarrow S_1 h_1 = S_2 h_2 \qquad \ldots\ldots (12)$$

PASCAL'S LAW

The pressure at any point in a fluid at rest has the same magnitude in all directions. In other words, when a certain pressure is applied at any point in a fluid at rest, the pressure is equally transmitted in all the directions and to every other point in the fluid.

Proof of Pascal's Law

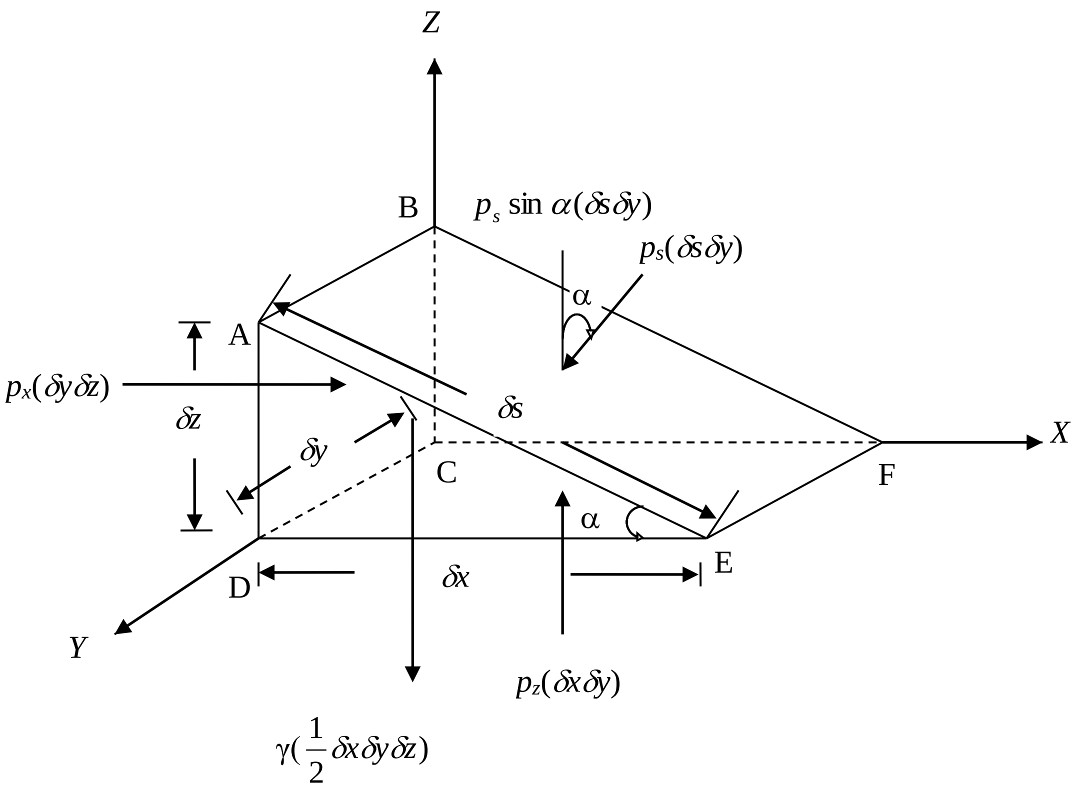

Figure 4 – Free body diagram of a wedge-shaped fluid element

Figure 4 shows an infinitesimal wedge shaped fluid element at rest. The element is arbitrarily chosen and has dimensions as shown in Figure. As the fluid element is at rest, it cannot sustain any shear forces. The only forces acting on the fluid element are the normal pressure forces exerted by the surrounding fluid on the plane surfaces of the fluid element, and the weight of the fluid element. Further, as the fluid element is at rest (in static equilibrium), the sum of force components on the fluid element in any direction must be equal to zero.

The algebraic sum of force components in X – direction is equal to zero. i.e.,

$$\sum F_x = 0$$

$$\Rightarrow \; p_x(\delta y\delta z) - p_s \sin\alpha(\delta s\delta y) = 0 \qquad \ldots\ldots(13)$$

where, p_x = average pressure intensity acting in X – direction, normal to the face *ABCD*

$\delta y\delta z$ = area of face *ABCD*

$p_x(\delta y\delta z)$ = pressure force acting normal to the face *ABCD* of the fluid element

p_s = average pressure intensity acting normal to the inclined face *ABFE*

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$p_s \sin\alpha$ = component of pressure intensity p_s in negative X – direction acting on face $ABFE$

$\delta s \delta y$ = area of face $ABFE$

$p_s \sin\alpha(\delta s \delta y)$ = Pressure force component in the negative X - direction (force acting right to left) acting on the face $ABFE$

Note: the sign convention adopted in writing equation (13) is that force acting rightward (left to right) in X – direction is taken as positive force and force acting leftward (right to left) in negative X –direction is taken as negative force.

From right-angled triangular face ADE of the fluid element, we have,

$$\sin\alpha = \frac{AD}{AE} = \frac{\delta z}{\delta s}$$

$$\Rightarrow \delta s = \frac{\delta z}{\sin\alpha}$$

Substituting $\delta s = \dfrac{\delta z}{\sin\alpha}$ in equation (13), we have,

$$p_x(\delta y \delta z) - p_s \sin\alpha\left(\frac{\delta z}{\sin\alpha}\delta y\right)$$

$$\Rightarrow p_x(\delta y \delta z) - p_s(\delta z \delta y)$$

$$\Rightarrow p_x(\delta y \delta z) = p_s(\delta z \delta y)$$

$$\Rightarrow p_x = p_s \qquad \ldots\ldots(14)$$

Similarly, the algebraic sum of force components in Z – direction is equal to zero. i.e.,

$$\sum F_z = 0$$

$$\Rightarrow p_z(\delta x \delta y) - p_s \cos\alpha(\delta s \delta y) - \gamma\left(\frac{1}{2}\delta x \delta y \delta z\right) = 0 \qquad \ldots\ldots(15)$$

where p_z = average pressure intensity acting in Z – direction, normal to the bottom face $CDEF$

$\delta x \delta y$ = area of bottom face $CDEF$

$p_z(\delta x \delta y)$ = pressure force acting normal to the face $CDEF$ of the fluid element

p_s = average pressure intensity acting normal to the inclined face $ABFE$

$p_s \cos\alpha$ = component of pressure intensity p_s in vertical downward direction (negative Z – direction) acting on face $ABFE$

$\delta s \delta y$ = area of face $ABFE$

$p_s \cos\alpha(\delta s \delta y)$ = Pressure force component in the negative Z - direction (force acting vertically downward) acting on the face $ABFE$

γ = specific weight of liquid

$\left(\frac{1}{2}\delta x\delta y\delta z\right)$ = volume of fluid element

= (area of the triangular face BCF of the fluid element) x (length of fluid element in Y – direction)

$= \left(\frac{1}{2} xCFxCB\right)xCD$

$= \left(\frac{1}{2}\delta x\delta z\right)\delta y$

$\gamma\left(\frac{1}{2}\delta x\delta y\delta z\right)$ = self-weight of the fluid element

Note: the sign convention adopted in writing equation (15) is that force acting vertically upward in Z – direction is taken as positive force and force acting vertically downward in negative Z –direction is taken as negative force.

From right-angled triangular face ADE of the fluid element, we have,

$$\cos\alpha = \frac{DE}{AE} = \frac{\delta x}{\delta s}$$

$$\Rightarrow \delta s = \frac{\delta x}{\cos\alpha}$$

Substituting $\delta s = \frac{\delta x}{\cos\alpha}$ in equation (14), we have,

$$p_z(\delta x\delta y) - p_s \cos\alpha\left(\frac{\delta x}{\cos\alpha}\delta y\right) - \gamma\left(\frac{1}{2}\delta x\delta y\delta z\right) = 0$$

$$\Rightarrow p_z(\delta x\delta y) - p_s(\delta x\delta y) - \gamma\left(\frac{1}{2}\delta x\delta y\delta z\right) = 0$$

The third term in the above equation is much smaller than the first two terms, since it involves the product of three infinitesimal quantities namely, $\delta x, \delta y, \delta z$. Hence, the third term $\gamma\left(\frac{1}{2}\delta x\delta y\delta z\right)$ can be neglected. So, the above equation reduces to

$$p_z(\delta x\delta y) - p_s(\delta x\delta y) = 0$$

$$\Rightarrow p_z(\delta x\delta y) = p_s(\delta x\delta y)$$

$$\Rightarrow p_z = p_s \qquad \text{......(16)}$$

From equations (14) and (16), we have,

$$p_x = p_z = p_s \qquad \text{......(17)}$$

Equation (17) proves that the pressure is the same in all directions.

RELATIONSHIP BETWEEN ATMOSPHERIC, ABSOLUTE AND GAGE AND VACUUM PRESSURES

Atmospheric pressure

It is the normal pressure exerted by atmospheric air on surfaces with which it comes in contact. The magnitude of atmospheric pressure varies with altitude. It can be measured using a barometer. Hence, atmospheric pressure is also called barometric pressure. At sea level, under normal conditions, the equivalent values of the atmospheric pressure are 10.1043×10^4 N/m^2 (or) 1.03 kg(f)/cm^2 (or) 10.3 m of water (or) 760 mm of mercury.

Generally, fluid pressure at a point can be measured with respect to two most common datum. They are:

(i) absolute zero pressure
(ii) local atmospheric pressure

Absolute zero pressure

It is the minimum pressure that can prevail at a point under natural or forced conditions. It is the zero pressure prevailing under complete vacuum condition.

When pressure at a point is measured with respect to absolute zero (or complete vacuum), it is called an ***absolute pressure***.

When pressure at a point is measured with respect to the local atmospheric pressure, it is called ***gage pressure***. It is called gage pressure because practically all pressure gages read zero when exposed to atmosphere. The pressure gages are calibrated such that they read only the difference between the pressure of fluid to which they are connected and the atmospheric pressure.

Gage negative pressure

If the pressure of a fluid is below atmospheric pressure, it is designated as ***vacuum pressure*** or ***suction pressure*** or ***negative gage pressure***. The value of the vacuum pressure is the amount by which it is below atmospheric pressure. A pressure gage intended to measure vacuum pressure is called a vacuum gage.

All values of absolute pressure are positive, since in case of fluids, the lowest absolute pressure that can possibly exist corresponds to absolute zero or complete vacuum.

Gage pressures are positive if they are above atmospheric pressure and they are negative if they if they are below atmospheric pressure. Figure 5 shows the relationship between absolute, atmospheric, gage and gage negative pressures (vacuum pressure).

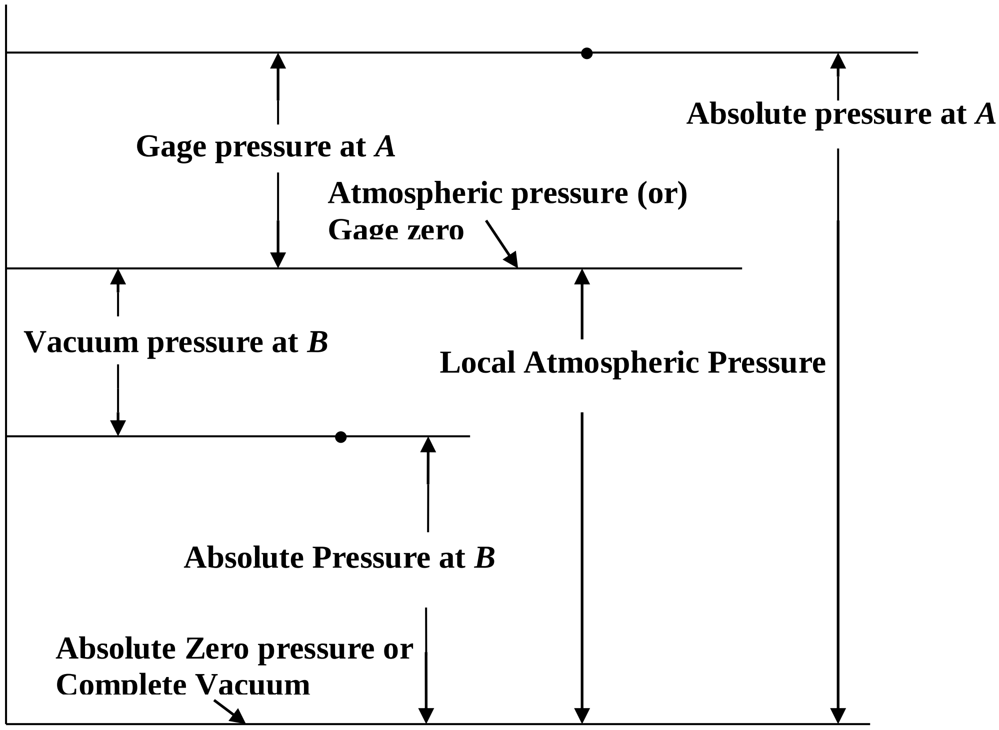


Figure 5 – Relationship between absolute, gage and vacuum pressures

Pressures at points A and B shown in above figure are respectively above and below atmospheric. That is, the pressure at A is gage positive and the pressure at B is gage negative (or vacuum pressure or suction pressure).

Absolute pressure at A = Atmospheric pressure + Gage pressure at A
Absolute pressure at B = Atmospheric pressure – Vacuum pressure at B

In general, the relationship between the various pressures can be stated as:

Absolute pressure = Atmospheric pressure ± Gage pressure

Note: The gage pressure may be positive or negative

Example 1. Express a pressure intensity of 5 $kg(f)/cm^2$ in all possible units. Take the barometric reading as 76 *cm* of mercury.

Solution.

Data given:
Pressure intensity (in excess of atmospheric pressure) = 5 $kg(f)/cm^2$
Barometer reading gives the atmospheric pressure in equivalent column of mercury
Atmospheric pressure measured in equivalent column of mercury = 76 *cm*

Required:
To express the given pressure intensity in all possible units.

