CHAPTER- 1
FLUID PROPERTIES

INTRODUCTION

- Matter
  - Solid state
  - Fluid state
    - Liquid state
    - Gaseous state

The same matter may exist in any one of these three states namely, solid, liquid and gaseous. For example, water exists in any of these three states under natural conditions. It exists in a solid state as ice and in a gaseous state as water vapour.

The solids, liquids and gases exhibit different characteristics. Why?

Because of their different molecular structure. Matter consists of vast number of molecules. These molecules are separated by empty space. The molecules tend to move continuously within the matter and they have an attraction for each other. When the distance between molecules becomes very small, there is a force of repulsion which pushes the molecules apart.

In solids, the molecules are closely spaced; in liquids, the spacing between the molecules is relatively large and in gases, the space between the molecules is still larger. Hence, in a given volume solid contains large number of molecules, a liquid contains relatively less number of molecules and a gas contains much less number of molecules.

Differences between Solids and Fluids

(1) Spacing between molecules

Solids – Molecules are closely spaced – force of attraction between molecules is large – very little movement of molecules within the mass of solid – solids possess compact and rigid form.

Liquids – molecules are widely spaced – force of attraction between molecules is relatively less – molecules can move freely within the mass of liquid – force of attraction is sufficient to keep the liquid together in a definite volume.

Gases – Molecules are very widely spaced – force of attraction between molecules is very less – molecules enjoy greater freedom to move freely such that they fill the entire volume of the vessel in which it is contained.

Fluids (Liquid and gases) – possess greater mobility (freedom to move) and large spacing of molecules.

Mechanical analysis of fluids – fluid is considered to be continuum.

What is continuum?

The matter is considered to be continuously distributed with no voids or empty spaces.

How this assumption of continuum in fluids is justifiable?

Because ordinarily the fluids involved in most of the engineering problem here large number of molecules and the distance between them are small.

(2) Ability to resist external forces

Solids – can resist tensile, compressive and shear forces upto a certain limit.

Fluids – have no or very little tensile strength (i.e., no ability or very little ability to resist tensile forces).

- can resist compressive forces only when contained in a vessel.
- cannot resist shear stress - undergoes deformation.

- When subjected to a shear force, as long as the shear force is applied, the fluid undergoes continuous deformation – this characteristic of fluid gives them the property to **<u>change shape</u>** or to **<u>flow</u>** – fluids do offer resistance to shearing (or tangential) stresses between adjacent fluid layers – this results in opposing the movement of one layer of fluid over the adjacent layer of fluid – the magnitude of shear stress to which a fluid is subjected to depends on the rate of deformation of the fluid element – when a fluid is at rest no shear force can exist in it.

Differences between gases and liquids

Gases – can be compressed easily under the action of external pressure – when the external pressure is removed, the gases tend to expand indefinitely.

Liquids – Incompressible under ordinary conditions – a liquid has a free surface.

What is free surface of a liquid?

A surface from which all pressure except atmospheric pressure is removed.

Definition of a Fluid

A substance which is capable of flowing – has no definite shape of its own – takes the shape of the vessel in which it is stored – undergoes deformation under the action of even a small magnitude of shear force and the deformation continues as long as the shear force continues to act on the fluid.

A fluid is a substance that

(a) is essentially incompressible
(b) has a viscosity that always decreases with temperature
(c) cannot remain at rest when subjected to a shearing stress
(d) cannot be subjected to shear forces

Definition of a Liquid

- A fluid which has a definite volume – the volume of a liquid varies only slightly with temperature and pressure – liquids are generally incompressible under ordinary conditions – any liquid forms a free surface separating it from the atmosphere overlying it.

Definition of a Gas

- A fluid which is compressible and it has no definite volume – always expands until its volume is equal to the volume of the container in which it is filled – undergoes significant variations in its volume and pressure even for a slight change in temperature.

Definition of Vapour

- A gas whose temperature and pressure are such that it is very near its liquid state. Example : Steam

Distinguish between Ideal fluids and real fluids.

Ideal fluids – have no viscosity and surface tension and are incompressible – ideal fluids do not have any resistance as they flow – in practice, there are no ideal fluids, but the fluids such as air, water, etc, which have low viscosity may be considered as ideal fluids.

Practical (or) Real fluids

- Fluids which actually exist in nature – real fluids have viscosity, surface tension and compressibility – they experience a certain amount of resistance when they flow.

Mass density of a fluid

Mass density or Specific mass of a fluid is defined as the mass which the fluid pressure per unit volume. It is denoted by Greek symbol 'ρ' (rho). The SI unit of mass density is $kg\ (mass)/m^3$.

Mass density of water at 4°C is 1000 $kg\ (mass)/m^3$

Factors affecting mass density of a fluid

A fluid is composed of molecules. Any molecule has a certain mass. Hence, the mass density of a fluid is proportional to the number of molecules present in unit volume of fluid.

Mass density depends upon the temperature of the fluid. As the temperature increases, the molecular mobility increases and the spacing between molecules increases. This results in only fewer molecules to exist in a unit volume of fluid. Therefore, the mass density of a fluid decreases with increase in temperature.

When a fluid (gas) is pressurized, a large number of molecules get packed in a given volume. Hence, it is to be expected that with increase in pressure, the mass density of fluid will increase.

Specific weight of a fluid

Specific weight or Weight density of a fluid is the weight that the fluid possesses per unit volume. It is denoted by the Greek symbol 'γ' (gama). In SI units, specific weight is expressed in N/m^3.

Specific weight of water at 4°C is 9810 N/m^3.

Relationship between mass density ρ and specific weight γ:

We know that weight of a fluid is the product of mass of fluid and acceleration due to gravity. Hence, specific weight can be expressed as

γ = (mass of fluid) x (acceleration due to gravity)

Volume of fluid

$$= \frac{\text{mass of fluid}}{\text{Volume of fluid}} \times \text{acceleration due to gravity}$$

$$= \rho g$$

where g = acceleration due to gravity in m/s^2

Factors influencing specific weight

From the above relationship, it is clear that specific weight depends upon mass density and acceleration due to gravity. As the acceleration due to gravity varies from place to place, the specific weight also varies. As the mass density of fluid depends upon temperature and pressure, the specific weight also depends upon temperature and pressure.

Specific Volume

Specific volume of a liquid is defined as the reciprocal of specific weight (weight density) of liquid.

That is, specific volume of liquid = $\frac{1}{\gamma}$

where γ = specific weight of liquid

Specific volume of liquid has units of m^3 / N

Specific volume of a gas is defined as the reciprocal of mass density (specific mass) of gas.

That is, specific volume of gas = $\frac{1}{\rho}$

where ρ = mass density of gas

Specific volume of gas has units of m^3 / kg (mass)

Specific Gravity of a Fluid

Specific gravity of a fluid is defined as the ratio between the specific weight of the fluid and the specific weight of the standard fluid. Also, specific gravity of a fluid can be defined as the ratio between the mass density of the fluid and the mass density of the standard fluid.

For determining the specific gravity of a liquid, the standard liquid is pure water at 4ºC. Similarly, for determining the specific gravity of a gas, the standard gas is air or hydrogen at standard temperature and pressure.

Example 1. If 5.27 m^3 of certain oil weighs 44 *kN*, calculate the specific weight, mass density and specific gravity of the oil.

Solution.

Data given: Volume of oil, $V = 5.27\ m^3$
Weight of oil, $W = 44\ kN$

Required: Specific weight of oil, γ
Mass density of oil, ρ
Specific gravity of oil

Specific weight of oil, γ = (Weight of oil) / (Volume of oil)
$= W / V$
$= 44\ kN / 5.27\ m^3$
$= \mathbf{8.349\ kN/m^3}$ (or) $\mathbf{8349\ N/m^3}$

Mass density of oil, ρ = Specific weight of oil / acceleration due to gravity
$= (8349\ N/m^3) / (9.81\ m/s^2)$
$= \mathbf{851.07\ kg\ (mass) / m^3}$

Specific gravity of oil = Mass density of oil / Mass density of water
$= \{851.07\ kg\ (mass) / m^3\} / \{1000\ kg\ (mass) / m^3\}$
= **0.851** (no unit)

Also, specific gravity of oil
= specific weight of oil / specific weight of water
$= (8349\ N/m^3) / (9810\ N/m^3)$
= **0.851** (no unit)

Example 2. The specific gravity of a liquid is 3.0. What are its specific weight, specific mass, and specific volume?

Data given: Specific gravity of liquid = 3.0

Required: Specific weight of liquid, γ
Specific mass (or) mass density of liquid, ρ
Specific volume of liquid, ϖ

Specific weight of liquid, γ = (specific gravity of liquid) x (specific weight of water)

= 3.0 x 9810 N/m^3
= **29430** $\boldsymbol{N/m^3}$

Specific mass (or mass density), ρ
= (Specific weight of liquid) / (acceleration due to gravity)
= (29430 N/m^3) / (9.81 m/s^2)
= **3000** $\boldsymbol{kg(mass) / m^3}$

Specific volume, ϖ = volume per unit weight of liquid
= 1 / specific weight of liquid
= 1 / (29430 N/m^3)
= **3.398 x** $\boldsymbol{10^{-5}\ m^3/N}$

VISCOSITY

It is the property of the fluid by which it offers resistance to its flow. It is the property by which the fluid offers resistance to the movement of one layer of fluid over the other layer of fluid.