(a) *In gage units:*

(i) pressure intensity (in excess of atmospheric pressure), $p = 5\ kg(f)/cm^2$

$$p = (5 \text{ x } 9.81\ N) / (1/100)\ m^2 = 490500\ N/m^2$$
$$= 490.5\ kN/m^2$$

(ii) pressure intensity in equivalent column of water

$$h_w = \frac{p}{\gamma_w}$$

where h_w = pressure head (in excess of atmospheric pressure) in meters of water

γ_w = specific weight of water = 9810 N/m^3

$$\therefore\ h_w = \frac{490500\ N/m^2}{9810\ N/m^3} = 50\ m \text{ of water}$$

(iii) pressure intensity in equivalent column of mercury

$$h_m = \frac{p}{\gamma_m}$$

where h_m = pressure head (in excess of atmospheric pressure) in meters of mercury

γ_m = specific weight of mercury = (specific gravity of mercury) x (specific weight of water)

$= 13.6 \text{ x } 9810\ N/m^3$

$= 133416\ N/m^3$

$$\therefore h_m = \frac{490500\ N/m^2}{133416\ N/m^3} = 3.6765\ m \text{ of mercury}$$

(b) *In absolute units*

(i) as intensity of pressure

absolute pressure = atmospheric pressure + gage pressure

Let us now determine the intensity of atmospheric pressure, p_{atm}

$p_{atm} = \gamma_m (h_{atm})_m$ = (133416 N/m^3) x (0.76 m of mercury)

= 101396.2 N/m^2

Gage pressure intensity = 490500 N/m^2

$\therefore$ absolute pressure = 101396.2 N/m^2 + 490500 N/m^2

= 591896.2 N/m^2

(ii) in equivalent column of water

absolute pressure = atmospheric pressure + gage pressure

Note: all the terms in the above expression are expressed in meters of water

Let us now determine the value of atmospheric pressure (in equivalent column of water)

$$p_{atm} = \gamma_m (h_{atm})_m = \gamma_w (h_{atm})_w$$

where, $(h_{atm})_m$ = atmospheric pressure head in meters of mercury

$(h_{atm})_w$ = atmospheric pressure head in meters of water

Hence, we have,

101396.2 N/m^2 = (133416 N/m^3) $(h_{atm})_m$ = (9810 N/m^3) $(h_{atm})_w$

$\Rightarrow (h_{atm})_w$ = 10.336 m of water

$\therefore$ absolute pressure = 10.336 m of water + 50 m of water

= 60.336 m of water

(iii) in equivalent column of mercury

absolute pressure = atmospheric pressure + gage pressure

Note: all the terms in the above expression are expressed in meters of mercury

atmospheric pressure (in equivalent column of mercury) = 0.76 m

gauge pressure (in equivalent column of mercury) = $\dfrac{p}{\gamma_m}$

$= \dfrac{490500\, N/m^2}{133416\, N/m^3}$

= 3.6765 m of mercury

$\therefore$ absolute pressure head in meters of mercury = 0.76 + 3.6765

= 4.4365 m of mercury

Example 2. Find the depth of a point below free surface in a tank containing oil where the pressure intensity is 9 $kg(f)/cm^2$. Specific gravity of oil is 0.9

Solution.

Data given:
Pressure intensity, $p = 9\ kg(f)/cm^2$
Specific gravity of oil, $S_{oil} = 0.9$

Required:
Depth of point below free surface of oil, h = ?

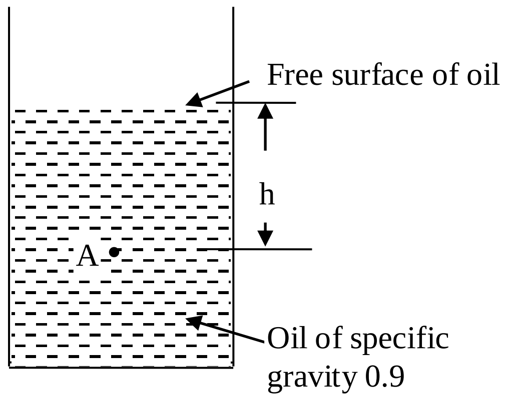


Figure 5

Let $p_A = 9\ kg(f)/cm^2$
$= 9 \times 9.81\ N / (1 / 100)\ m^2$
$= 882900\ N/m^2$

$$h = \frac{p_A}{\gamma_{oil}}$$

γ_{oil} =
(specific gravity of oil, S_{oil}) x
(specific weight of water, γ_w)
$= 0.9 \times 9810\ N/m^3$

Hence, $h = \dfrac{882900\ N/m^2}{8829\ N/m^3} = 100\ m$

Example 3. Convert a pressure head of 15 m of water to (a) meters of oil of specific gravity 0.75 and (b) meters of mercury of specific gravity 13.6

Solution.

Data given:
Pressure head (in meters of water), $h_w = 15\ m$ of water

Required:
To find (a) Pressure head, h_{oil}, in meters of oil of specific gravity 0.75
(b) Pressure head, h_m, in meters of mercury of specific gravity 13.6

(a) We have,

$$p = \gamma_w h_w = \gamma_{oil} h_{oil}$$

where, p = intensity of pressure in gage units in N/m^2
γ_w = specific weight of water = 9810 N/m^3

h_w = equivalent column of water in meters to exert the pressure intensity of p

γ_{oil} = specific weight of oil = (specific gravity of oil) x (specific weight of water)

= 0.75 x 9810 N/m^3 = 7357.5 N/m^3

h_{oil} = equivalent column of oil of specific gravity 0.75, in meters, to exert the pressure intensity of p

$$\therefore h_{oil} = \frac{\gamma_w h_w}{\gamma_{oil}} = \frac{(9810\,N/m^3)(15m)}{7357.5\,N/m^3} = \mathbf{20\ m\ of\ oil}$$

(b) We have,

$$p = \gamma_w h_w = \gamma_m h_m$$

where, p = intensity of pressure in N/m^2

γ_w = specific weight of water = 9810 N/m^3

h_w = equivalent column of water in meters to exert the pressure intensity of p

γ_m = specific weight of mercury = (specific gravity of mercury) x (specific weight of water)

= 13.6 x 9810 N/m^3 = 133416 N/m^3

h_m = equivalent column of mercury of specific gravity 13.6, in meters, to exert the pressure intensity of p

$$\therefore h_m = \frac{\gamma_w h_w}{\gamma_m} = \frac{(9810\,N/m^3)(15m)}{133416\,N/m^3} = \mathbf{1.1029\ m\ of\ mercury}$$

MEASUREMENT OF PRESSURE

The broad classification of devices used for measurement of fluid pressure is:

1. Manometers
2. Mechanical Pressure Gages

Manometers

Manometers are based on the principle of balancing the column of liquid whose pressure is to be determined by the same or another column of liquid.

Classification of Manometers

(a) Simple Manometers
(b) Differential Manometers

Simple manometers measure pressure at a point in a fluid contained in a pipe or a vessel. Differential manometers measure the pressure difference between any two points in a fluid contained in a pipe or a vessel.

(a) ***Simple Manometers***

Construction: It consists of a glass tube. One of the ends of the glass tube is connected to the gage point where the pressure is to be measured and the other end is kept open to the atmosphere.

Common types of simple manometers:

(i) Piezometer
(ii) U – tube manometer
(iii) Single column manometer

(i) *Piezometer*

One of the simplest ways of measuring pressure in a pipeline is to drill a hole in it and connect a tube of small diameter to the hole. The connection should be flush with the inside surface of the pipe. The diameter must be small to prevent any disturbance to flow in the pipe, and to ensure that the kinetic energy of the flow does not contribute to the level of liquid in the tube. The tube is called the piezometer. It is the simplest form of manometer. It can be used for measuring moderate pressures of liquids. It consists of a glass tube, one end of which is inserted in the wall of the pipe or the vessel containing the liquid whose pressure is to be measured. The other end of the tube extends vertically upward to a height such that the liquid can freely rise in it without overflowing. The pressure at any point in the liquid is indicated by the height of liquid in the tube above that point. This is illustrated in Figure 6 below.

Assuming that the pipe is flowing full and that the liquid is under pressure, the liquid will be forced up the piezometer tube to some height measured from the centerline of the pipe. This height is known as the piezometric height or the piezometeric head. The elevation to which the liquid rises is the piezometric level. The principle of the piezometer is that the liquid rises the piezometric tube until atmospheric pressure, p_{atm}, and the weight of column of liquid in the tube generate a pressure that equals the pressure in the pipeline.

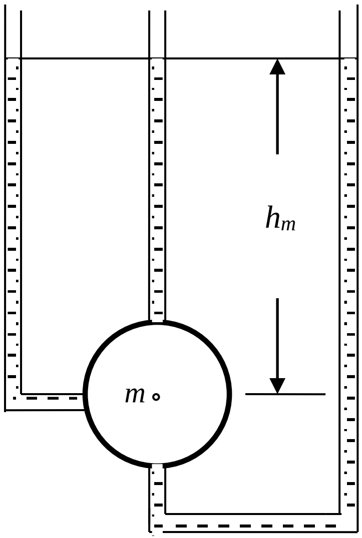


Figure 6 Piezometer

The pressure at point 'm' in the pipe is indicated by the height 'h_m' of liquid in the piezometer above the point m.

$p_m = \gamma h_m$

where p_m = pressure intensity at point 'm'
γ = specific weight of liquid

In other words, h_m is the pressure head at 'm'. Piezometers can measure only gauge pressures as the liquid surface in the piezometer is exposed to atmospheric pressure.

Measurement of negative gage pressures by piezometer

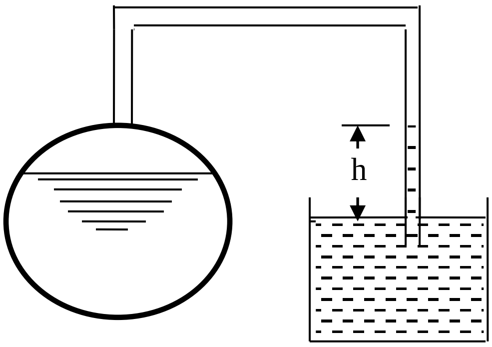


Figure 7 Measurement of Negative Pressure by Piezometer

If the pressure inside the container is less than atmospheric pressure, liquid cannot raise in the ordinary piezometer shown in Figure above. But if the top of the tube is bent downward and its lower end is dipped into a vessel containing water or any other suitable liquid as shown in Figure 7, the atmospheric pressure prevailing at the free surface of liquid in the vessel will drive the liquid up in the piezometeric tube to a height h. The rise of liquid in the tube indicates the magnitude of negative pressure head in the container. Neglecting the weight of air caught in the portion of the tube, the pressure on the liquid surface in the container is the same as the pressure at the free surface of liquid in the tube.

Piezometers can also be used to measure pressure heads in pipes where the liquid is in motion. In such cases, the piezometer tube should enter the pipe in a direction at right angles to the direction of flow and the connecting end w should be flush with the inner surface of the pipe.

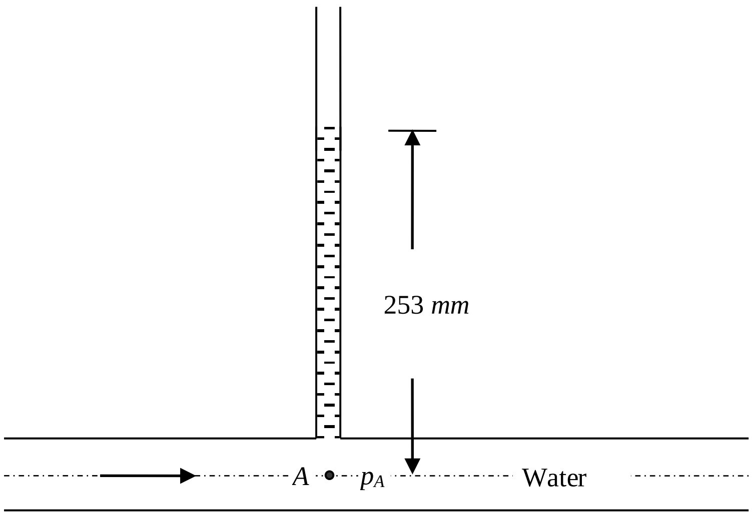
Glass tubes used for piezometers should not have a diameter less than 12 mm. *Why*? If the glass tube has a diameter less than 12 mm, the effect of capillarity will affect the height of column of liquid. Hence, in order to prevent the capillary action, the glass tube should have an internal diameter not less than 12 mm. For precise work at low heads, the internal diameter of glass tube should be 25 mm.

Main disadvantages with the piezometer are:

(1) Piezometers cannot be used to measure large pressures of lighter liquids (liquids with lesser specific weight). Because, this would require very long tubes and they cannot be handled conveniently.
(2) Piezometers cannot be used to measure gas pressures as a gas forms no free atmospheric surface.

Example. A piezometer measures the pressure in pipeline carrying water. The piezometer reading is 253 *mm* measured from the centerline of the pipe. At this point, what are the gauge pressure and the absolute pressure in $N\ m^{-2}$? Take atmospheric pressure as equivalent of 10.3 *m* of water absolute.

Solution.



Figure

Let the pressure at point A in the centerline of the pipeline be p_A

Atmospheric pressure, $p_{atm} = \gamma_w h$

where γ_w = specific weight of water = 9810 $N\ m^{-3}$

h = column of water equivalent of atmospheric pressure = 10.3 m

Hence, p_{atm} = (9810 $N\ m^{-3}$) x (10.3 m) = 101043 $N\ m^{-2}$

In gauge units:

$p_A = \gamma_w h_A$

where γ_w = specific weight of water = 9810 $N\ m^{-3}$
h_A = rise of water in the piezometer measured from the point A
= 253 mm = 0.253 m

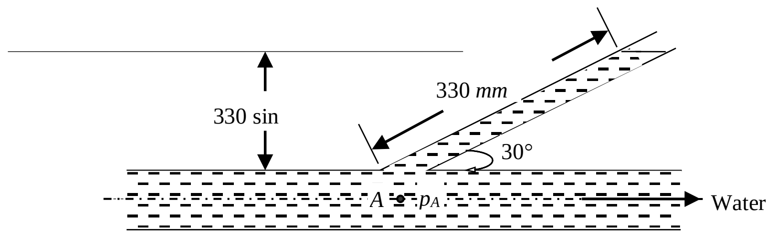
Hence, p_A = (9810 $N\ m^{-3}$) x (0.253 m) = 2482 N m^{-2} (gauge)

In absolute units:

Absolute pressure at A = atmospheric pressure + gauge pressure at A
= 101043 $N\ m^{-2}$ + 2482 $N\ m^{-2}$
= 103525 $N\ m^{-2}$

Example. An inclined piezometer measures the pressure in pipeline carrying water. The piezometer is inclined at an angle of 30° to the horizontal and has an inclined reading of 330 *mm*. What is the gauge pressure in the pipeline?

Solution.



Figure

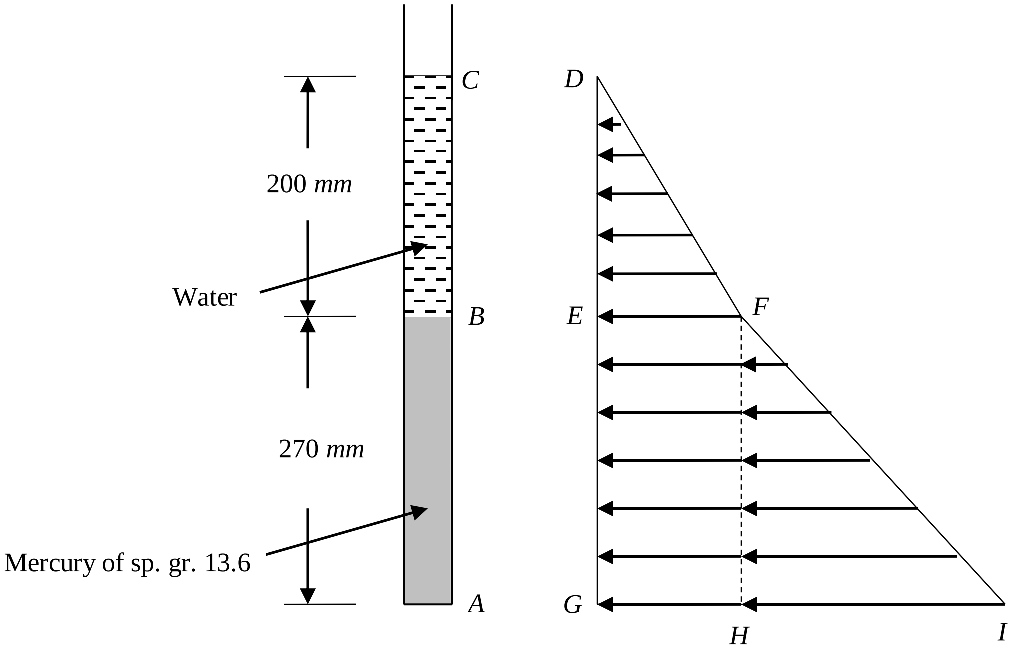
Let the gauge pressure at A be p_A $N\ m^{-2}$

The gauge pressure at A depends upon the vertical height, above A, of the free surface of water in the inclined piezometer. Hence,

$p_A = \gamma_w$ (0.33 sin 30°) = 9810 x (0.33 sin 30°) = 9810 x 0.165 = 1619 $N\ m^{-2}$

Example. A vertical tube contains 200 *mm* of water on top of 270 *mm* of mercury of relative density 13.6. What is the pressure at the bottom of the tube?