What are the physical phenomena which are responsible for the property of viscosity?

Cohesion (inter-molecular attraction)
Exchange of momentum between molecules of adjacent fluid layers

How the above factors contribute to the viscosity?

When the fluid is considered to flow in layers with one layer of molecules over the other layer of molecules, the effects of cohesion and exchange of momentum between molecules of adjacent layers appear as shearing stresses between the moving layers of fluid. These shearing stresses offer resistance to the free flow of fluid.

Let us consider two large plates which are placed parallel to each other as shown in Figure 1. Let the two plates be placed a small distance Y apart. Let the space between the two plates be filled with a fluid. Let the lower plate be fixed. Let the upper plate be moved parallel to the lower plate with a velocity V under the application of a shear force F. The shear force F acts parallel to the surface area A of the upper plate. Fluid particles in contact with the bottom surface of the upper moving plate would also move with the same velocity V as that of the plate, while fluid particles in contact with the top surface of the lower fixed plate would not move and be stationary (i.e., at rest or zero velocity). If the velocity of the upper plate V is not too great and as the distance between the two plates Y is small, it can be assumed that the variation in velocity of fluid particles in the space Y between the two plates is linear with zero velocity at the level of lower fixed plate and a velocity V at the level of the upper plate. Let v be the velocity of fluid particles at any distance y from the lower fixed plate. The variation in v is uniform with distance y from the lower plate.

From experiments on large variety of fluids, it is observed that the shear stress to which the fluid particles adhering to the bottom surface of the upper moving plate is subjected to is proportional to the ratio of the velocity V of the moving flat plate and the distance Y between the two parallel plates. That is,

$$\frac{F}{A} \alpha \frac{V}{Y} \qquad \ldots\ldots \qquad (1)$$

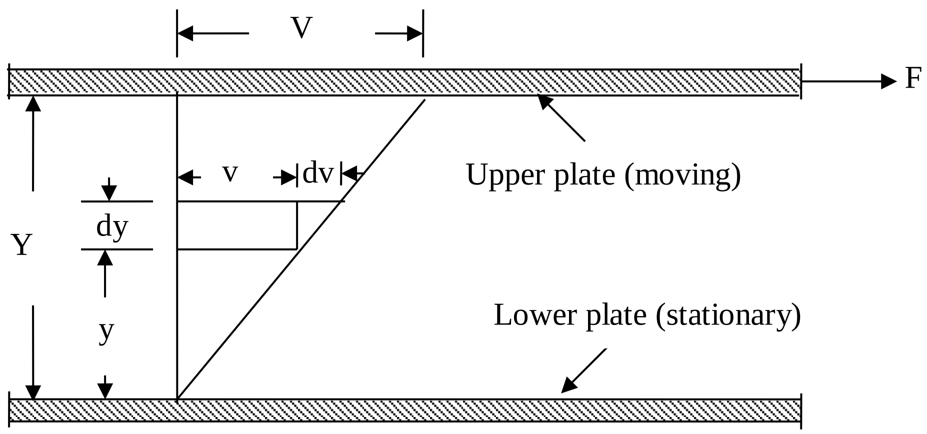


Figure 1 – Fluid motion between two parallel plates

From similar triangles, the ratio $\frac{V}{Y}$ can be replaced by the velocity gradient $\left(\frac{dv}{dy}\right)$. The velocity gradient represents the rate of angular deformation of the fluid.

Now, equation (1) becomes

$$\frac{F}{A} \alpha \left(\frac{dv}{dy}\right) \qquad \ldots\ldots \qquad (2)$$

Let us represent the shear stress $\left(\frac{F}{A}\right)$ by the symbol τ (tau). Introducing a constant of proportionality μ (mew), equation (2) becomes

$$\tau = \mu\left(\frac{dv}{dy}\right) \qquad \ldots\ldots \qquad (3)$$

Equation (3) gives the shear stress τ between any two thin layers of flowing fluid. Equation (3) is called the ***Newton's law of viscosity***. The constant of proportionality μ is called the ***coefficient of viscosity*** or the ***dynamic viscosity*** or simply the ***viscosity of the fluid***. From equation (3), we have,

$$\mu = \frac{\tau}{\left(\frac{dv}{dy}\right)} \qquad \ldots\ldots \qquad (4)$$

Newton's law of viscosity relates (a) intensity of pressure and rate of angular deformation **(b) shear stress and rate of angular deformation** (c) shear stress, viscosity and temperature (d) viscosity and rate of angular deformation

Define dynamic viscosity of the fluid.

The dynamic viscosity μ of the fluid is defined as the shear stress required producing unit rate of angular deformation.

SI units of viscosity μ:

The SI unit of shear stress τ is N/m^2. The SI units of velocity dv is m/s. The SI units of distance dy is m. Hence, the SI unit of viscosity is

$$\frac{N/m^2}{\left(\frac{m/s}{m}\right)} = Ns/m^2$$

The mass units of viscosity can be derived as below.

The mass unit of *Newton* is *kg* (*mass*).m/s^2. Hence, replacing N by *kg* (*mass*).m/s^2 in the above expression, we have

$$\frac{\dfrac{kg(mass)m/s^2}{m^2}}{\left(\dfrac{m/s}{m}\right)} = kg(mass)/m.s$$

Viscosity is also expressed in terms of ‘poise’.

1 poise = 0.1 N.s/m^2 (or)
1 N.s/m^2 = 10 poise
1 centipoise = one-hundredth of a poise = 0.01 poise

Kinematic viscosity

Kinematic viscosity of a fluid is defined as the ratio between the viscosity μ of the fluid and mass density ρ of the fluid. Kinematic viscosity is denoted by the symbol υ (nu).

$$\upsilon = \frac{\mu}{\rho}$$

SI units of kineamtic viscosity:

The mass unit of viscosity μ is kg(mass) / m.s and the SI unit of mass density ρ is kg (mass)/m^3. Hence, υ has units of

$$\frac{\left(\dfrac{kg(mass)}{ms}\right)}{\left(\dfrac{kg(mass)}{m^3}\right)} = \frac{m^2}{s}$$

Kinematic viscosity is also expressed in terms of ‘stoke’.

1 stoke = 1 cm^2/sec = 1 x 10^{-4} m^2/s
1 centistoke = one-hundredth of a stoke = 0.01 stoke

Factors affecting dynamic viscosity of fluids

For gases, with increase in temperature, viscosity increases while for liquids, with increase in temperature, viscosity decreases? Why?

This is because of the fundamentally different intermolecular characteristics. In case of liquids, the viscosity is governed by the cohesive forces (i.e., intermolecular attraction). Once the temperature of a liquid is increased, the molecular mobility increases resulting in weakening the cohesive forces between the molecules. This weakening of cohesion results in decreasing the viscosity of liquid. In case of gases, the viscosity is governed by the molecular activity. As the temperature of a gas is increased, the molecular activity increases, thereby bringing more molecules in a given space. This increases the cohesion between molecules thereby increasing the viscosity.

The viscosity of a gas

(a) decreases with increase in temperature
(b) increases with increase in temperature
(c) is independent of temperature
(d) is independent of pressure for very high pressure intensities

At room temperature, the dynamic and kinematic viscosity of water

(a) are both greater than that of air
(b) are both less than that of air
(c) are respectively greater than and less than that of air
(d) are respectively less than and greater than that of air

What are Newtonian Fluids?

Newtonian fluids are those fluids that obey Newton's law of viscosity given by equation (3). Examples of Newtonian fluids are air, water, glycerine, kerosene, etc., For Newtonian fluids, there is a linear relationship between the shear stress τ and the resulting rate of angular deformation of fluid (velocity gradient), $\left(\frac{dv}{dy}\right)$. That is, the ratio between τ and $\left(\frac{dv}{dy}\right)$ which is the constant of proportionality μ (dynamic viscosity) does not vary with the rate of deformation.

What are Non-Newtonian Fluids?

Non-Newtonian fluids are those which do not obey Newton's law of viscosity; instead, there is a non-linear relationship between the magnitude of the applied shear stress τ and the rate of angular deformation $\left(\frac{dv}{dy}\right)$. i.e., the ratio between the shear stress τ and the rate of angular deformation $\left(\frac{dv}{dy}\right)$ is not a constant and varies with the applied shear stress.

What is a plastic substance?

A ***plastic substance*** is a non-Newtonian fluid in which an initial yield stress has to be overcome to cause a continuous deformation. Plastic substances can be divided into two classes as *ideal plastic substances* and *thixotropic substances*.