Solution.



Figure

Point C in the tube denotes the free surface of water standing over the column of mercury in the tube. Pressure at C is atmospheric. That is, in gauge units, pressure at C, $p_C = 0$

The triangle DEF represents the pressure distribution diagram for the 200 *mm* column of water standing over the column of mercury in the tube. The rectangle $EFHG$ represents the uniform pressure distribution due to 200 *mm* column of water exerted in the column AB of the tube. The triangle FHI represents the pressure distribution diagram for the 270 *mm* column of mercury.

Pressure intensity at B, $p_B = \gamma_w h_{BC} = 9810\ N\,m^{-3} \times 0.2\ m = 1962\ N\,m^{-2}$

Pressure at the bottom of tube, p_A = uniform pressure intensity due to 200 *mm* column of water + pressure intensity due to 270 *mm* column of mercury

$= p_B + \gamma_m h_{AB}$

$= 1962\ N\,m^{-2} + (9810 \times 13.6 \times 0.27)\ N\,m^{-2}$

$= 37984\ N\,m^{-2}$

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(ii) U – tube manometers

The limitations of a piezometer are overcome by a U-tube manometer. It consists of a glass tube bent in the form of alphabet 'U'. One end of the U-tube is connected to the gage point (point at which the pressure is required to be measured) and the other end is kept open to the atmosphere. Refer Figure 8 below.

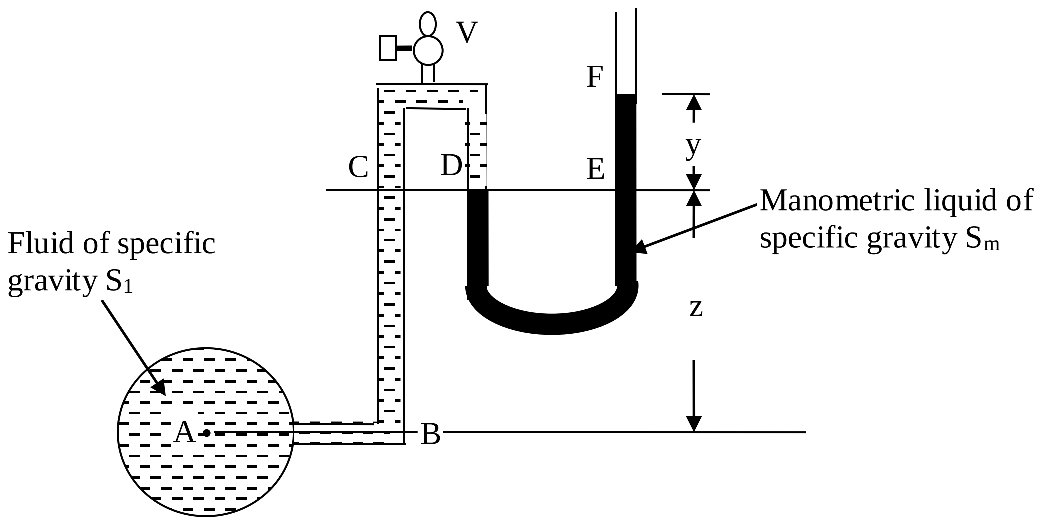


Figure 8 U – tube manometer

The tube contains a liquid of specific gravity greater than that of the fluid whose pressure is to be measured. Sometimes more than one liquid may also be used in the manometer.

Liquids frequently used in manometers:

Mercury, oil, salt solution, carbon disulphide, carbon tetrachloride, bromoform and alcohol. Water is also used as a manometric liquid when it is required to measure pressures of gases or certain coloured liquids that are immiscible with water are to be measured.

The liquids used in manometers should be such that they do not mix with the fluids whose pressures are to be determined.

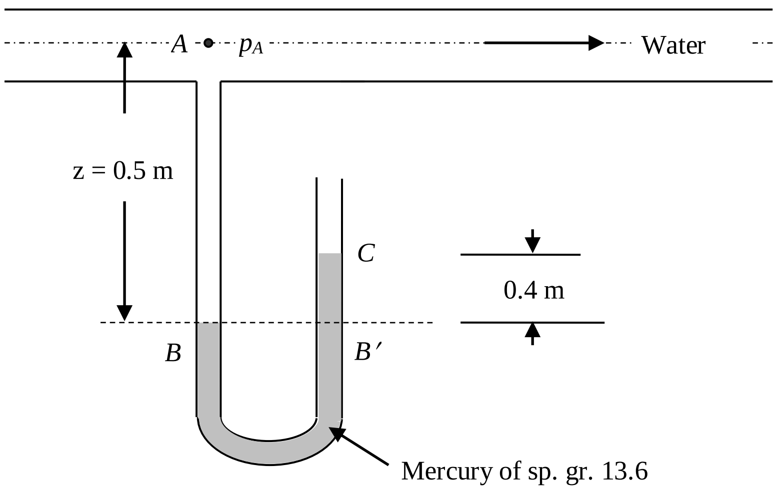
The choice of the manometric liquid depends on the range of pressure to be measured. For low pressure range, liquids of lower specific gravities are used. For high pressure range, generally, mercury is used as the manometric liquid.

In Figure 8, the left limb of the U-tube manometer is connected to the gage point A where the pressure is required to be measured. Because of the pressure of liquid at A, it enters the connected limb of the manometer thereby pushing the manometric liquid and making it to rise in the other limb (right limb). An air relief valve V is provided at the top of the connecting tube which expels the air in the portion A'B and its place is taken by the fluid at A. The expulsion of air is important from the point of view of accurate pressure measurement.

Pressure at point A can be determined by writing a gage equation (manometric equation).

U-tube manometers can be used to measure pressures of not only liquid but also gases. Further, they can be used to measure negative gage pressures of liquids and gases.

Example. The lower part of the U – tube manometer shown below contains mercury of specific gravity 13.6. The pipe contains water. Determine the gauge pressure, *p*, at the centre of the pipe if the manometer readings are as shown in the figure.



Figure

Solution.

Let the pressure at the centre of pipe, *A*, be p_A $N\ m^{-2}$

Pressure intensity at *B*, $p_B = p_A + \gamma_w z = \{p_A + (9810 \text{ x } 0.5)\}\ N\ m^{-2}$

Pressure intensity at *B′* = Pressure intensity at *B*

(since, *B* and *B′* lie in the same horizontal plane)

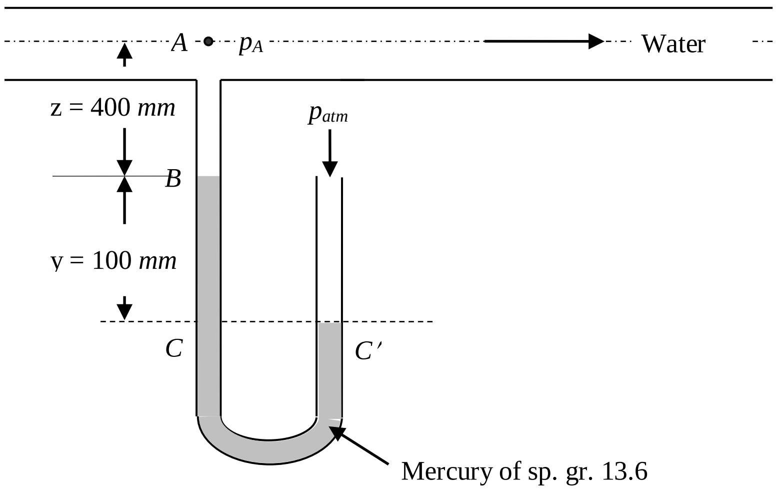
$= \{p_A + (9810 \text{ x } 0.5)\}\ N\ m^{-2}$

Pressure intensity at C, p_C = atmospheric pressure (or) zero gauge pressure
$= 0$

Also, $p_C = p_{B'} - \gamma_m \overline{B'C} = \{p_A + (9810 \text{ x } 0.5)\} – (0.4 \text{ x } 13.6 \text{ x } 9810) = 0$

$\Rightarrow p_A = (0.4 \text{ x } 13.6 \text{ x } 9810) – (9810 \text{ x } 0.5) = 48461\ N\ m^{-2}$

Example. The U – tube manometer shown in Figure below is used to measure the pressure in a pipeline as it passes over the crest of a hill. The pipeline carries freshwater. The bottom of the U – tube contains mercury of specific gravity 13.6. Taking atmospheric pressure as equivalent of 10.3 m of water, with the readings shown in the figure, calculate the absolute pressure at the centre of the pipeline.



Figure

Solution.

Let the pressure at the centre of pipe, A, be $p_A\ N\ m^{-2}$

Pressure intensity at B, $p_B = p_A + \gamma_w z = \{p_A + (9810 \text{ x } 0.4)\}\ N\ m^{-2}$

Pressure intensity at C, $p_C = p_B + \gamma_w y$

$= \{p_A + (9810 \text{ x } 0.4) + (9810 \text{ x } 0.1)\}\ N\ m^{-2}$

Pressure intensity at C' = Pressure intensity at C

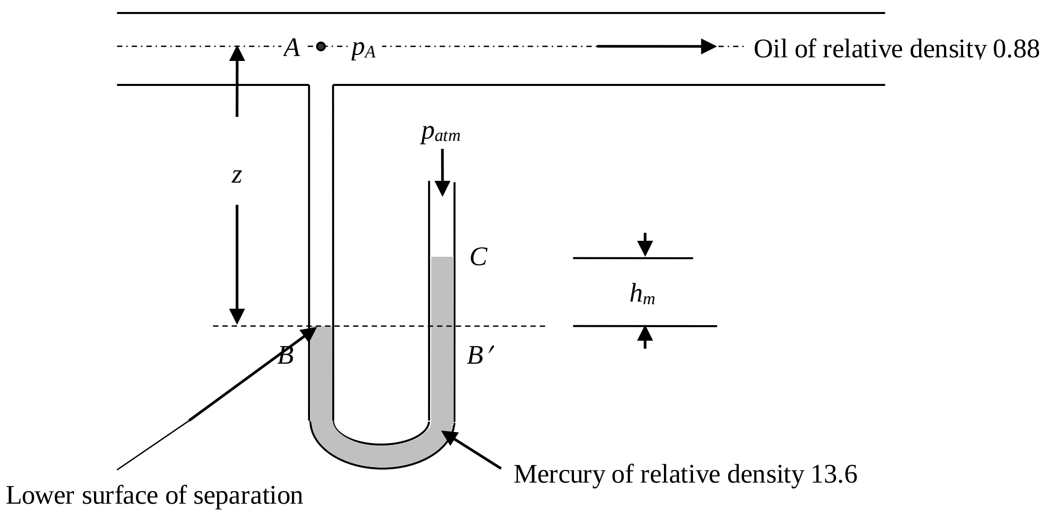
(since, C and C' lie in the same horizontal plane)

$= \{p_A + (9810 \text{ x } 0.4) + (9810 \text{ x } 0.1)\}\ N\ m^{-2}$

As C' in the right limb of the manometer marks the free surface of mercury, pressure intensity at C is atmospheric. The pressure head at C' is 10.3 m of water absolute.

$p_{C'}$ = 9810 x 10.3 = 101043 $N\ m^{-2}$
= $\{p_A$ + (9810 x 0.4) + (9810 x 0.1)$\}$ $N\ m^{-2}$
$\Rightarrow p_A$ = 101043 – (9810 x 0.4) – (9810 x 0.1) = 96138 $N\ m^{-2}$

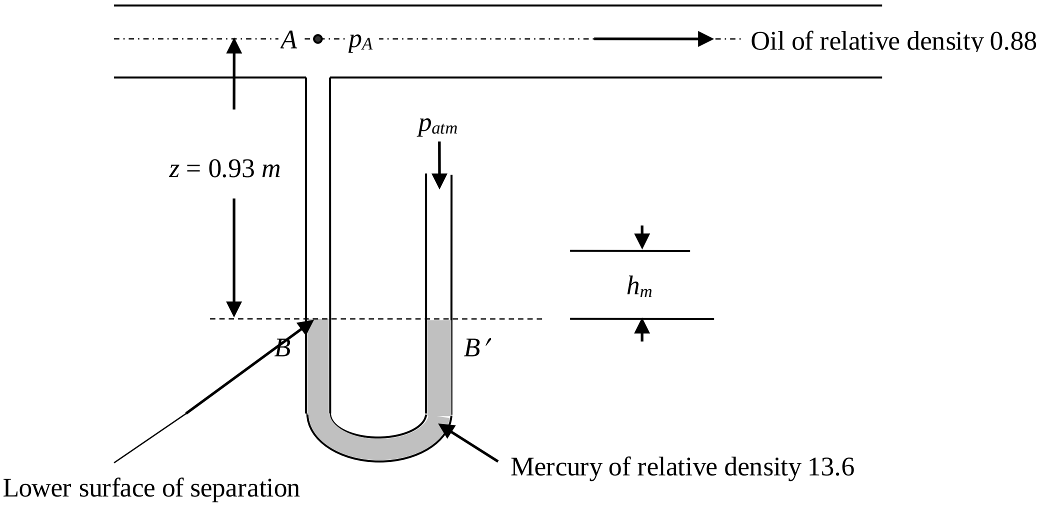
Example. A U – tube manometer as shown in Figure below is used to measure the pressure in a pipe which carries oil of relative density 0.88. The lower manometer liquid is mercury of relative density 13.6. The surface of separation between the oil and mercury is 0.93 m below the centerline of the pipe. (a) If the differential head, h_m, is zero, what is the gauge and absolute pressure at the centerline of the pipe? (b) If z in figure remains at 0.93 m but h_m is now 0.037 m of mercury, what is the gauge pressure in the pipe? Take atmospheric pressure equal to101,404 N m^{-2}.



Figure

Solution.