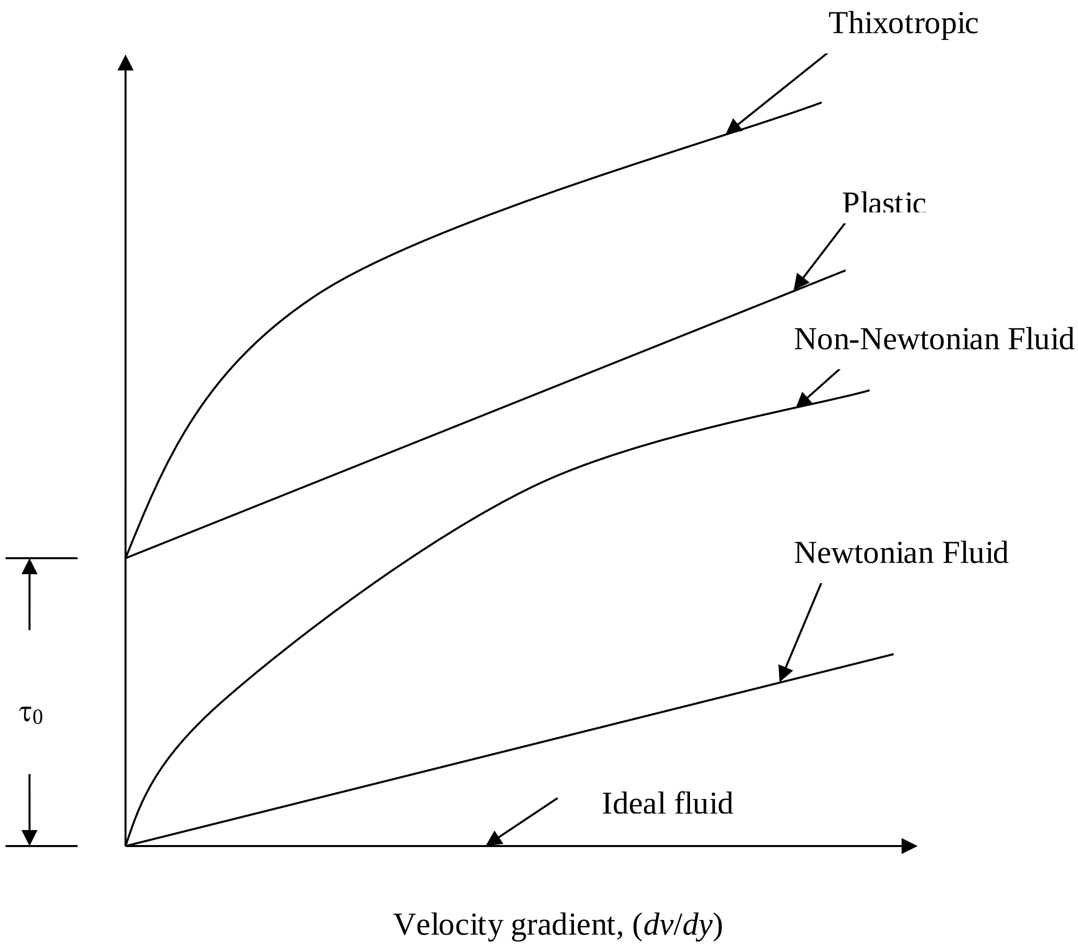
In case of an ***ideal plastic substance***, there is a definite yield stress which has to be overcome before the substance undergoes continuous deformation and the ratio between the applied shear stress and the rate of angular deformation is a constant.

In case of a ***thixotropic substance***, which is also a Non-Newtonian fluid, has a non-linear relationship between the applied shear stress and the rate of angular deformation, after overcoming an initial yield stress. Example: Printer's ink.

What is an ideal fluid?

It is the fluid which has zero viscosity, i.e, the ratio between the shear stress and rate of deformation is zero, or in other words, the shear stress is always zero regardless of the rate of deformation of the fluid (motion of fluid).

Figure below shows the diagrammatic representation of the Newtonian, non-Newtonian, ideal plastic, thixotropic and ideal fluids.



Consider the following fluids: (1) Blood (2) Glycerine (3) Molasses (4) Slurry of clay in water (5) Kerosene Among these, non-Newtonian fluids would include (a) 2, 4 and 5 (b) 2, 3 and 4 **(c) 1, 3 and 4** (d) 1, 4 and 5

Example 3. A plate 2 *m* x 2 *m*, 0.25 *mm* distant apart from a fixed plate, moves at 40 *cm* s^{-1} and requires a force of 1 *N*. Determine the dynamic viscosity of the fluid in between the plates.

Data given:

Area of plate in contact with fluid, A = 2 *m* x 2 *m* = 4 m^2
Distance between the fixed plate and the moving plate, y = 0.25 *mm*
= 0.25 x 10^{-3} *m*
Velocity of moving plate, v = 40 *cm* s^{-1} = 0.4 *m* s^{-1}
Force required to drag the plate at a velocity of 40 *cm* s^{-1}, F = 1 *N*

Required:

Dynamic viscosity of the fluid, μ

Solution.

As per Newton's law of viscosity,

$$\tau = \mu \frac{dv}{dy}$$

As the distance between the plates is very small, the variation in velocity from the fixed plate to the moving plate can be assumed to be linear. Hence, we can replace the term (dv/dy) in the above equation by (v/y). Therefore, the above equation becomes

$$\tau = \mu \frac{v}{y}$$

$$\Rightarrow \mu = \frac{\tau}{\left(\frac{v}{y}\right)}$$

$\tau = F/A$ = 1 *N* / 4 m^2 = 0.25 *N* m^{-2}
v/y = 0.4 *m* s^{-1} / (0.25 x 10^{-3} *m*) = 1600 s^{-1}
Hence, μ = (0.25 *N* m^{-2}) / (1600 s^{-1}) = 1.5625 x 10^{-4} *N s* m^{-2}

Example 4. At a certain point in oil, the shear stress is 0.2 *N* m^{-2} and the velocity gradient is 0.21 s^{-1}. If the mass density of oil is 950 *kg* m^{-3}, find the kinematic viscosity.

Data given:

Shear stress, $\tau = 0.2\ N\ m^{-2}$
Velocity gradient, $dv/dy = 0.21\ s^{-1}$
Mass density of oil, $\rho = 950\ kg\ m^{-3}$

Required:

Kinematic viscosity, ν

Solution.

As per Newton's law of viscosity,

$$\tau = \mu \frac{dv}{dy}$$

$\Rightarrow 0.2\ N\ m^{-2} = \mu \times (0.21\ s^{-1})$
$\Rightarrow \mu = (0.2\ N\ m^{-2}) / (0.21\ s^{-1}) = 0.95238\ N\ s\ m^{-2}$

Kinematic viscosity, $\nu = \frac{\mu}{\rho} = (0.95238\ N\ s\ m^{-2}) / (950\ kg\ m^{-3})$
$= 1 \times 10^{-3}\ m^2\ s^{-1}$

We know that 1 *stoke* = $1\ cm^2\ s^{-1}$

Hence, $1 \times 10^{-3}\ m^2\ s^{-1} = 1 \times 10^{-3} \times 100^2\ cm^2\ s^{-1} = 10$ *stokes*

$\nu = 10$ *stokes*

Example 5. If the equation of a velocity profile over a plate is $v = 5y^2 + y$ (where v is the velocity in $m\ s^{-1}$), determine the shear stress at $y = 0$ and $y = 7.5\ cm$. Given the viscosity of the liquid is 8.35 *poise*.

Data given:

Velocity profile, $v = 5y^2 + y$
Viscosity of the liquid, $\mu = 8.35$ *poise* $= 8.35 \times 0.1\ N\ s\ m^{-2} = 0.835\ N\ s\ m^{-2}$

Required:

Shear stress, τ, at $y = 0$ and $y = 7.5\ cm$

Solution:

Velocity gradient, $\frac{dv}{dy} = 10y + 1$

Shear stress, $\tau = \mu \frac{dv}{dy}$

At y = 0, shear stress, $\tau_{@\, y\, =\, 0} = \mu \left(\frac{dv}{dy}\right)_{@\, y=0}$

$\left(\frac{dv}{dy}\right)_{@\, y=0}$ = (10 x 0) + 1 = 0 + 1 = 1 s^{-1}

Hence, $\tau_{@\, y\, =\, 0}$ = (0.835 $N\, s\, m^{-2}$) x (1 s^{-1}) = 0.835 $N\, m^{-2}$

At y = 7.5 *cm* = 0.075 *m*, shear stress, $\tau_{@\, y\, =\, 0.075\, m} = \mu \left(\frac{dv}{dy}\right)_{@\, y=0.075\, m}$

$\left(\frac{dv}{dy}\right)_{@\, y=0.075\, m}$ = (10 x 0.075) + 1 = 0.75 + 1 = 1.75 s^{-1}

Hence, $\tau_{@\, y\, =\, 0.075\, m}$ = (0.835 $N\, s\, m^{-2}$) x (1.75 s^{-1}) = 1.461 $N\, m^{-2}$

Example 6. If the equation of velocity distribution over a plate is given by $v = 2y - y^3$ where v is the velocity in *m/s* at a distance y in *m* above the plate, what is the velocity gradient at the boundary and at 75 *mm* and 150 *mm* from it? Also determine the shear stress at these points if the absolute viscosity μ = 8.6 *poise*?

Solution.