(a) When the differential head, h_m, is zero, the free surface at C in the right limb of the U – tube manometer coincides with B'.
The surface of separation between the oil column and mercury column at B in the left limb of the U – tube manometer is located at a vertical distance z = 0.93 m below the centerline of the pipe. Hence, the above figure becomes



Figure

Let the pressure at the centre of pipe, A, be p_A $N\ m^{-2}$

Pressure intensity at B, $p_B = p_A + \gamma_{oil}z = [p_A + \{(9810 \times 0.88) \times 0.93\}]\ N\ m^{-2}$

Pressure intensity at B' = Pressure intensity at B

(since, B and B' lie in the same horizontal plane)

$= [p_A + \{(9810 \times 0.88) \times 0.93\}]\ N\ m^{-2}$

As B' in the right limb of the manometer marks the free surface of mercury, pressure intensity at B is atmospheric. The pressure head at B' is 101,404 $N\ m^{-2}$.

Hence, we have,

$p_{B'} = 101404\ N\ m^{-2}$

$= [p_A + \{(9810 \times 0.88) \times 0.93\}]\ N\ m^{-2}$

$\Rightarrow p_A = 101404 - \{(9810 \times 0.88) \times 0.93\} = 93375\ N\ m^{-2}$ (in absolute units)

$p_A = 93375 - 101404 = -8029\ N\ m^{-2}$ (in gauge units)

Note: The negative sign indicates that the pressure at A is below atmospheric pressure, that is, pressure at A is gauge negative or suction pressure or vacuum pressure.

(b) Differential head, $h_m = 0.037\ m$

$z = 0.93\ m$

Pressure intensity at B, $p_B = p_A + \gamma_{oil}\, z = [p_A + \{(9810 \times 0.88) \times 0.93\}]\ N\ m^{-2}$

Pressure intensity at B' = Pressure intensity at B

(since, B and B' lie in the same horizontal plane)
$= [p_A + \{(9810 \text{ x } 0.88) \text{ x } 0.93\}]\ N\ m^{-2}$

Pressure intensity at C, p_C = atmospheric pressure (or) zero gauge pressure
$= 0$

Also, $p_C = p_{B'} - \gamma_m h_m = [p_A + \{(9810 \text{ x } 0.88) \text{ x } 0.93\}] - (13.6 \text{ x } 9810 \text{ x } 0.037)$
$= 0$

$\Rightarrow p_A = (13.6 \text{ x } 9810 \text{ x } 0.037) - \{(9810 \text{ x } 0.88) \text{ x } 0.93\} = -3092\ N\ m^{-2}$

Problem 4. A U-tube containing mercury has its right limb open to atmosphere. The left limb is full of water and is connected to a pipe containing water under pressure, the center of which is in level with the free surface of mercury; find the pressure of water in the pipe above atmosphere, if the difference in level of mercury in the limbs is 5.08 *cm*.

Solution.

Figure 9 shows the schematic representation of the problem statement.

Required: To determine the pressure of water in the pipe above atmospheric

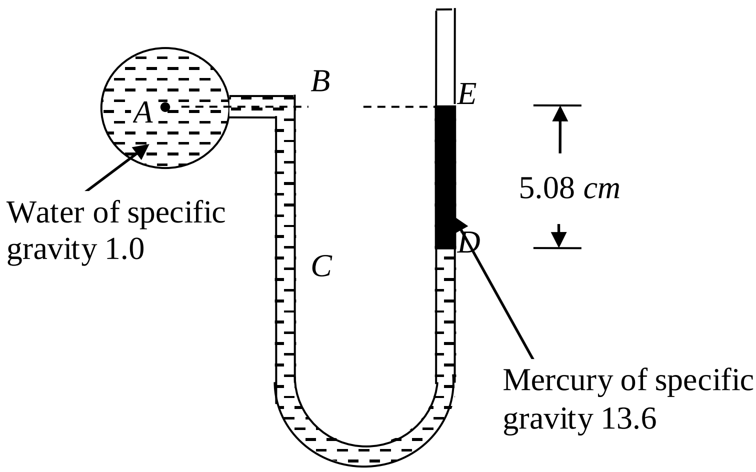


Figure 9 Problem 4

Note: The pressures at all points noted in Figure 9 are expressed in equivalent column of water of specific gravity 1.0.

Now, let us write the manometric equation in gage units.

Let the pressure intensity (in gage units) at point A in pipe be p_A. The pressure head at point A in equivalent column of water is expressed as $\frac{p_A}{\gamma_w}$, where γ_w is the specific weight of water equal to 9810 N/m^3.

Pressure head at B = Pressure head at A = $\frac{p_A}{\gamma_w}$ (as points A and B lie in the same horizontal plane)

It should be noted that for points lying in the same horizontal plane, pressures at all points are the same.

Pressure head at C (in meters of water) = Pressure head at B (in meters of water) + 0.0508 m of water

$$= \frac{p_A}{\gamma_w} + 0.0508 \text{ } m \text{ of water}$$

Pressure head at D = Pressure head at C = $\frac{p_A}{\gamma_w}$ + 0.0508 m of water (as points C and D lie in the same horizontal plane)

Pressure head at E = Pressure head at D – 0.0508 m of mercury expressed in equivalent column of water

Pressure head at E = Pressure head at D – $0.0508\frac{S_m}{S_w}$ = atmospheric pressure

where S_m = Specific gravity of mercury = 13.6
S_w = Specific gravity of water = 1.0

Note: As the pressures at all points are expressed in gage units, atmospheric pressure is treated as zero pressure.

$\therefore$ Pressure head at E, $\frac{p_E}{\gamma_w} = 0 = \frac{p_A}{\gamma_w}$ + 0.0508 m of water - $0.0508\frac{S_m}{S_w}$

$\Rightarrow \frac{p_A}{\gamma_w}$ = - 0.0508 m of water + $0.0508\frac{S_m}{S_w}$

$$= -0.0508 + 0.0508\,\frac{13.6}{1.0} = 0.64008 \text{ } m \text{ of water}$$

p_A = 0.64008 x γ_w = (0.64008 m of water) x 9810 N/m^3 = **6279.2 N/m^2**

= **0.064 $kg(f)/cm^2$ (gage positive)**

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(a) ***Differential Manometers***

Differential manometers are those manometers that are employed for measuring the difference of pressure between any two points in a pipe line or in two pipes or containers.

It consists of a bent glass tube, the two ends of which are connected to the gage points between which the pressure difference is required to be measured.

Common types of differential manometers

- (i) Two-piezometer manometer
- (ii) Inverted U – tube manometer
- (iii) U – tube differential manometer
- (iv) Micro manometer

(i) Two – piezometer manometer

It consists of two separate piezometers that are inserted at the two gage points between which the pressure difference is to be measured. The difference in levels of liquid raised in the two piezometers denotes the pressure difference between these two points. Figure below shows the schematic arrangement of two-piezometer manometer for measuring pressure difference between two points in a pipe line.

Figure 10 shows a pipe line conveying a liquid of certain specific weight. It is required to measure the pressure difference between points A and B in the pipe line. Two piezometers are connected, one to the gage point A and the other to the gage point B. The flow direction in the pipe line is from A to B. Let h_A and h_B be the respective heights to which the liquid has raised in the piezometers at points A and B respectively. h_A and h_B indicate the pressures at points A and B in equivalent column of the flowing liquid. Now, the pressure difference, h, in equivalent column of flowing liquid, between points A and B is simply the difference between h_A and h_B.

Demerits of two-piezometer manometer

- (i) This method is useful only if the pressures at the two gage points are small. If the magnitudes of pressures are high, it requires long piezometers and measurement becomes inconvenient with long piezometers.
- (ii) It cannot be used to measure pressure difference in flow of gases.

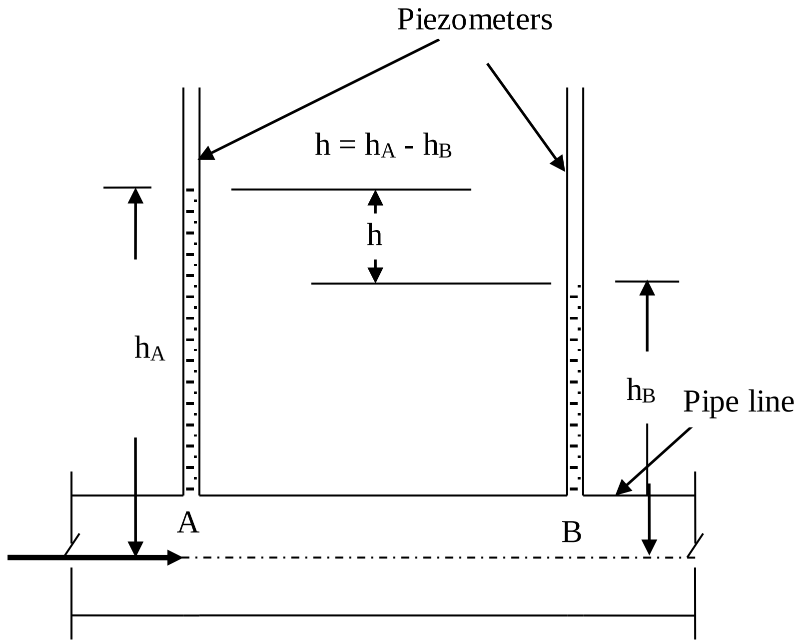


Figure 10 Two-Piezometer Manometer

(ii) Inverted U – tube manometer

It consists of a glass tube bent in U – shape and held inverted as shown in Figure 11.

The two ends of the manometer are connected to the points *A* and *B* in the pipe line between which the pressure difference is required to be measured. When flow occurs in the pipe line, because of the pressures prevailing at points *A* and *B*, liquid will rise in the two limbs of the manometer thereby pushing the air in the manometer to go up and get compressed. The compressed air restricts the heights of columns of liquid raised in the two limbs of the manometer. An air cock is provided at the top of the inverted U - tube which facilitates the raise of liquid in the two limbs to suitable levels by driving out a portion of the compressed air. The air cock also allows the expulsion of air bubbles that might have been entrapped in the pipe line.

Let p_A and p_B be the pressure intensities at the points *A* and *B*. Corresponding to the intensities of pressure at these two points, the liquid will rise above the points *A* and *B* up to points *C* and *D* respectively in the two limbs of the inverted U – tube manometer. Let *S* be the specific gravity of the liquid flowing in the pipe line. Let γ_w be the specific weight of water.

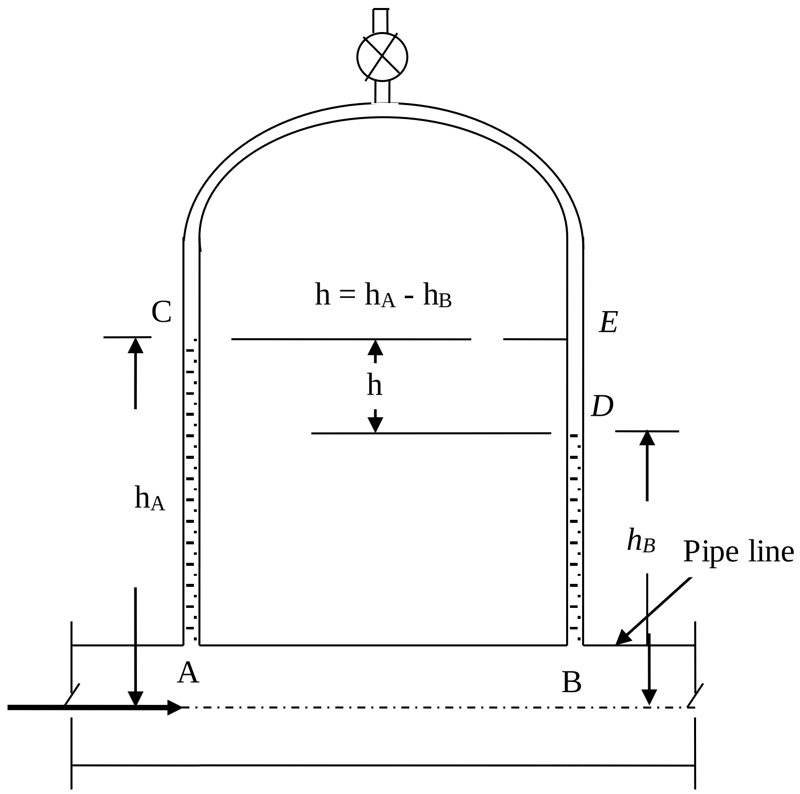


Figure 11 – Inverted U – Tube Manometer

Let us develop the manometric equation (pressure equation) commencing from point A in pipe line. Let us express the pressures at various points in equivalent columns of flowing liquid of specific gravity S.

Pressure head at A = $\dfrac{p_A}{\gamma_w S}$ meters of flowing liquid of specific gravity S

Pressure head at C = Pressure head at A in meters of liquid of specific gravity S – h_A meters of liquid of specific gravity S

$$= \frac{p_A}{\gamma_w S} - h_A$$

Pressure head at E = Pressure head at C (since, both points C and E lie in the same horizontal plane)

$$= \frac{p_A}{\gamma_w S} - h_A$$

Pressure head at D = Pressure head at E + Pressure head due to column of air in the portion ED of the right limb of manometer

As the pressure exerted by column of air is negligible due to its low specific weight, the pressure head at D = Pressure head at E = $\dfrac{p_A}{\gamma_w S}$ - h_A

Pressure head at B = Pressure head at D + h_B meters of liquid of specific gravity S

$$= \frac{p_A}{\gamma_w S} - h_A + h_B$$

From Figure above, $h_B = h_A - h$; therefore,

Pressure head at B = $\frac{p_A}{\gamma_w S} - h_A + (h_A - h) = \frac{p_A}{\gamma_w S} - h$

i.e., $\frac{p_B}{\gamma_w S} = \frac{p_A}{\gamma_w S} - h$

$\Rightarrow \frac{p_A}{\gamma_w S} - \frac{p_B}{\gamma_w S} = h$ metres of liquid of specific gravity S

Applicability of Inverted U – tube manometers

For measurement of small difference of pressure in liquids

Problem 5. The pressure between two points M and N in a pipe conveying oil of specific gravity 0.9 is measured by an inverted U – tube and the column connected to point N is 1.5 m higher than that at point M. A commercial pressure gage directly attached to the pipe at M reads 2 $kg(f)/cm^2$, determine the reading of the gage when directly attached to the pipe at N.

Solution.

Figure 12 shows the schematic representation of the problem statement.

Required: To determine the reading of the commercial pressure gage directly attached to point N of pipe.

Note: The pressures at all points noted in the Figure below are expressed in equivalent column of oil of specific gravity 0.9

Now, let us write the manometric equation in gage units.

Let the pressure intensity (in gage units) at point M in pipe be p_M. The pressure head at point M in equivalent column of oil is expressed as $\frac{p_M}{\gamma_{oil}}$, where γ_{oil} is the specific weight of oil equal to (0.9 x 9810 N/m^3 = 8829 N/m^3).