Data given: velocity distribution, $v = 2y - y^3$
Absolute viscosity, μ = 8.6 *poise* = 8.6 x 0.1 Ns/m^2
= 0.86 Ns/m^2

Required:

Velocity gradient at the boundary of the plate, i.e., $\frac{dv}{dy}$ at y = 0 m from plate

Velocity gradient at y = 75 *mm* from plate, i.e., $\frac{dv}{dy}$ at y = 0.075 *m* from plate

Velocity gradient at y = 150 *mm* from plate, i.e., $\frac{dv}{dy}$ at y = 0.150 *m* from plate

Velocity gradient, $\frac{dv}{dy} = \frac{d}{dy}(2y - y^3) = 2 - 3y^2$

$\frac{dv}{dy}$ at y = 0 m from plate = 2 – 3 (0)^2 = 2 – 0 = **2 s^{-1}**

$\frac{dv}{dy}$ at y = 0.075 m from plate = 2 – 3(0.075)^2 = 2 – 0.016875 = **1.983 s^{-1}**

$\frac{dv}{dy}$ at y = 0.150 m from plate = 2 – 3(0.150)^2 = 2 – 0.675 = **1.325 s^{-1}**

Hence, shear stress, τ, at y = 0 m from plate is given by

$$\tau\,(@\ y = 0) = \mu\left(\frac{dv}{dy}\right)_{@\ y=0} = (0.86\ Ns/m^2) \text{ x } (2\ s^{-1}) = \mathbf{1.72\ N/m^2}$$

$$\tau\,(@\ y = 0.075\ m) = \mu\left(\frac{dv}{dy}\right)_{@\ y=0.075\,m} = (0.86\ Ns/m^2) \text{ x } (1.983\ s^{-1})$$

$$= \mathbf{1.705\ N/m^2}$$

$$\tau\,(@\ y = 0.150\ m) = \mu\left(\frac{dv}{dy}\right)_{@\ y=0.15\,m} = (0.86\ Ns/m^2) \text{ x } (1.325\ s^{-1})$$

$$= \mathbf{1.1395\ N/m^2}$$

Example 7. As shown in Figure, a cubical block of 20 cm side and of 20 kg weight is allowed to slide down along a plane inclined at 30° to the horizontal on which there is a film of oil having a viscosity 2.16 x 10^{-3} N s m^{-2}. What will be the terminal velocity of the block if the film thickness is 0.025 mm?

Data given:

Dimensions of cubical block = 20 *cm* x 20 *cm* x 20 *cm*
Area of cubical block in contact with oil, A = 20 *cm* x 20 *cm* = 400 cm^2 = 400 x 10^{-4} m^2
Note: 1 *cm* = 10^{-2} *m*
Weight of cubical block, W = 20 kg_f = 20 x 9.81 = 196.2 N
Inclination of the plane with the horizontal, θ = 30°
Viscosity of oil, μ = 2.16 x 10^{-3} $N\ s\ m^{-2}$
Thickness of oil film, y = 0.025 *mm*

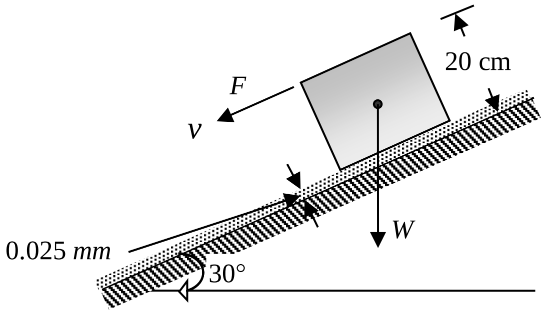


Figure: Cubical block sliding down along the inclined plane

Required:

Terminal velocity of the block, v = ?

Solution:

Let the drag force (shear force) causing the sliding of the block down the inclined plane be F. This force acts parallel to the inclined plane.

The weight, W, of the block acts in the vertical downward direction through the centre of gravity, G, of the block.

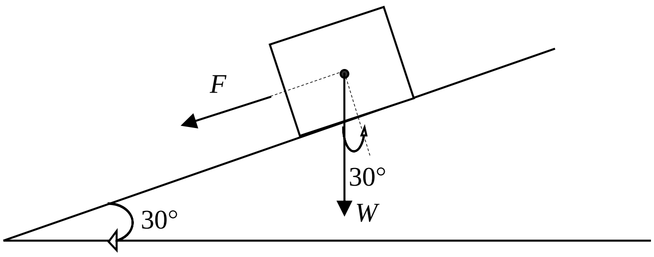


Figure: Free body diagram (Example)

$$F = W \sin 30° = 196.2 \sin 30° = 98.1\ N$$

Shear stress, $\tau = \dfrac{F}{A} = \dfrac{98.1\,N}{0.04\,m^2} = 2452.5\ N\,m^{-2}$

As per Newton's law of viscosity,

$$\tau = \mu \frac{dv}{dy}$$

As the distance between the inclined plane and the bottom face of the sliding cubical block is very small (y = 0.025 mm), the variation in velocity from the inclined plane (velocity of oil on the surface of the inclined plane = 0) to the bottom face of the moving block (velocity of oil is maximum on the bottom face plate) can be assumed to be linear. Hence, we can replace the term (dv/dy) in the above equation by (v/y). Therefore, the above equation becomes

$$\tau = \mu \frac{v}{y}$$

$$\Rightarrow 2452.5\ N\ m^{-2} = (2.16 \times 10^{-3}\ N\ s\ m^{-2}) \times \frac{v}{(0.025 \times 10^{-3}\ m)}$$

$$\Rightarrow v = 28.38\ m\ s^{-1}$$

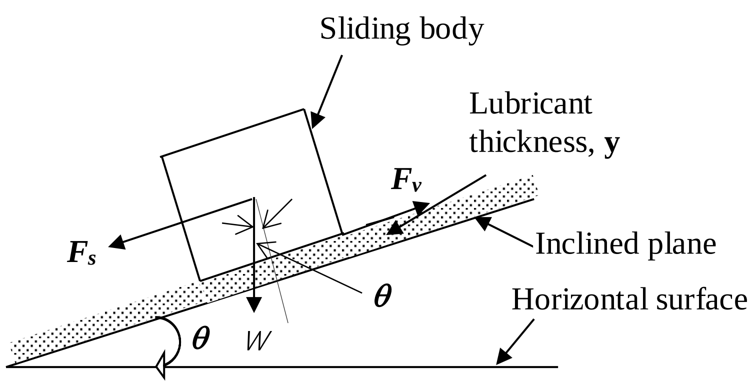
Example 8. A body weighing 441.45 N with a flat surface area of 0.093 m^2 slides down lubricated inclined plane making a 30° angle with the horizontal. For a viscosity of 0.1 Ns/m^2 and body speed of 3 m/s, determine the lubricant film thickness.

Solution.

Data given: Weight of body, W = 441.45 N
Surface area of body, A = 0.093 m^2
Inclination of plane with horizontal, θ = 30°
Viscosity of lubricant, μ, in between the inclined plane surface and the flat surface area of the body sliding down the inclined plane = 0.1 Ns/m^2
Speed of body, v = 3 m/s

Required:

Lubricant film thickness, y



For dynamic equilibrium of the body sliding down the lubricated inclined plane,

Force due to sliding, F_s = Resisting force due to viscosity of lubricant on the inclined surface, F_v

F_s acts parallel to the inclined plate surface through the center of gravity of the body.

$F_s = W \cos(90° - \theta) = W \sin\theta = 441.45 \sin 30° = 441.45 \times 0.5 = 220.725\ N$

The force due to viscosity of lubricant can be determined using the Newton's law of viscosity, $\tau = \mu.\frac{dv}{dy}$

where τ = shear stress at the flat surface area of body in contact with the lubricant

$= F_v / A = (220.725\ \text{N}) / (0.093\ \text{m}^2) = 2373.387\ N/m^2$

μ = viscosity of the lubricant = 0.1 Ns/m^2

$\frac{dv}{dy} = \frac{v}{y}$ = (assuming the velocity gradient to vary linearly with the distance y from the inclined surface)

Velocity gradient at the base of the body, $\frac{v}{y} = \frac{3}{y}$, where y is the lubricant film thickness

$$\tau = \mu.\frac{dv}{dy}$$

$\Rightarrow 2373.387\ N/m^2 = (0.1\ Ns/m^2)\ \{(\frac{3}{y})s^{-1}\}$

$\Rightarrow y = 0.000126\ m = 0.126\ mm$

Example 9. A hydraulic lift consists of a 50 *cm* diameter ram and slides in a cylinder of diameter 50.015 *cm* while the annular space is being filled up with oil having kinematic viscosity of 0.025 $cm^2\ s^{-1}$ and specific gravity of 0.85. If the rate of travel of the ram is 9.15 $m\ min^{-1}$, find the frictional resistance when 3.85 *m* of ram is engaged in cylinder.

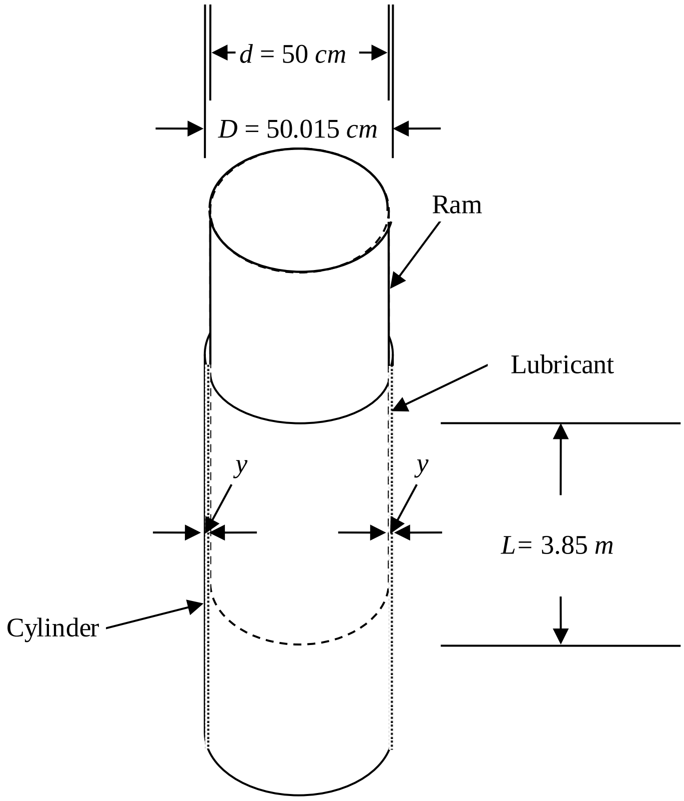


Figure: Hydraulic Lift with ram and cylinder

Data given:

Diameter of cylinder, D = 50.015 cm = 0.50015 m
Diameter of ram, d = 50 cm = 0.50 m
Thickness of annular space between the outer cylinder and the inner ram = $\frac{(D-d)}{2} = \frac{(0.50015-0.50)}{2} = \frac{0.00015}{2}$
= 0.000075 m
Kinematic viscosity of oil, υ = 0.025 cm^2/s = 0.025 x 10^{-4} m^2/s
Specific gravity of oil = 0.85
Rate of travel of ram, v = 9.15 m/min = 9.15 m / (1 x 60 s)
= 0.1525 m/s
Length of ram engaged in cylinder, i.e., length of ram in contact with oil, L = 3.85 m

Required:

Frictional resistance (due to viscosity of oil in the annular space between the cylinder and the ram), F_v

As per Newton's law of viscosity, we have,

$$\tau = \mu . \frac{dv}{dy}$$

where τ = shear stress at the outer peripheral surface of the ram in contact with the oil in the annular space between the ram and the outer cylinder

$$= \frac{\text{(Force due to viscosity of oil, } F_v \text{, or the frictional resistance)}}{\text{Area of outer peripheral surface of ram in contact with oil, } A}$$

A = (Circumference of ram) x (length of ram in contact with cylinder, L)
= $(\pi d)L$
= π x 0.50 x 3.85 = 6.0476 m^2

μ = viscosity of oil = $\upsilon\rho$

where υ = kinematic viscosity of oil = 0.025 x 10^{-4} m^2/s

ρ = mass density of oil = (specific gravity of oil) x (mass density of water)
= 0.85 x {1000 $kg(mass)/m^3$} = 850 $kg(mass)/m^3$

Hence, μ = (0.025 x 10^{-4} m^2/s) x (850 $kg(mass)/m^3$) = 0.002125 Ns/m^2

Assuming the velocity gradient $\frac{dv}{dy}$ to be linear in the annular space between the ram and the outer cylinder, we have $\frac{dv}{dy} = \frac{v}{y}$, where v is the velocity of travel of ram = 0.1525 m/s at a distance y = 0.000075 m measured from the inner wall of the outer cylinder; hence, $\frac{v}{y} = \frac{0.1525 m/s}{0.000075 m}$ = 2033.33 s^{-1}

$$\tau = \mu . \frac{dv}{dy}$$

$\Rightarrow F_v$ / (6.0476 m^2) = (0.002125 Ns/m^2) x (2033.33 s^{-1}) $\quad\quad \Rightarrow F_v$ = **26.13 *N***

Example 10. Determine the torque and power required to run a 15 *cm* long and 5 *cm* diameter shaft running at the rate of 500 *rpm* in a 5.1 *cm* diameter concentric bearing flooded with oil of dynamic viscosity 100 *centipoise*.

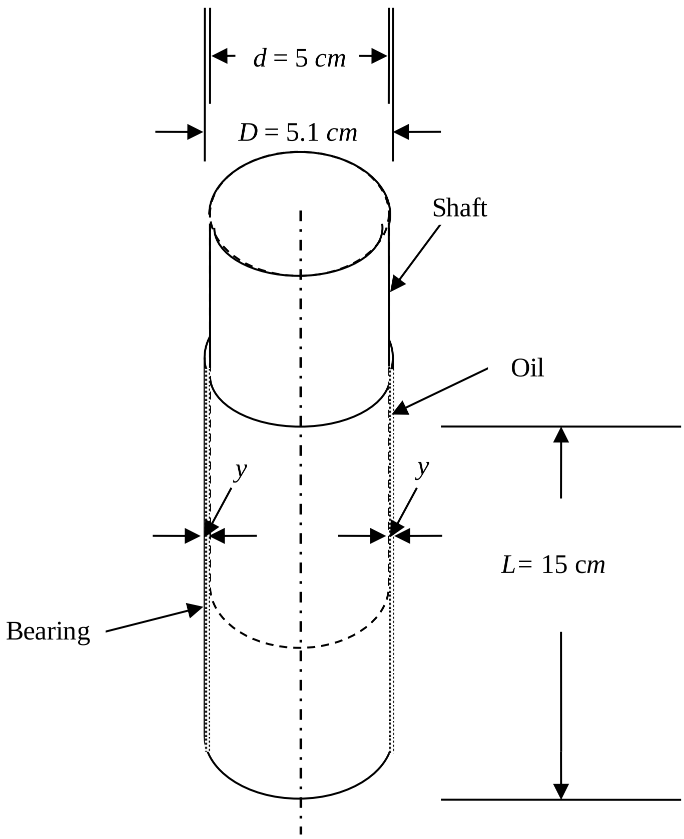


Figure: Shaft rotating inside a concentric bearing

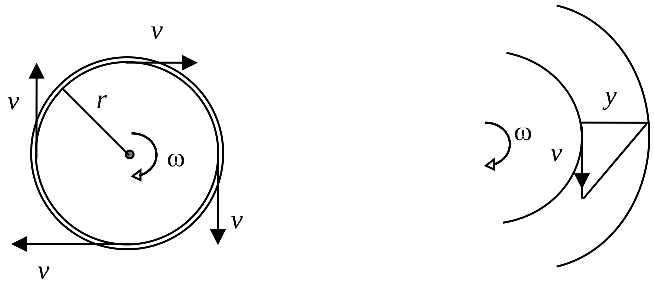


Figure: Plan view of shaft rotating inside a concentric bearing

Data given:

Length of shaft, L = 15 cm = 0.15 m
Diameter of shaft, d = 5 cm = 0.05 m
Rotational speed of shaft, N = 500 rpm
Diameter of concentric bearing inside which the shaft revolves, D = 5.1 cm
= 0.051 m

Viscosity of oil, μ = 100 *centipoise* = 100 x 10^{-2} *poise* = 1 *poise*

$= 0.1\ N\ s\ m^{-2}$

Required:

Torque required revolving the shaft, T
Power required revolving the shaft, P

Solution.

Angular velocity of shaft, $\omega = 2\pi N / 60 = 2\pi \text{ x } 500\ rpm / 60 = 52.39\ rad\ s^{-1}$
Circumferential velocity or peripheral velocity or velocity of shaft tangential to its circumference, $v = r\omega$, where r is the radius of the shaft.

Radius of shaft, $r = d/2 = (0.05\ m) / 2 = 0.025\ m$

Hence, peripheral velocity of shaft, $v = 0.025\ m \text{ x } 52.39\ rad\ s^{-1} = 1.31\ m\ s^{-1}$

Thickness of annular gap between the shaft and the bearing, $y = (D\text{-}d) / 2$
$= (0.051 – 0.05) / 2$
$= 0.001 / 2 = 0.0005\ m$

As per Newton's law of viscosity,

$$\tau = \mu.\frac{dv}{dy}$$

As the thickness of annular gap between the shaft and the bearing is very small, it can be assumed that the velocity gradient is linear. Hence, the term (dv / dy) in the above expression can be replaced by the term (v / y). The equation becomes

$$\tau = \mu\frac{v}{y}$$

$\Rightarrow \tau = (0.1\ N\ s\ m^{-2}) \text{ x } \{(1.31\ m\ s^{-1}) / (0.0005\ m)\} = 262\ N\ m^{-2}$

This is the shear stress exerted on the peripheral surface of the rotating shaft in contact with oil filled in the annular gap between the shaft and the concentric bearing.

Area over which the shear stress is distributed, A =
(circumference of the shaft) x (length of shaft in contact with oil)
$= (2\pi r) \text{ x } L = (2\pi) \text{ x } (0.025\ m) \text{ x } (0.15\ m) = 0.02356\ m^2$

Peripheral force on this area, $F = \tau A = (262\ N\ m^{-2}) \text{ x } (0.02356\ m^2)$

$= 6.17\ N$

Torque required T = Peripheral force x radius of shaft = F x r

$= 6.17\ N \times 0.025\ m$

$= 0.154\ N\ m$

Total power required, P = torque x angular velocity of shaft = $T \times \omega$

$= (0.154\ N\ m) \times (52.39\ rad\ s^{-1})$

$= 8.068\ watts$

Example 11. A hydraulic lift consists of a 250 *mm* diameter ram which slides in a 250.15 *mm* diameter cylinder, the annular space being filled with oil having a kinematic viscosity of $0.025 \times 10^{-4}\ m^2/s$ and specific gravity of 0.85. If the rate of travel of the ram is 9.15 *m*/*min*, find the frictional resistance when 3.05 *m* of the ram is engaged with the cylinder.

Solution.