Pressure head at A (in meters of oil of specific gravity 0.9) =

Pressure head at M (in meters of oil) $- y$ meters of oil

$$= \frac{p_M}{\gamma_{oil}} - y$$

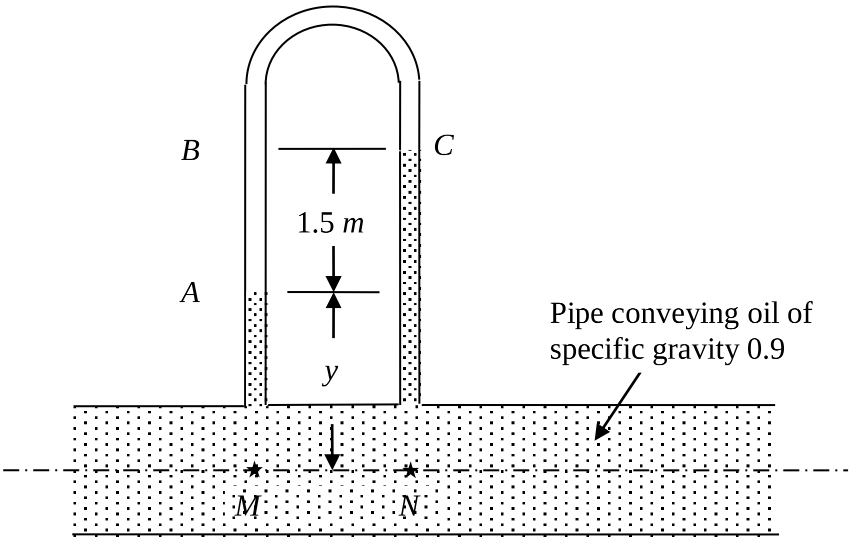


Figure 12 – Problem 5

Pressure head at *B* =
Pressure head at *A* – 1.5 *m* of air expressed in equivalent column of oil of specific gravity 0.9

Note: the column of air in the portion of manometer between *A* and *B* can be neglected as the pressure due to air is negligible compared to that of oil.

Hence, Pressure head at *B* = Pressure head at *A* = $\frac{p_M}{\gamma_{oil}} - y$

Pressure head at *C* = Pressure head at *B* = $\frac{p_M}{\gamma_{oil}} - y$ (as points *B* and *C* lie in the same horizontal plane)

Pressure head at *N* = Pressure head at *C* + (1.5 + *y*)

$$= \frac{p_M}{\gamma_{oil}} - y + 1.5 + y$$

$$\frac{p_N}{\gamma_{oil}} = \frac{p_M}{\gamma_{oil}} + 1.5$$

$p_M = 2\ kg(f)/cm^2 = 2 \times 9.81\ N / (1/100)\ m^2 = 196200\ N/m^2$

Hence, $\frac{p_N}{\gamma_{oil}} = \frac{p_N}{8829\ N/m^3} = \frac{196200\ N/m^2}{8829\ N/m^3} + 1.5$

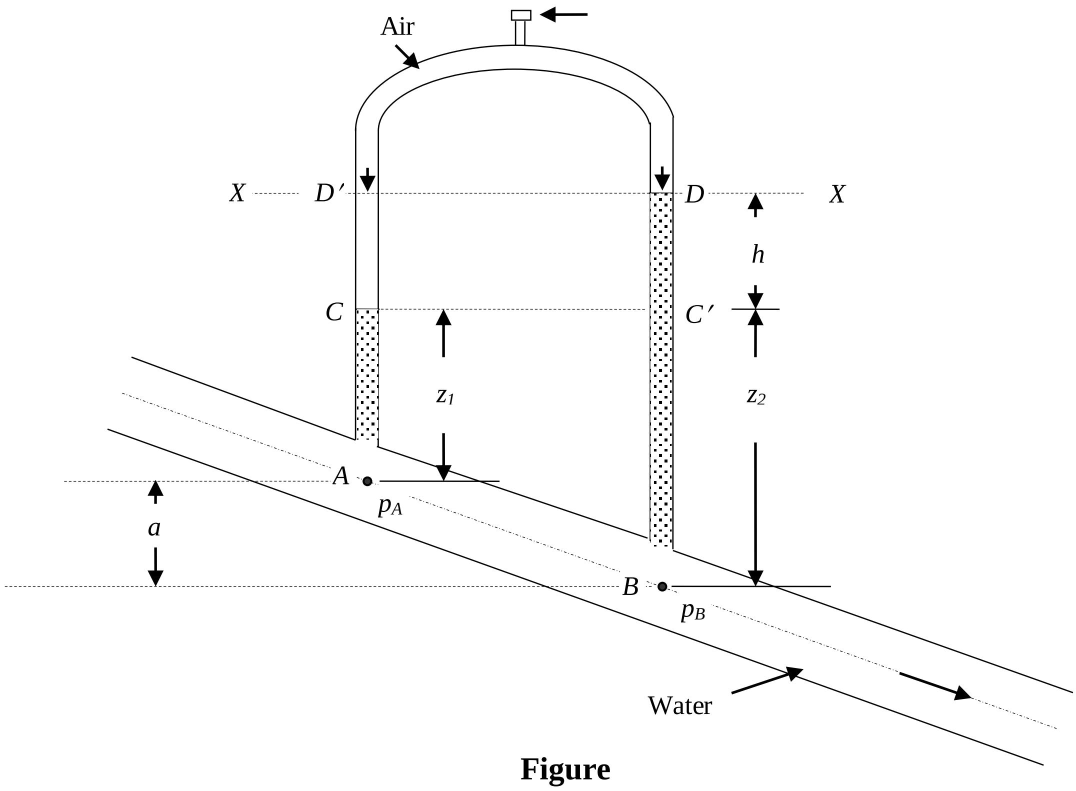
$= 22.222 + 1.5 = 23.722\ m$ of oil

$\Rightarrow p_N = (23.722\ m \text{ of oil}) \times (8829\ N/m^3) = 209441.5\ N/m^2$

$= (209441.5 / 9.81)\ kg(f) / (100\ cm)^2$

$= \mathbf{2.135\ kg(f)/cm^2}$

Example. A manometer is used to measure the pressure difference between two points in a pipe. The pipe carries water. Above the water in the manometer is air, which is pressurized. The manometer readings are z_1 = 100 *mm*, z_2 = 250 *mm* and h = 90 *mm*. What is the value of $(p_A - p_B)$ in $N\ m^{-2}$?



Figure

Solution.

Let the pressure at A be $p_A\ N\ m^{-2}$

Pressure intensity at C, $p_C = p_A - (\gamma_w\ z_1) = p_A - (9810 \times 0.1)$
$= p_A - 981$

Pressure at D' = Pressure at C – pressure due to air column $\overline{D'C}$ +
pressure due to pressurized air above the level of D', p_x

Neglecting the pressure due to air column $\overline{D'C}$ and pressure due to pressurized air above the level of D', p_x , we have,

Pressure at D' = Pressure at C
i.e., $p_{D'} = p_C = (p_A - 981)$

Pressure at D = Pressure at D'
(since, D and D' lie in the same horizontal plane)

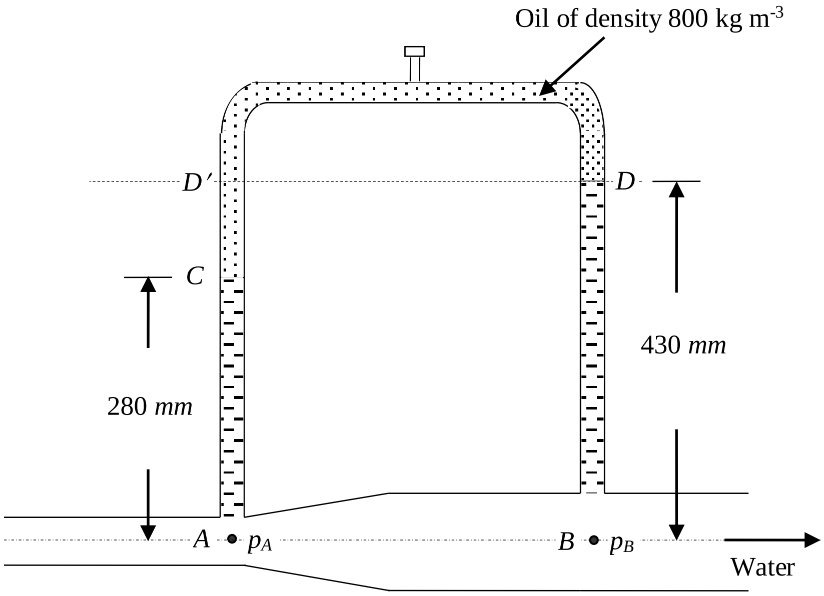
i.e., $p_D = p_{D'} = (p_A - 981)$

Pressure at C' = Pressure at $D + \gamma_w h$
$= (p_A - 981) + (9810 \text{ x } 0.09)$
$= (p_A - 981) + 883 = p_A + 98$

Pressure at B = Pressure at $C' + \gamma_w z_2$
$p_B = (p_A - 98) + (9810 \text{ x } 0.25) = (p_A - 98) + 2453 = p_A + 2355$
$\Rightarrow p_B - p_A = 2355\ N\ m^{-2}$
$\Rightarrow p_A - p_B = -\ 2355\ N\ m^{-2}$

Example. The Figure below shows an inverted U – tube manometer with oil of density 800 $kg\ m^{-3}$ above the pipe liquid which is water. The pipeline is horizontal. What is the value of $(p_A - p_B)$?

(b) If the manometer readings are the same but the oil is replaced by air, what is $(p_A - p_B)$ now?



Figure

Solution.

Let the pressure at A be $p_A\ N\ m^{-2}$

Pressure intensity at C, $p_C = p_A - (\gamma_w \text{ x } 0.280) = p_A - (9810 \text{ x } 0.280)$
$= p_A - 2747$

Pressure at D' = Pressure at C – pressure due to oil column $\overline{CD'}$
$= (p_A - 2747) - \gamma_{oil}\ \overline{CD'}$
$= (p_A - 2747) - \{(800 \times 9.81) \times (0.430 - 0.280)\}$
$= (p_A - 2747) - 1177$
$= p_A - 3924$

Pressure at D = Pressure at D'
(since, D and D' lie in the same horizontal plane)
i.e., $p_D = p_{D'} = (p_A - 3924)$

Pressure at B = Pressure at $D + \gamma_w \times 0.430$
$p_B = (p_A - 3924) + (9810 \times 0.430) = (p_A - 3924) + 4218$
$= p_A + 294$
$\Rightarrow (p_B - p_A) = 294\ N\ m^{-2}$
$\Rightarrow (p_A - p_B) = -294\ N\ m^{-2}$
(b) Let us now consider the case where oil is replaced by air in the above case and all other things remain the same

Let the pressure at A be $p_A\ N\ m^{-2}$

Pressure intensity at C, $p_C = p_A - (\gamma_w \times 0.280) = p_A - (9810 \times 0.280)$
$= p_A - 2747$

Pressure at D' = Pressure at C – pressure due to air column $\overline{CD'}$

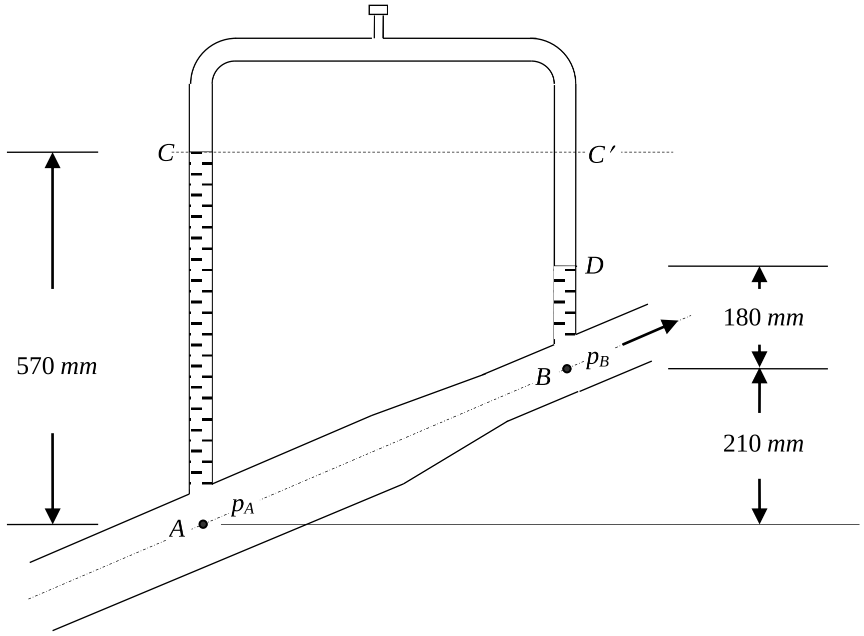
Neglecting the pressure due to air column $\overline{CD'}$ due to its insignificance compared to the pressure due to water, we have,

Pressure at D' = Pressure at $C = p_A - 2747$

Pressure at D = Pressure at D'
(since, D and D' lie in the same horizontal plane)
i.e., $p_D = p_{D'} = (p_A - 2747)$

Pressure at B = Pressure at $D + \gamma_w \times 0.430$
$p_B = (p_A - 2747) + (9810 \times 0.430) = (p_A - 2747) + 4218$
$= p_A + 1471$
$\Rightarrow (p_B - p_A) = 1471\ N\ m^{-2}$
$\Rightarrow (p_A - p_B) = -1471\ N\ m^{-2}$

Example. An inverted U – tube manometer is connected to a pipeline which slopes upward as shown in Figure below. The pipeline carries water. Calculate (p_A – p_B) when (a) the upper part of the manometer is filled with air and (b) when oil of relative density 0.8 is introduced above the water.



Figure

Solution.

(a) When the upper part of the manometer is filled with air

Let the pressure at A be p_A $N\ m^{-2}$

Pressure intensity at C, $p_C = p_A - (\gamma_w \times 0.570) = p_A - (9810 \times 0.570)$
$= p_A – 5592$

Pressure at C' = Pressure at C

(since, C and C' lie in the same horizontal plane)

i.e., $p_{C'} = p_C = (p_A - 5592)$

Pressure at D = Pressure at C′ + Pressure due to air column $\overline{C'D}$

Neglecting the pressure due to air column $\overline{C'D}$ due to its insignificance compared to the pressure due to water, we have,

Pressure at D = Pressure at C' = (p_A - 5592)

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Pressure at B = Pressure at $D + \gamma_w$ x 0.180

$p_B = (p_A - 5592) + (9810 \text{ x } 0.180) = (p_A - 5592) + 1766$

$= p_A - 3826$

$\Rightarrow (p_B - p_A) = - 3826\ N\ m^{-2}$

$\Rightarrow (p_A - p_B) = 3826\ N\ m^{-2}$

(b) When oil of relative density 0.8 is introduced above the water

Let the pressure at A be $p_A\ N\ m^{-2}$

Pressure intensity at C, $p_C = p_A - (\gamma_w \text{ x } 0.570) = p_A - (9810 \text{ x } 0.570)$

$= p_A - 5592$

Pressure at C' = Pressure at C

(since, C and C' lie in the same horizontal plane)

i.e., $p_{C'} = p_C = (p_A - 5592)$

Pressure at D = Pressure at C' + Pressure due to oil column $\overline{C'D}$

Pressure at $D = (p_A - 5592) + \{\gamma_{oil} \text{ x } (0.57 - 0.18 - 0.21)\}$

$= (p_A - 5592) + \{(9810 \text{ x } 0.8) \text{ x } 0.18\}$

$= (p_A - 5592) + 1413$

$= p_A - 4179$

Pressure at B = Pressure at $D + (\gamma_w$ x 0.180)

$p_B = (p_A - 4179) + (9810 \text{ x } 0.180) = (p_A - 4179) + 1766$

$= p_A - 2413$

$\Rightarrow (p_B - p_A) = - 2413\ N\ m^{-2}$

$\Rightarrow (p_A - p_B) = 2413\ N\ m^{-2}$

(iii) U – Tube Differential Manometer

Construction: It consists of a glass tube bent in the shape of English alphabet '*U*'. The two ends of the *U* – tube are connected to the two gage points between which the pressure difference is required to be measured. Figure 13 below shows an arrangement for measuring pressure difference between two points *A* and *B* in a pipe line carrying a fluid of specific gravity *S*.

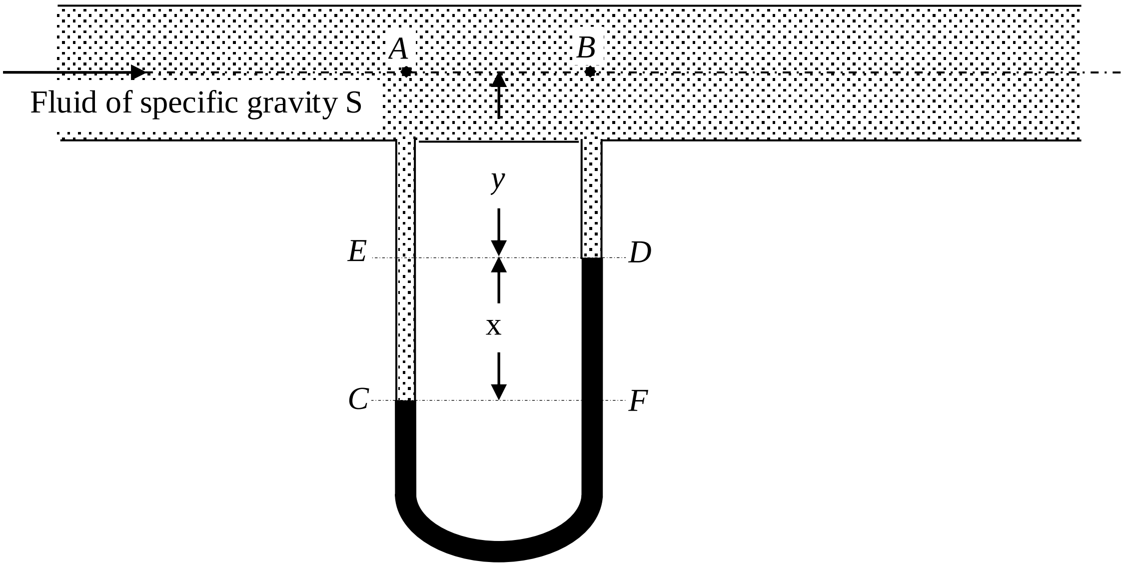


Figure 13 – U – Tube Differential Manometer

The manometer contains a liquid (manometric liquid) which is heavier than the liquid flowing in pipe line. The manometric liquid should be immiscible with the liquid flowing in pipe line. Before measurement, the level of manometric liquid in the two limbs of the manometer should be made to lie in the same horizontal line by releasing the air cocks. When the two limbs of the *U* – tube manometer are connected to the gage points *A* and *B*, because of the pressure of the flowing fluid, the level of mercury in the two limbs will be pushed down. Corresponding to the difference in pressure intensities p_A and p_B at points *A* and *B* respectively, the levels of mercury will get displaced through a distance *x* as shown in Figure. By measuring this difference in levels of the manometric liquid, the pressure difference ($p_A - p_B$) may be computed as below.

Let *S* be the specific gravity of the liquid flowing through the pipe line. Let S_m be the specific gravity of the manometric liquid. Let us develop the pressure equation (manometric equation) starting from point *A* in pipe line. Let us express all pressure quantities in terms of equivalent columns of liquid of specific gravity *S* flowing in pipe line.

Let pressure head at $A = \frac{p_A}{\gamma_w S}$

where γ_w = specific weight of water = 9810 N/m^3

$\gamma_w S$ = specific weight of liquid of specific gravity *S*

p_A = pressure intensity in N/m^2 at *A*

Pressure head at *C* = pressure head at *A* + {($y + x$) meters of liquid of specific gravity *S*}

$= [\frac{p_A}{\gamma_w S} + (y + x)]$ meters of liquid of specific gravity S

Pressure head at F = Pressure head at C

$= [\frac{p_A}{\gamma_w S} + (y + x)]$ meters of liquid of specific gravity S

(since, both points C and F lie in the same horizontal plane)

Pressure head at D = Pressure head at F –

$\left(x \frac{S_m}{S} \right)$ meters of liquid of specific gravity S

$= [\frac{p_A}{\gamma_w S} + (y + x) - \left(x \frac{S_m}{S} \right)]$ meters of liquid of specific gravity S

Pressure head at B = Pressure head at D –

y meters of liquid of specific gravity S

$= [\frac{p_A}{\gamma_w S} + (y + x) - \left(x \frac{S_m}{S} \right) - y]$ meters of liquid of specific gravity S

$\frac{p_B}{\gamma_w S} = [\frac{p_A}{\gamma_w S} + x - \left(x \frac{S_m}{S} \right)]$ meters of liquid of specific gravity S

$\Rightarrow \frac{p_A}{\gamma_w S} - \frac{p_B}{\gamma_w S} = \left(x \frac{S_m}{S} \right) - x = [x\left(\frac{S_m}{S} - 1 \right)]$ meters of liquid of specific gravity S

Problem 6. A pipe containing water at 172 kN/m^2 pressure s connected by a differential manometer to another pipe 1.5 m lower than the first pipe and containing water at high pressure. If the difference of height of two mercury columns of the manometer is equal to 75 mm, what is the pressure in the lower pipe? Specific gravity of mercury is 13.6.

Solution:

Pressure intensity in pipe 1, p_1 = 172 kN/m^2 = 172 x 10^3 N/m^2

Pressure head in pipe 1 (in meters of water) = $\frac{p_1}{\gamma_w} = \frac{172 x 10^3 N/m^2}{9810 N/m^3}$

= 17.53313 m of water

Note: all pressure terms are expressed in meters of water

Pressure head at point A = Pressure head in pipe 1 + 1.5 m

= 17.53313 + 1.5 = 19.03313 m of water

Pressure head at B = Pressure head at A + y m of water

= 19.03313 *m* of water + *y m* of water
= (19.03313 + *y*) *m* of water

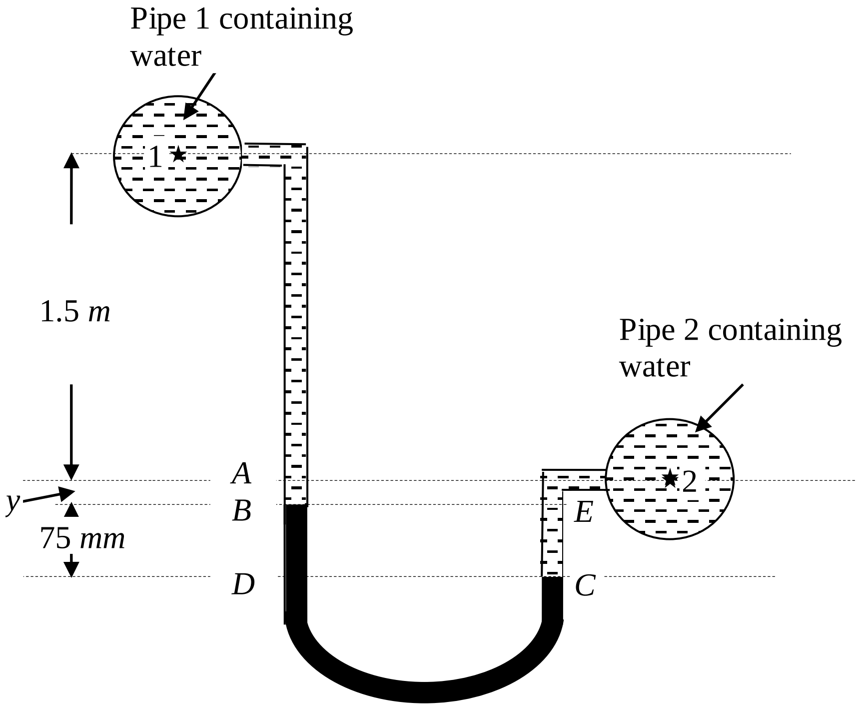


Figure 14 – Problem 6

Pressure head at *D* = Pressure head at *B* + 0.075 *m* of mercury expressed in equivalent meters of water

$$= (19.03313 + y)\ m \text{ of water} + (0.075)\ \frac{S_m}{S_w}$$

$$= 19.03313 + y + (0.075)\ \frac{13.6}{1.0}$$

$$= 19.03313 + y + 1.02$$

= (20.05313 + *y*) m of water

Pressure head at *C* = Pressure head at *D* (as both *C* and *D* lie in the same horizontal plane)
= (20.05313 + *y*) *m* of water

Pressure head at *E* = Pressure head at *C* – 0.075 *m* of water
= (20.05313 + *y*) *m* of water - 0.075 *m* of water
= (19.97813 + *y*) *m* of water

Pressure head in pipe 2 = Pressure head at *E* – *y m* of water
= (19.97813 + *y*) *m* of water - *y m* of water

$= 19.97813\ m$ of water

i.e., $\dfrac{p_2}{\gamma_w} = 19.97813\ m$ of water

$\Rightarrow p_2 = 19.97813 \times 9810 = 195985.4\ N/m^2 = \mathbf{1.998\ kg(f)/cm^2}$

Problem 7. A *U* – tube differential gage is attached to two sections *A* and *B* in a horizontal pipe in which oil of specific gravity 0.8 is flowing. The deflection of mercury in the gage is 60 *cm*, the level nearer to *A* being the lower one. Calculate the difference of pressure in $kg(f)/cm^2$ between the sections *A* and *B*.

Solution.

Figure below shows the schematic representation of the problem statement.

Let us express all pressure terms in meters of oil of specific gravity 0.8.

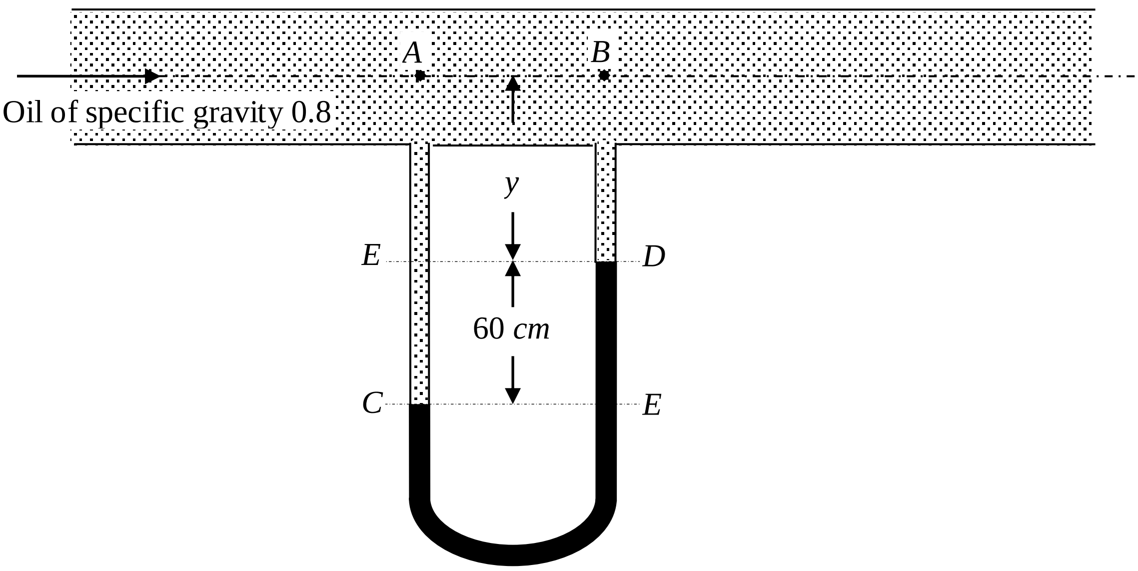


Figure 15 – Problem 7

Required: To determine the pressure difference between sections *A* and *B* of the pipeline, in $kg(f)/cm^2$.

Let the pressure head in meters of oil at section *A* of pipeline be expressed as $\dfrac{p_A}{\gamma_{oil}}$, where γ_{oil} is the specific weight of oil.

γ_{oil} = specific gravity of oil x specific weight of water

$= 0.8 \times 9810\ N/m^3 = 7848\ N/m^3$

Pressure head at E = Pressure head at $A + y$ meters of oil

$$= \frac{p_A}{\gamma_{oil}} + y$$

Pressure head at C = Pressure head at E + 0.6 meter of oil

$$= \frac{p_A}{\gamma_{oil}} + y + 0.6$$

Pressure head at E = Pressure head at C (as both points C and E lie in the same horizontal plane)

$$= \frac{p_A}{\gamma_{oil}} + y + 0.6$$

Pressure head at D = Pressure head at E – 0.6 m of mercury expressed in equivalent column of oil of specific gravity 0.8

$$= \frac{p_A}{\gamma_{oil}} + y + 0.6 - 0.6\left(\frac{S_m}{S_{oil}}\right)$$

where S_m = specific gravity of mercury = 13.6
S_{oil} = specific gravity of oil = 0.8

Pressure head at $D = \frac{p_A}{\gamma_{oil}} + y + 0.6 - 0.6\left(\frac{13.6}{0.8}\right)$

$$\frac{p_D}{\gamma_{oil}} = \frac{p_A}{\gamma_{oil}} + y + 0.6 - 10.2 = \frac{p_A}{\gamma_{oil}} + y - 9.6$$

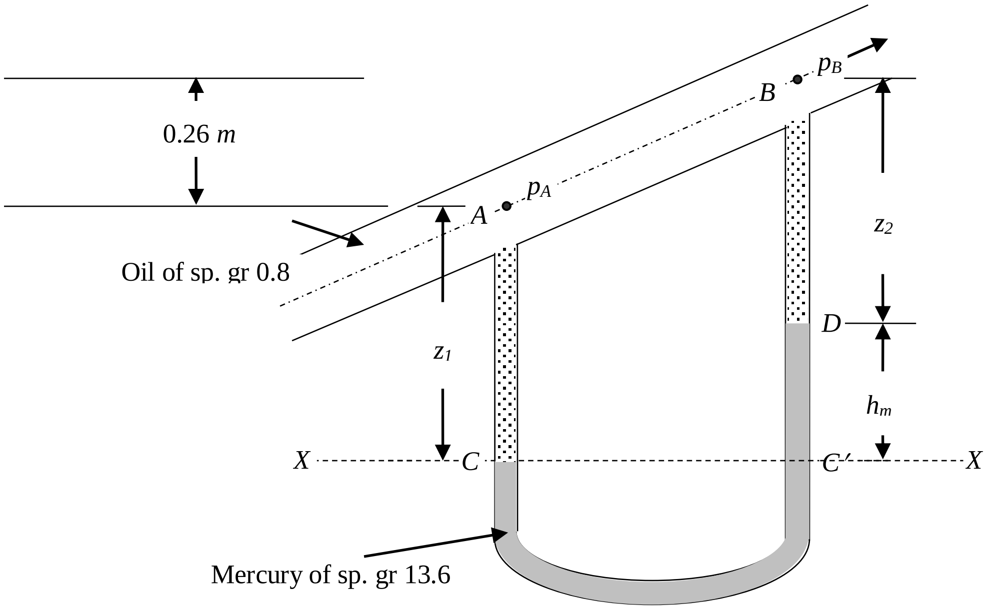
Pressure head at B = Pressure head at $D - y$

$$\frac{p_B}{\gamma_{oil}} = \frac{p_A}{\gamma_{oil}} + y - 9.6 - y$$

$$\Rightarrow \frac{p_A}{\gamma_{oil}} - \frac{p_B}{\gamma_{oil}} = 9.6 \text{ meters of oil}$$

$\Rightarrow p_A - p_B = (9.6\ m) \times (7848\ N/m^3) = 75340.8\ N/m^2 =$ **0.768 $kg(f)/cm^2$**

Example. A differential U – tube is used to measure the change in pressure between two points in a pipeline which carries oil of relative density 0.8. The lower part of the U – tube contains mercury of relative density 13.6. There is an increase in elevation between the two points of 0.26 m. If z_1 = 0.60 m, z_2 = 0.73 m and h_m = 0.13 m, calculate the difference in pressure.