Data given: Diameter of cylinder, D = 250.15 *mm* = 0.25015 *m*

Diameter of ram, d = 250 *mm* = 0.250 *m*

Thickness of annular space between the outer cylinder and the inner ram = $(D - d) / 2 = (0.25015 - 0.250) / 2$

$= 0.00015 / 2 = 0.000075\ m$

Kinematic viscosity of oil, $\upsilon = 0.025 \times 10^{-4}\ m^2/s$

Specific gravity of oil = 0.85

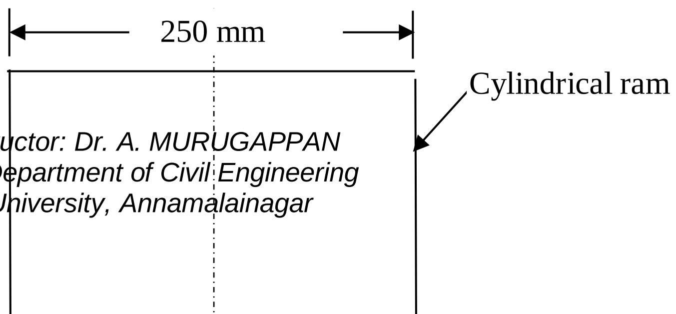
Rate of travel of ram, v = 9.15 *m*/*min* = 9.15 *m* / (1 *min* x 60 *s*)

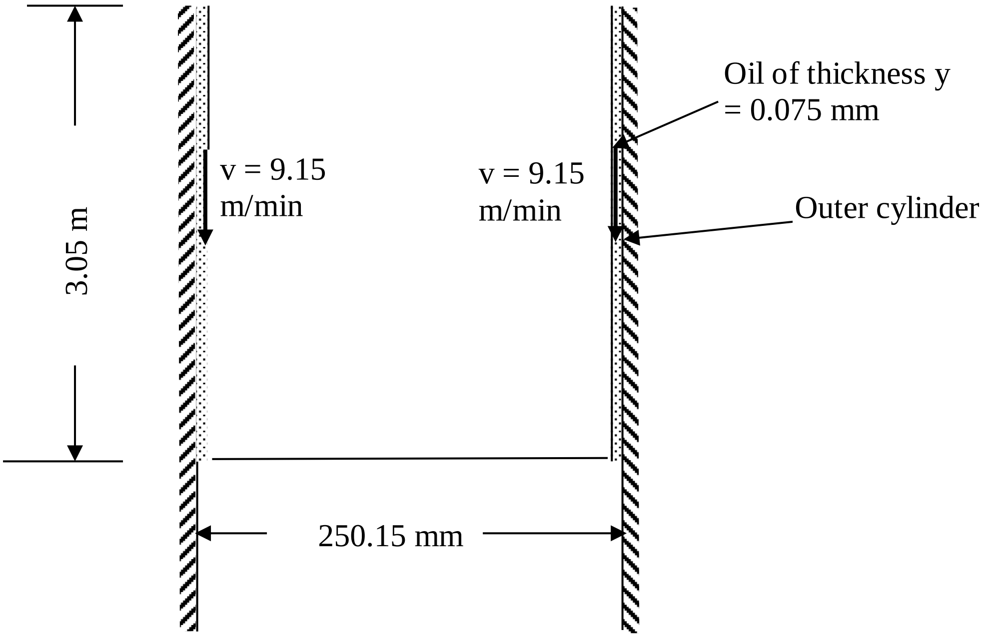
$= 0.1525\ m/s$

Length of ram engaged in cylinder, i.e., length of ram in contact with oil, L = 3.05 *m*

Required:

Frictional resistance (due to viscosity of oil in the annular space between the cylinder and the ram), F_v





As per Newton's law of viscosity, we have,

$$\tau = \mu.\frac{dv}{dy}$$

where τ = shear stress at the outer peripheral surface of the ram in contact with the oil in the annular space between the ram and the outer cylinder

$$= \frac{\text{(Force due to viscosity of oil, } F_v\text{, or the frictional resistance)}}{\text{Area of outer peripheral surface of ram in contact with oil, } A}$$

A = (Circumference of ram) x (length of ram in contact with cylinder, L)
$= (\pi d)L$
$= \pi$ x 0.250 x 3.05 = 2.3964 m^2

μ = viscosity of oil = $\upsilon\rho$

where υ = kinematic viscosity of oil = 0.025 x 10^{-4} m^2/s

ρ = mass density of oil = (specific gravity of oil) x (mass density of water)
= 0.85 x {1000 $kg(mass)/m^3$} = 850 $kg(mass)/m^3$

Hence, μ = (0.025 x 10^{-4} m^2/s) x (850 $kg(mass)/m^3$) = 0.002125 Ns/m^2

Assuming the velocity gradient $\frac{dv}{dy}$ to be linear in the annular space between the ram and the outer cylinder, we have $\frac{dv}{dy} = \frac{v}{y}$, where v is the velocity of travel of ram = 0.1525 *m/s* at a distance y = 0.000075 *m* measured from the inner wall of the outer cylinder; hence, $\frac{v}{y} = \frac{0.1525 m/s}{0.000075 m}$ = 2033.33 s^{-1}

$$\tau = \mu . \frac{dv}{dy}$$

$$\Rightarrow \frac{F_v}{2.3964\, \text{m}^2} = (0.002125\ Ns/m^2) \text{ x } (2033.33\ s^{-1})$$

$\Rightarrow F_v$ = **10.35 *N***

Example 12. A tape of 0.015 *cm* thick and 1.00 *cm* wide is to be drawn through a gap with a clearance of 0.01 *cm* on each side. A lubricant of dynamic viscosity 0.021 $N\ s\ m^{-2}$ completely fills the gap for a length of 80 *cm* along the tape. If the tape can withstand a maximum tensile force of 7.5 *N* calculate the maximum speed with which it can be drawn through the gap.

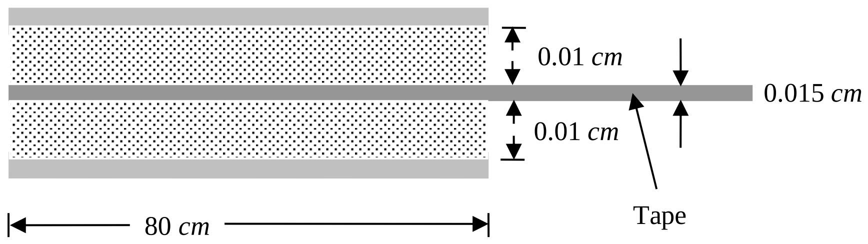


Figure Tape sliding through the gap filled with lubricant

Data given:

Clearance on each side, dy = 0.01 *cm* = 0.01 x 10^{-2} *m* = 0.0001 *m*
Dynamic viscosity of oil, μ = 0.021 $N\ s\ m^{-2}$
Width of tape (perpendicular to the plane of paper), b = 1 *cm* = 0.01 *m*
Length of tape in contact with oil, L = 80 *cm* = 0.8 *m*
As the gap is completely filled with the lubricant, both the bottom and top faces of the tape are in contact with the lubricant. Hence,

Contact area of the tape, A =
2 x (width of tape) x (length of tape in contact with lubricant)
= 2 x b x L = 2 x 0.01 *m* x 0.8 *m* = 0.016 m^2
Maximum tension the tape can withstand, F_{max} = 7.5 *N*
This is the maximum shear force acting on both the bottom and top faces of the tape. Hence, maximum shear stress, $\tau_{max} = F_{max} / A$ = 7.5 *N* / 0.016 m^2
= 468.75 $N\ m^{-2}$

As per Newton's law of viscosity,

$$\tau = \mu.\frac{dv}{dy}$$

$\Rightarrow \tau_{max} = 468.75\ N\ m^{-2} = (0.021\ N\ s\ m^{-2})\ \text{x}\ \{dv\ /\ 0.0001\ m\}$
$\Rightarrow dv = 2.23\ m\ s^{-1}$

Maximum speed with which the tape can be drawn through the gap,
$v = 2.23\ m\ s^{-1}$

VAPOUR PRESSURE

The tendency of any liquid is to evaporate or vaporize. That is, the liquid changes to the gaseous state. We know that all liquids possess a free surface. There is a continuous escaping of molecules of liquid through the free surface.

Let us consider the case of a liquid stored in a closed vessel. Let the volume of liquid in the vessel confine only partially to the volume of the vessel. Because of the tendency of the liquid, there is continuous escaping of molecules through the free surface of the liquid. As the vessel is closed, the ejected vapour molecules get accumulated in the space of the vessel over the free surface of the liquid. This accumulated vapour of the liquid exerts a partial pressure on the liquid surface. This partial pressure is known as the vapour pressure of the liquid.

Why the pressure exerted by ejected vapour on the free surface of liquid is called partial pressure?

We know that atmosphere prevails over the free surface of liquid in the closed vessel. Hence, the atmosphere is already exerting a pressure (atmospheric pressure). Atmospheric pressure is due to the pressure exerted by the constituents of the atmosphere. The major constituents of the atmosphere are Nitrogen and Oxygen. The molecules of vapour ejected through the free surface liquid become part of the atmosphere overlying the free surface of liquid in the vessel. Hence, the magnitude of atmospheric pressure is the sum of the pressures exerted individually by the different constituents of the atmosphere including the ejected vapour molecules. Therefore, the pressure exerted by ejected vapour molecules of the liquid is a part of the atmospheric pressure and so it is called the ***partial pressure***.

Factors influencing vapour pressure

Influence of temperature: With increase in temperature of a liquid, the molecular activity increases, i.e., the molecules have more freedom to move from one place to another place within the liquid. Because of increased molecular activity, more molecules get the opportunity to escape through the free liquid surface as vapour. With increase in temperature, as more vapour molecules get accumulated over the free surface of liquid, there is an increase in vapour pressure.