Figure

Solution.

Specific weight of oil, γ_{oil} = (specific gravity of oil, S_{oil}) x (specific weight of water, γ_w)
= 0.8 x 9810 $N\ m^{-3}$ = 7848 $N\ m^{-3}$

Specific weight of mercury, γ_m = (specific gravity of mercury, S_m) x (specific weight of water, γ_w)
= 13.6 x 9810 $N\ m^{-3}$ = 133416 $N\ m^{-3}$

Let the pressure at A be p_A $N\ m^{-2}$

Pressure intensity at C, $p_C = p_A + \gamma_{oil} z_1 = p_A + (7848 \text{ x } 0.60) = p_A + 4708.8$

Pressure intensity at C' = Pressure intensity at C
(since, C and C' lie in the same horizontal plane)
$= p_A + 4708.8$

Pressure intensity at D, p_D = Pressure intensity at C' - $\gamma_m h_m$
$= (p_A + 4708.8) – (133416 \text{ x } 0.13)$
$= p_A + 4708.8 – 17344 = p_A – 12635$

Pressure intensity at B, p_B = Pressure intensity at D - $\gamma_{oil} z_2$
$= (p_A – 12635) – (7848 \text{ x } 0.73) = p_A – 18364.3$

Pressure difference between points A and B in pipeline,

$p_A - p_B = 18364.3\ N\ m^{-2}$

Example. Calculate the value of ($p_A - p_B$) if the manometer in Figure above has exactly the same readings but now carries (a) oil of relative density 0.88 (b) fresh water.

Solution.

(a) ***When the pipeline carries oil of relative density 0.88***

Specific weight of oil, γ_{oil} = (specific gravity of oil, S_{oil}) x
(specific weight of water, γ_w)
= 0.88 x 9810 $N\ m^{-3}$ = 8632.8 $N\ m^{-3}$

Specific weight of mercury, γ_m = (specific gravity of mercury, S_m) x
(specific weight of water, γ_w)
= 13.6 x 9810 $N\ m^{-3}$ = 133416 $N\ m^{-3}$

Let the pressure at A be $p_A\ N\ m^{-2}$

Pressure intensity at C, $p_C = p_A + \gamma_{oil}\, z_1 = p_A + (8632.8 \text{ x } 0.60) = p_A + 5179.7$

Pressure intensity at C' = Pressure intensity at C
(since, C and C' lie in the same horizontal plane)
$= p_A + 5179.7$

Pressure intensity at D, p_D = Pressure intensity at C' - $\gamma_m h_m$
$= (p_A + 5179.7) - (133416 \text{ x } 0.13)$
$= p_A + 5179.7 - 17344 = p_A - 12164$

Pressure intensity at B, p_B = Pressure intensity at D - $\gamma_{oil} z_2$
$= (p_A - 12164) - (8632.8 \text{ x } 0.73) = p_A - 18466$

Pressure difference between points A and B in pipeline,

$p_A - p_B = 18466\ N\ m^{-2}$

(b) ***When the pipeline carries fresh water***

Let the pressure at A be $p_A\ N\ m^{-2}$

Pressure intensity at C, $p_C = p_A + \gamma_w z_1 = p_A + (9810 \times 0.60) = p_A + 5886$

Pressure intensity at C' = Pressure intensity at C
(since, C and C' lie in the same horizontal plane)
$= p_A + 5886$

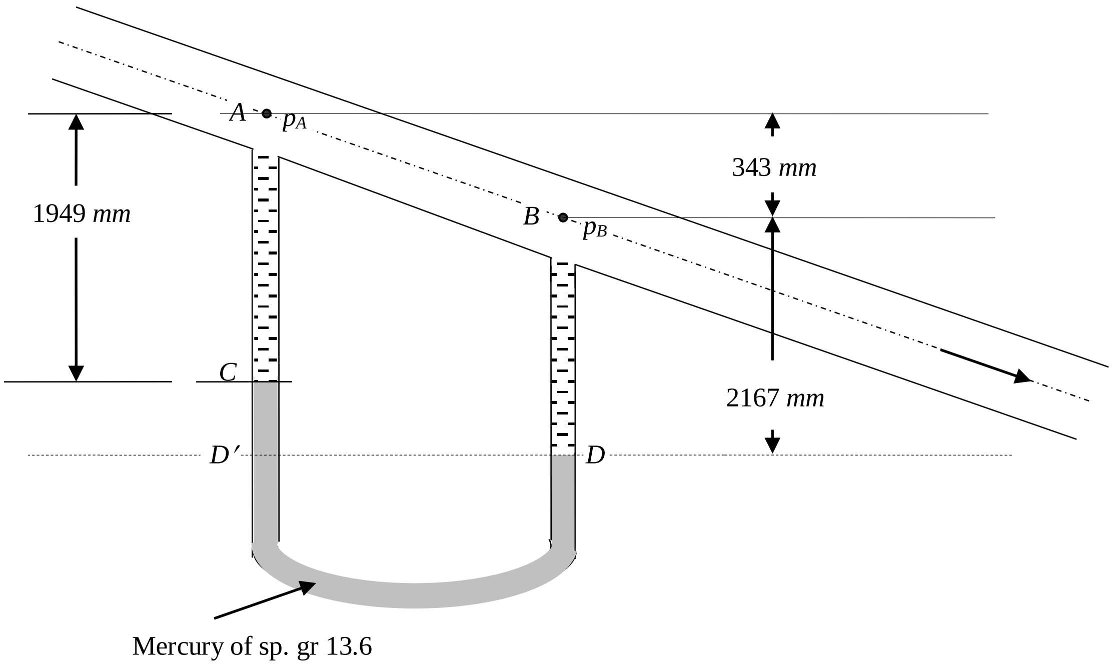
Pressure intensity at D, p_D = Pressure intensity at C' - $\gamma_m h_m$
$= (p_A + 5886) - (133416 \times 0.13)$
$= p_A + 5886 - 17344 = p_A - 11458$

Pressure intensity at B, p_B = Pressure intensity at D - $\gamma_{oil} z_2$
$= (p_A - 11458) - (9810 \times 0.73) = p_A - 18619$

Pressure difference between points A and B in pipeline,

$p_A - p_B = 18619\ N\ m^{-2}$

Example. A U – tube manometer has readings as shown in Figure below. The pipe liquid is water, and the manometer liquid is mercury. Calculate (p_A – p_B).



Figure

Solution.

Specific weight of mercury, γ_m = (specific gravity of mercury, S_m) x

(specific weight of water, γ_w)
$= 13.6 \text{ x } 9810\ N\,m^{-3} = 133416\ N\,m^{-3}$

Let the pressure at A be $p_A\ N\,m^{-2}$

Pressure intensity at C, $p_C = p_A + (\gamma_w \text{ x } 1.949) = p_A + (9810 \text{ x } 1.949)$
$= p_A + 19120$

Pressure intensity at D′ = Pressure intensity at C +
$\{\gamma_m \text{ x } (2.167 + 0.343 - 1.949)\}$
$= (p_A + 19120) + \{133416 \text{ x } (2.167 + 0.343 - 1.949)\}$
$= p_A + 93966$

Pressure intensity at D = Pressure intensity at D'
(since, D and D' lie in the same horizontal plane)
$= p_A + 93966$

Pressure intensity at B, p_B = Pressure intensity at $D - (\gamma_w \text{ x } 2.167)$
$= (p_A + 93966) - (9810 \text{ x } 2.167) = p_A + 72708$

Pressure difference between points A and B in pipeline,

$p_A - p_B = -\ 72708\ N\,m^{-2}$

Problem 8. Vessels A and B contain water under pressures of 274.68 kN/m^2 and 137.34 kN/m^2 respectively. What is the deflection of mercury in the differential gage shown in the accompanying figure?

Solution.

Let us express all pressures in terms of equivalent column of water of specific gravity 1.0.

Let the pressure head in vessel A be denoted as $\frac{p_A}{\gamma_w}$, where p_A is the intensity of pressure at A equal to 274.68 kN/m^2 and γ_w is the specific weight of water equal to 9810 N/m^3. Hence,

Pressure head at A = $\frac{p_A}{\gamma_w} = \frac{(274.68x1000)N}{9810N/m^3}$ = 28 m of water

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Pressure head at C = Pressure head at A = 28 m of water (as both points A and C lie in the same horizontal plane)

Pressure head at D = Pressure head at C + x m of water = (28 + x) m of water

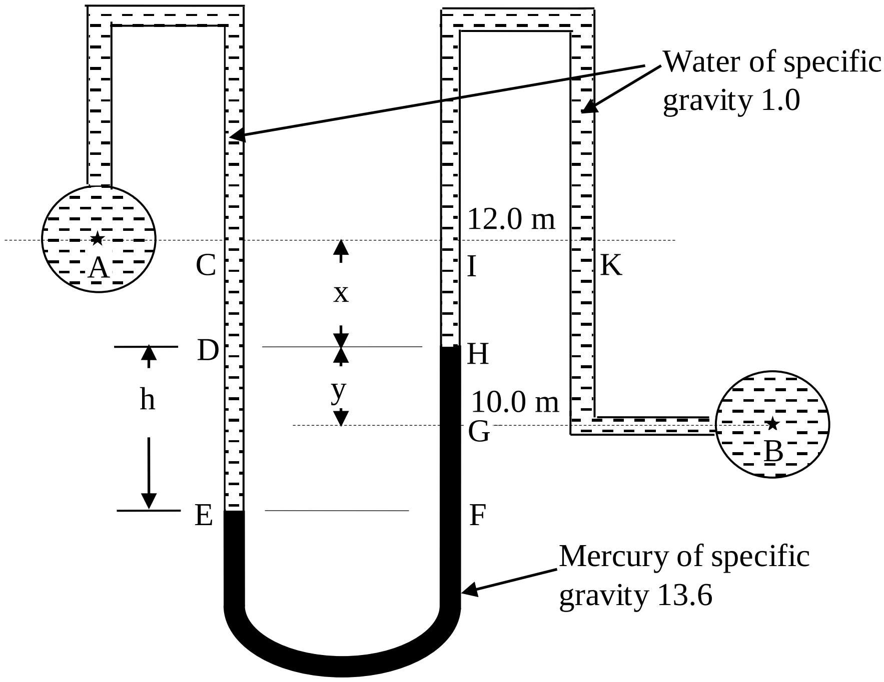

Figure 16 – Problem 8

Pressure head at E = Pressure head at D + h m of water
= {(28 + x) + h} m of water

Pressure head at F = Pressure head at E = {(28 + x) + h} m of water (as both E and F lie in the same horizontal plane)

Pressure head at G = Pressure head at F –
(h – y) m of mercury of specific gravity 13.6 expressed in equivalent column of water of specific gravity 1.0
= [{(28 + x) + h} – (h – y) (13.6 / 1.0)] m of water

Pressure head at H = Pressure head at G –
y m of mercury of specific gravity 13.6 expressed in equivalent column of water of specific gravity 1.0
= [{(28 + x) + h} – {(h – y) (13.6 / 1.0)} – {y (13.6 / 1.0)}] m of water

Pressure head at I = Pressure head at H – x m of water

= [{(28 + x) + h} – {(h – y) (13.6 / 1.0)} – {y (13.6 / 1.0)} – x] m of water

Pressure head at K = Pressure head at I
= [{(28 + x) + h} – {(h – y) (13.6 / 1.0)} – {y (13.6 / 1.0)} – x] m of water
(as both I and K lie in the dame horizontal plane)

Pressure head at B = Pressure head at K + (12.0 – 10.0) m
= [{(28 + x) + h} – {(h – y) (13.6 / 1.0)} – {y (13.6 / 1.0)} – x + {(12.0 – 10.0)}] m of water
= [28 + x + h – 13.6 h + 13.6 y – 13.6 y – x + 2.0] m of water
= [30 – 12.6 h] m of water

$$= \frac{p_B}{\gamma_w} = \frac{(137.34 x 1000) N/m^2}{9810 N/m^3} = 14 \text{ m of water}$$

$\therefore \frac{p_B}{\gamma_w}$ = 14 m of water = [30 – 12.6 h] m of water

→ h = deflection of mercury = **1.270 m**

Problem 9. The tank in the accompanying figure contains oil of specific gravity 0.75. Determine the reading of pressure gage A in (a) kg(f)/cm^2 and (b) kN/m^2.