A liquid may start boiling even at ordinary temperature. How?

If the external absolute pressure imposed on the free surface of liquid is reduced by some means to such an extent it becomes equal to or less than the vapour pressure of the liquid, the boiling of liquid starts, whatever may be the temperature. Thus a liquid may start boiling even at ordinary temperature, if the pressure above the free surface of liquid is reduced so as to be equal to or less than the vapour pressure of the liquid at that temperature.

Influence of pressure: If at any point in a flow system the pressure of the liquid approaches vapour pressure, vapourization of liquid starts, resulting in pockets of dissolved gases and vapours.

Why mercury is chosen to be the liquid for use in the barometer measuring atmospheric pressure?

As mercury has a very low vapour pressure, it becomes an excellent fluid to be used in the barometer.

State an example for a liquid with a very high vapour pressure.

Benzene

COMPRESSIBILITY

Fluids can be compressed by the application of external force. When the external force applied on the fluid is removed, the compressed volumes of fluids expand to their original volumes. Thus fluids also possess elastic characteristics like elastic solids.

Define compressibility of a fluid.

It is quantitatively expressed as the reciprocal of the bulk modulus of elasticity *K* of the fluid.

$$K = \frac{\textit{stress}}{\textit{strain}} = \frac{dp}{\left(\frac{dV}{V}\right)} = \frac{\textit{change in pressure}}{\left(\frac{\textit{change in volume}}{\textit{original volume}}\right)} \quad \ldots\ldots \quad (6)$$

Let the original volume of fluid be V. Let the initial pressure of the fluid be p. Let the pressure of the fluid be increased by dp, i.e., the pressure of the fluid becomes ($p + dp$). As the fluid undergoes compression because of the increased pressure, the change (decrease) in volume is dV, i.e., the final volume of the fluid becomes ($V - dV$).

SI Units of Bulk Modulus of Elasticity:

Let us derive the units of bulk modulus of elasticity K.

The SI units of stress is N/m^2. Strain which is the ratio of change in volume to original volume has no units, as it is the ratio between two similar quantities. Hence, bulk modulus of elasticity has the same units as that of stress, i.e., N/m^2.

Factors influencing bulk modulus of elasticity of a fluid

What happens to the bulk modulus of elasticity of fluid with increase in pressure of fluid?

When the pressure is increased, say by Δp units, the fluid mass gets compressed, that is, the molecules of fluid come close together and the volume of fluid gets decreased, say by ΔV units. When the pressure on the fluid is further increased, say by another Δp units, the further decrease in volume of fluid is less than ΔV units. *Why?* This is because, as the molecules of fluid have already become close together, with further increase in pressure, the molecules have lesser freedom to come still closer together. Or in other words, the molecules which are held close together offer more resistance to further compression with further increase in pressure. Therefore, with increase in pressure, the compressibility of fluid decreases. As bulk modulus of elasticity is the reciprocal of compressibility of fluid, with decrease in compressibility with increase in pressure, the bulk modulus of elasticity of fluid increases.

For example, the bulk modulus of water roughly gets doubled as the pressure is increased from 1 atmosphere to 3500 atmosphere.

What is the influence of temperature on bulk modulus of elasticity of fluid?

As far as the gases are concerned, since the pressure and temperature are inter-related, as the temperature increases, pressure also increases, and hence an increase in temperature results in an increase in the bulk modulus of elasticity.

In case of liquids, with increase in temperature, there is a decrease of bulk modulus of elasticity.

In most of the problems involving flow of liquids, the effect of compressibility is neglected. That is, *the liquids are generally considered incompressible. Why?*

Liquids, in general, have very high bulk modulus of elasticity. Therefore, the change in density (or volume) with increase in pressure is very small even when the pressure exerted is very large. Therefore, in most problems involving flow of liquids, they are treated as incompressible. However, in some special cases where the change in pressure is either very large or sudden, it is necessary to consider the effect of compressibility of liquids.

In problems involving flow of gases, the compressibility of gases needs to be considered since with change in pressure, the gases undergo considerable change in their mass density.

Example 13. A fluid compressed in a cylinder has a volume of 0.01132 m^3 at a pressure of 70.3 $kg(f)/cm^2$. What should be the new pressure in order to make its volume 0.01121 m^3? Assume bulk modulus of elasticity K of the liquid as 7030 $kg(f)/cm^2$.

Solution.

Data given: Let V_1 = volume of compressed fluid = 0.01132 m^3

p_1 = pressure of compressed fluid corresponding to volume V_1 = 70.3 $kg(f)/cm^2$

$$= \frac{[70.3 kg(f)/cm^2][9.81 N]}{\left[\frac{1}{100}\right]^2 m^2} = 6896430\ N/m^2$$

Let V_2 = new volume of compressed fluid = 0.01121 m^3

Let p_2 = pressure of compressed fluid corresponding to volume V_2

As $V_2 < V_1$, $p_2 > p_1$

Bulk modulus of elasticity, $K = \dfrac{dp}{\left(\dfrac{dV}{V}\right)}$

dp = change in pressure = increase in pressure = $p_2 - p_1$

dV = change in volume = decrease in volume = $V_1 - V_2$

= 0.01132 m^3 - 0.01121 m^3

= 0.00011 m^3

V = original volume of compressed fluid = V_1 = 0.01132 m^3

$K = 7030\ kg(f) / cm^2 = \dfrac{[7030 kg(f)/cm^2][9.81 N]}{\left[\dfrac{1}{100}\right]^2 m^2} = 689643000\ N/m^2$

$K = \dfrac{dp}{\left(\dfrac{dV}{V}\right)} = \dfrac{p_2 - p_1}{\left(\dfrac{V_1 - V_2}{V_1}\right)} = \dfrac{p_2 - 6896430\ \mathrm{N/m^2}}{\left(\dfrac{0.00011\,\mathrm{m^3}}{0.01132\,\mathrm{m^3}}\right)} = 689643000\ N/m^2$

$\Rightarrow p_2 =$ ____________ N/m^2

SURFACE TENSION AND CAPILLARITY

What are the properties possessed by liquids due to molecular attraction?

Cohesion and Adhesion

What is Cohesion?

It is the attraction between molecules of the same liquid.

What is adhesion?

It is the attraction between molecules of a liquid and the molecules of a solid surface with which it is in contact.

What is the characteristic possessed by a liquid because of the property of cohesion?

The liquid has the ability to resist tensile stress because of the property of cohesion.

What is the characteristic possessed by a liquid because of the property of adhesion?

The liquid has the characteristic of sticking to the surface of another body with which it comes in contact.

Surface Tension

Let us consider a vessel containing a liquid. The liquid has a free surface (exposed to atmosphere). Consider a liquid molecule at a point *A* located inside the liquid body. The liquid molecule at point A is surrounded by other liquid molecules in all directions, so that the molecule is equally attracted on all sides. Hence the net force of attraction exerted by molecules surrounding the molecule at A is zero.

On the other hand, if we consider a liquid molecule at the free surface of liquid at a point B, it is attracted by a molecule below it while it has no molecule above it. Consequently, there is a net downward force on each molecule at free surface due to the force of attraction by molecule below it. This net downward force on each molecule in the liquid surface acts normal (perpendicular) to the liquid surface. Due to the attraction of liquid molecules below the free surface of liquid, a film or special layer seems to form on the free surface of the liquid, which is in tension. This tension enables the liquid surface to support small loads coming over it. For example, if a small needle is placed gently over the water surface, it will not sink into water but will be supported due to the tension in the water surface.

Hence, surface tension can be explained as the property by which the liquid surface film exerts a tension. It is denoted by the Greek symbol σ. Surface tension is defined as the force required to maintain unit length of the liquid film at the free surface in equilibrium.

SI Units of Surface tension

It has units of N/m

Factors influencing surface tension

We have already discussed that surface tension is due to the cohesive (inter-molecular attraction) force exerted by the molecules of liquid. When the temperature of a liquid rises, the cohesive forces get weakened and hence surface tension gets decreased.

Surface tension is also dependent on the fluid in contact with the free surface of liquid. Usually, surface tension for water quoted in contact with air varies from 0.0736 N/m at 19^0C to 0.0589 N/m at 100^0C. These values clearly show the decrease in magnitude of surface tension with increase in temperature. Many organic liquids in contact with air have values of surface tension between 0.026 N/m and 0.0304 N/m. Mercury has a value of about 0.4944 N/m at normal temperature, with the fluid in contact with mercury being air.

Surface tension of water

(a) increases with decrease in temperature
(b) decreases with decrease in temperature
(c) independent of temperature
(d) none of the above

Illustration of effect of surface tension in a liquid droplet

When a liquid droplet gets initially separated from the surface of main body of liquid, due to surface tension there is a net inward force exerted over the entire surface of droplet. This makes the droplet to contract from all sides and results in increasing the internal pressure within the droplet.

How far the contraction of droplet continues?

The contraction (reduction in size) of the droplet continues till the force due to surface tension is in balance with the internal pressure. In such equilibrium condition, the droplet attains a spherical shape (the spherical shape has the minimum surface area).

At equilibrium, the force due to surface tension balances the force due to internal pressure. Let us consider a spherical liquid droplet of radius r having internal pressure intensity p in excess of the outside pressure intensity (if the liquid droplet is considered to be present in atmosphere, the outside pressure intensity is equal to the atmospheric pressure).