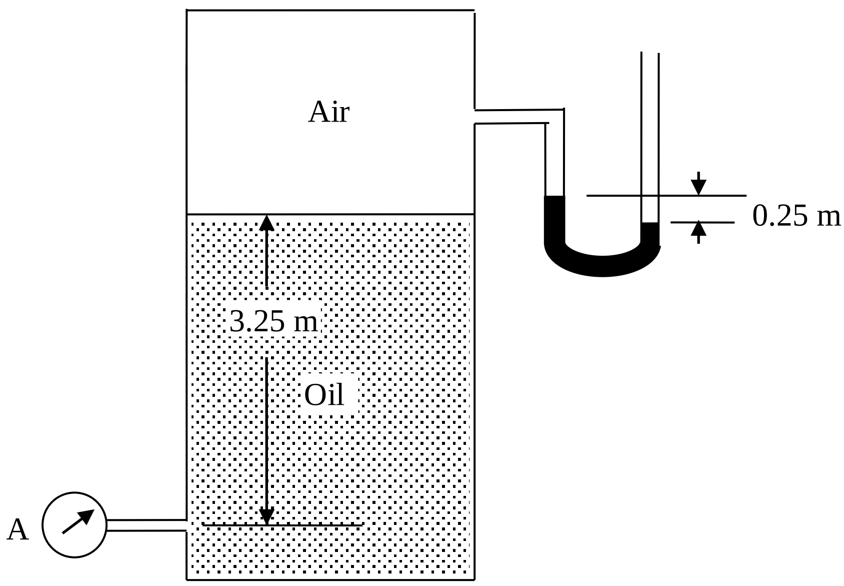

Figure 17 – Problem 9

Solution.

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Let us express all pressures in terms of equivalent column of oil of specific gravity 0.75.

The U – tube measures the pressure exerted by the air overlying the oil. As the level of mercury in the left limb of the manometer is lower compared to the level of mercury in the right limb, it is understood that the overlying air is exerting a negative pressure (i.e., pressure below atmospheric, or gage negative pressure). The magnitude of negative pressure exerted by air is 0.25 m of mercury, i.e., the pressure head due to air lying above the oil column expressed in equivalent column of oil of specific gravity 0.75 is $\frac{(-0.25m)S_m}{S_{oil}}$ = $\frac{(-0.25m)13.6}{0.75}$ = - 4.533 m of oil.

Hence, pressure head at A = Negative pressure head due to overlying air column + Pressure head due to oil column of 3.25 m
= - 4.533 + 3.25 = - 1.283 m of oil

Let the pressure intensity at A be represented as p_A. We have,

$p_A = \gamma_{oil} h_A = (0.75 x 9810 N/m^3)(-1.283m)$ = - 9439.6 N/m^2
= - (9439.6 / 9.81) kg(f) / (100 cm)^2
= **- 0.0962 kg(f)/cm^2**

Problem 10. In the left hand tank shown in Figure 18, the air pressure is – 0.23 *m* of mercury. Determine the elevation of the gage liquid in the right hand column at A, if the liquid in the right hand tank is water.

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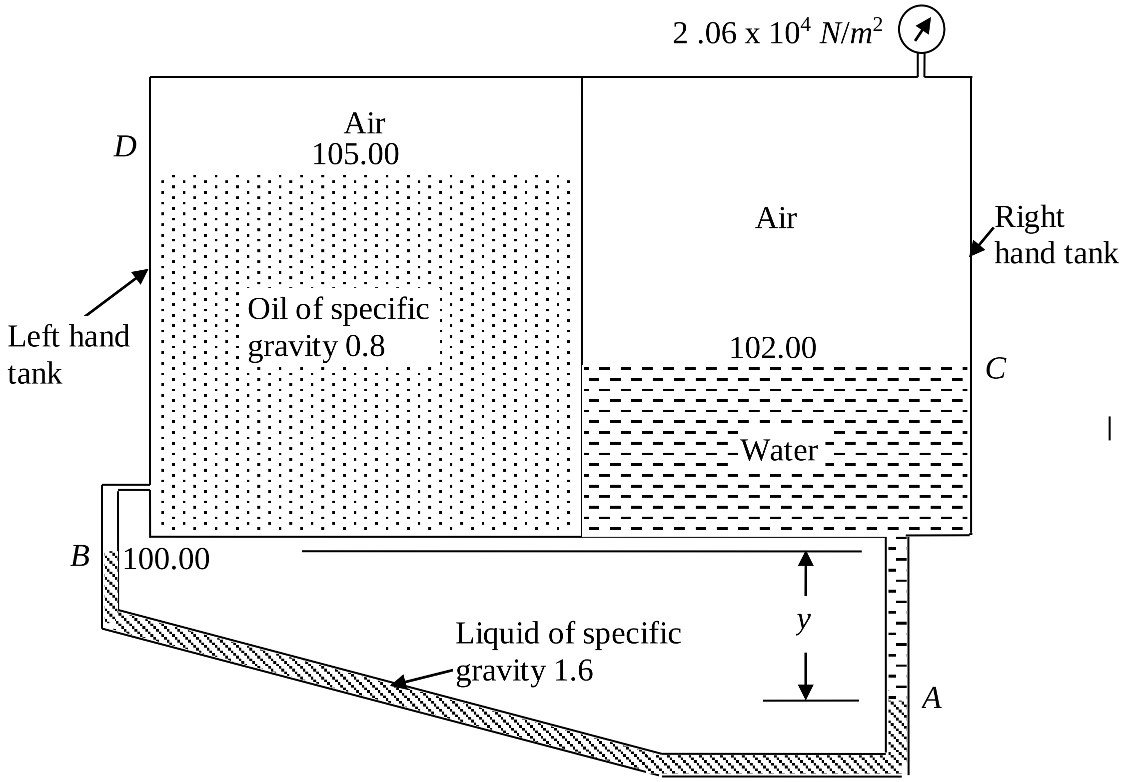

Figure 18 – Problem 10

Solution.

Let us express all pressures in terms of N/m^2.

Pressure head due to air overlying oil in left hand tank = - 0.23 m of mercury

Let the pressure intensity due to this pressure head of – 0.23 m of mercury be represented as p_D.

$p_D = \gamma_m.h_m$
where γ_m = specific weight of mercury = 133416 N/m^3
h_m = equivalent column of mercury to exert a pressure intensity of p_D
= - 0.23 m of mercury (given)
Hence, p_D = (133416 N/m^3) x (- 0.23 m) = - 30685.7 N/m^2

Pressure intensity at point B = Pressure intensity at point D + pressure intensity due to column of oil between D and B
= - 30685.7 N/m^2 + $\gamma_{oil}\ h_{oil}$
where γ_{oil} = specific weight of oil = 0.8 x 9810 N/m^3 = 7848 N/m^3

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h_{oil} = height of oil column in left hand tank (i.e., vertical distance between points B and D)
= 105.00 – 100.00 = 5.00 m

Hence, p_B = - 30685.7 N/m^2 + (7848 N/m^3) x (5.00 m)
= - 30685.7 N/m^2 + 39240 N/m^2
= 8554.3 N/m^2

Pressure intensity at point A = Pressure intensity at point B + Pressure intensity due to column of liquid of specific gravity 1.6 vertical depth y m below point B

$p_A = p_B + \gamma_{liquid}.y$
= 8554.3 N/m^2 + {(1.6 x 9810 N/m^3) x y}
= [8554.3 + 15696 y] N/m^2

Pressure intensity at point C = Pressure intensity at point A – Pressure intensity due to column of water between points A and C

$p_C = p_A - \gamma_w h_w$
= [8554.3 + 15696 y] – (9810) x {y + (102.00 – 100.00)}
= [8554.3 + 15696 y] – 9810 y – 19620
= - 11065.7 + 5886 y
= pressure exerted by air overlying water column in right hand tank
= 2 .06 x 10^4 N/m^2
$\Rightarrow y$ = 5.3798 m

Hence, elevation of gage liquid of specific gravity 1.6 in the right hand column at A = Elevation of point B – y
= 100.00 – 5.3798
= 94.6202 m

Problem 11. As show in Figure 19, the pipe and connection B are full of oil of specific gravity 0.9 under pressure. If the U-tube contains mercury, find the elevation of point A in meters.

Solution.

Let us express all pressures in terms of N/m^2.

Pressure intensity at A, p_A = 3 x 10^5 N/m^2

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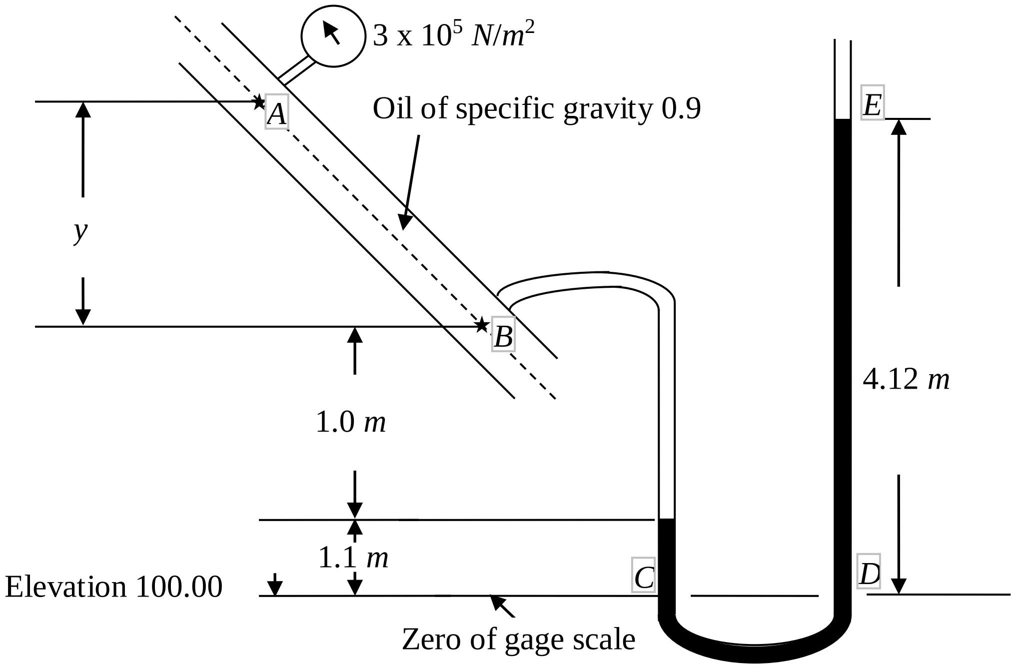

Figure 19 – Problem 11

Pressure intensity at *B*, $p_B = p_A$ + pressure intensity due to *y* meters of oil of specific gravity 0.9

$$= 3 \times 10^5 + \gamma_{oil}\, y$$
$$= 3 \times 10^5 + (0.9 \times 9810)y$$
$$= [3 \times 10^5 + 8829y]\ N/m^2$$

Pressure intensity at *C*, $p_C = p_B$ + pressure intensity due to 1 *m* of oil column of specific gravity 0.9 + pressure intensity due to 1.1*m* of gage liquid of specific gravity 13.6

$$= [3 \times 10^5 + 8829y] + \gamma_{oil}\,(1.0) + \gamma_m\,(1.1)$$
$$= [3 \times 10^5 + 8829y] + (0.9 \times 9810)\,(1.0) + (13.6 \times 9810)\,(1.1)$$
$$= [3 \times 10^5 + 8829y] + 8829 + 146757.6$$
$$= 455586.6 + 8829y$$

Pressure intensity at *D*, $p_D = p_C = 455586.6 + 8829y$ (since, both points *D* and *C* lie in the same horizontal plane)

Pressure intensity at *E*, $p_E = p_D$ – pressure intensity due to 4.12 *m* of gage liquid in right limb of *U* - tube

$= 455586.6 + 8829y - \gamma_m\ (4.12)$
$= 455586.6 + 8829y - (13.6 \times 9810)\ (4.12)$
$= 455586.6 + 8829y - 549673.92$
$= -\ 94087.32 + 8829y$
$= 0$ (since, point E corresponds to free surface of gage liquid in the right limb of the U-tube and hence is open to atmosphere)

Therefore, we have, $y = 10.657\ m$

Elevation of point A = 100.00 + 1.1 + 1.0 + 10.657 = **112.757 *m***

Problem 12. Neglecting the friction between the piston A and the gas tank, find the gage reading at B in meters of water. Assuming gas and air to be of constant specific weight and equal to 5.501 N/m^3 and 12.267 N/m^3 respectively.

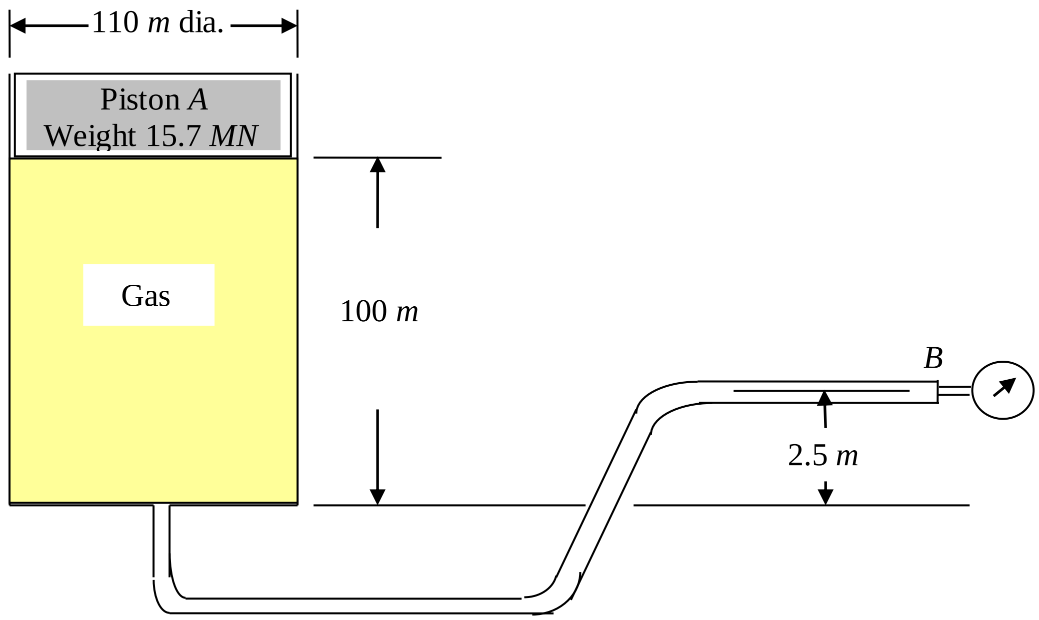

Figure 20 – Problem 12

Pressure intensity at the base of piston = $\dfrac{\text{Weight of piston}}{\text{Area of piston}}$

$$= \frac{(15.7 \times 10^6)N}{\frac{\pi}{4}(110m)^2} = 1651.4\ N/m^2$$

Pressure intensity at bottom of gas tank = Pressure intensity at the base of piston + Pressure intensity due to the gas column of 100 m

Pressure due to gas column of 100 m =

Specific weight of gas x height of column of gas
= (5.501 N/m^3) x (100 m)
= 550.1 N/m^2

$\therefore$ Pressure intensity at bottom of gas tank = 1651.4 N/m^2 + 550.1 N/m^2
= 2201.5 N/m^2

Pressure intensity at B, p_B = Pressure intensity at bottom of gas tank –
(5.501 N/m^3) x (2.5 m)
= 2201.5 N/m^2 – 13.75 N/m^2
= 2187.8 N/m^2

Equivalent column of water, h_w, corresponding to a pressure intensity of 2187.8 N/m^2 is given by

$$\boldsymbol{h_w = \frac{p_B}{\gamma_w} = \frac{2187.8\,N/m^2}{9810\,N/m^3}} = \textbf{0.223 m of water}$$