If the droplet is cut into two equal halves, then the forces acting on one- half of the droplet (refer figure below) are:

i) internal pressure intensity p on the projected area πr^2 (p acts perpendicular to the projected area)

ii) Tensile force due to surface tension σ acting around the circumference $2\pi r$.

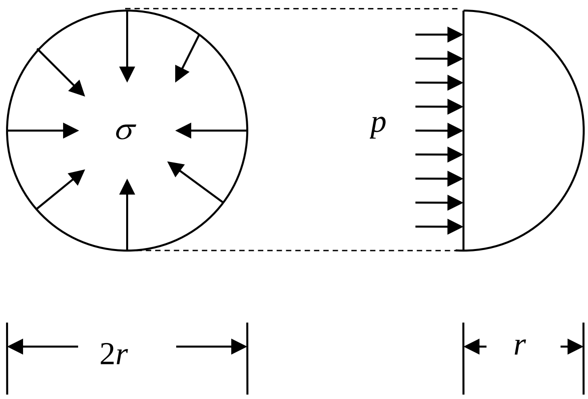


Figure – Two views of one-half of spherical droplet showing the action of surface tension and internal pressure intensity

The force due to surface tension and the force due to the internal pressure must be equal in magnitude and opposite in direction for equilibrium of the spherical droplet.

Then we have,

Fore due to surface tension = Force due to internal pressure

$\sigma(2\pi r) = p(\pi r^2)$

$$\Rightarrow\ p = \frac{2\sigma}{r} \qquad \ldots\ldots\ (7)$$

Equation (7) indicates that the internal pressure intensity increases with decrease in size of the droplet.

Pressure intensity inside a liquid jet

Let us consider a liquid jet of radius *r*, length *l* and having internal pressure *p* in excess of outside pressure intensity.

If the jet is cut into two halves along the length of the jet, then the forces acting on one-half of the jet (refer figure below) are:

1. Internal pressure intensity *p* acting on the projected area 2*rl*, normal to it (the projected area is a rectangle of dimensions 2*r* x *l*).

2. Force due to surface tension *σ* along the length of the jet on two sides (2*l*)

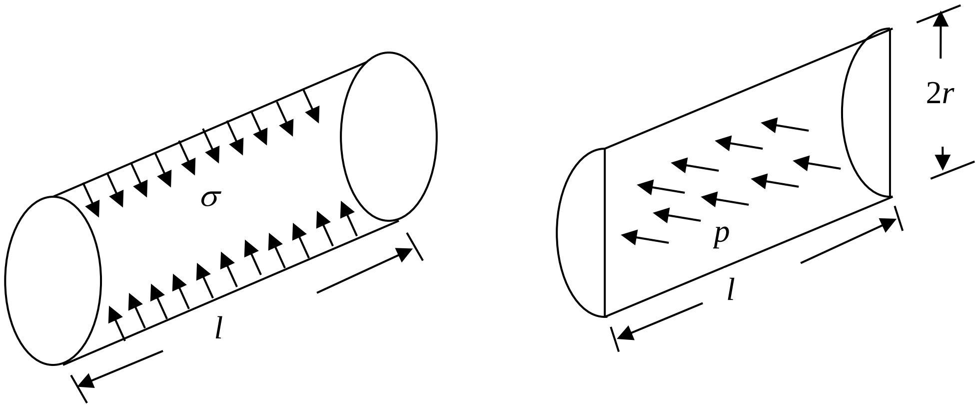


Figure: Forces on a liquid jet

For equilibrium of the liquid jet, the force due to surface tension must be equal and opposite to the force due to internal pressure intensity

$$\sigma(2l) = p(2rl)$$

$$\Rightarrow p = \frac{\sigma}{r} \qquad \ldots\ldots \quad (8)$$

Illustration of the effect of surface tension in a soap bubble

A spherical soap bubble has both its outer surface and inner surface in contact with air. Therefore, both outer and inner liquid surfaces of the soap bubble are subjected to surface tension of same magnitude.

Let us consider a spherical soap bubble of radius r having internal pressure p in excess of the outside pressure intensity. If the bubble is cut into two equal halves, then the forces acting on one half of soap bubble are:

i) Force due to surface tension acting on both outer and inner circumference of the projected area of one –half of the bubble.

ii) Internal pressure intensity on the projected area πr^2

Equating (i) and (ii)

$$2\sigma(2\pi r) = p(\pi r^2)$$

$$\Rightarrow\ p = \frac{4\sigma}{r} \qquad \ldots\ldots \quad (9)$$

Capillarity

If molecules of certain liquid possess relatively greater affinity (attraction) for solid molecules than the attraction between molecules of the concerned liquid, or in other words, the liquid has greater adhesion than cohesion, then the liquid will wet the solid surface with which it comes in contact and the liquid will tend to rise at the point of contact. Hence the liquid surface becomes concave upward and the angle of contact θ is less than 90^0.

Let us consider a glass tube of small diameter which is partially immersed in water present in a container. Water wets the surface of glass tube and it rises in the tube to some height, above the normal level of water surface in the container. The angle of contact of water with glass surface is zero.

How the rise of liquid takes place inside the glass when a liquid wets the solid boundary? It results in a decrease of pressure energy within the liquid, and hence the liquid rises in the glass tube such that the pressure of liquid within the glass column at the elevation of the surrounding liquid surface is the same as the pressure at this elevation outside the glass column.

On the other hand, if the molecules of certain liquid possess relatively lesser attraction for solid molecules than intermolecular attraction, or in other words, the liquid has greater cohesion than adhesion, then the liquid will not wet the solid surface with which it comes in contact and the liquid surface will tend to fall at the point of contact. Hence, the liquid surface is concave downward and the angle of contact is greater than 90^0.

For example, if a glass tube of small diameter is partially immersed in vessel containing mercury, as mercury does not wet the glass surface with which it is in contact, the level of mercury inside the glass tube will be lower than the adjacent mercury level in the vessel, with the angle of contact θ equal to about 130^0.

How the fall of mercury inside the glass tube partially dipped in a container with mercury takes place?

As mercury does not adhere to the solid surface, it results in creating an increase in pressure across the liquid surface. Because of this increased internal pressure, the elevation of the meniscus (curved liquid surface) in the glass tube is lowered to the level where the pressure is the same as that in the surrounding liquid.

Now, let us define the term ***capillarity***. It is defined as the phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid. The rise of liquid surface relative to the adjacent general level of liquid is known as ***capillary rise***. The fall of liquid surface relative to the adjacent general level of liquid is known as ***capillary depression***.

***SI units of capillary rise or capillary depression*:**

meters (or) mm of liquid

Derivation of expression for capillary rise (or capillary depression)

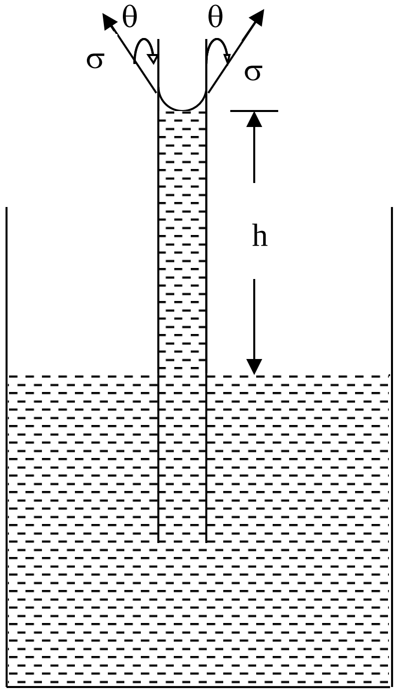
Let us consider the conditions of equilibrium of a circular glass tube of small diameter inserted in a liquid. Let the level of liquid in the glass tube has risen (or fallen) by h above (or below) the general level of liquid surface in the container in which the glass tube of radius r is partially inserted. Please see the figure below.

What are the forces acting on the mass of liquid lying above (or below) the general level of liquid surface in the container?

1. The weight of liquid column h in case of capillary rise (or) the total internal pressure in case of capillary depression.

2. Force due to surface tension σ at the surface of the liquid in the glass tube.

For the column of liquid above the general level of liquid surface to be in static equilibrium, the algebraic sum of vertical forces acting on the mass of the liquid column h must be equal to zero. As there are only two forces acting on the liquid column h, the weight of liquid column h acting in the vertical downward direction must be balanced by the upward vertical component of surface tension of liquid.



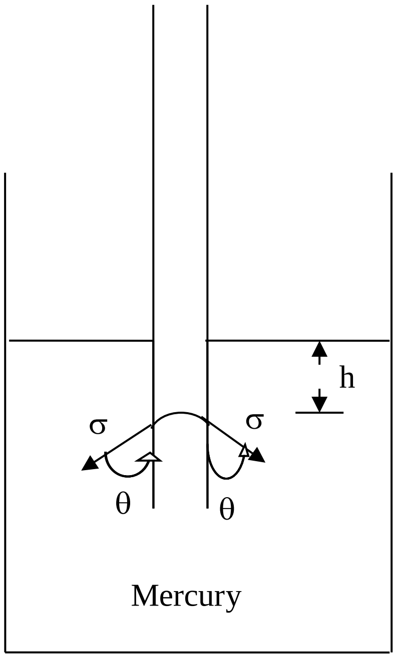


Figure – Capillary rise **Figure – Capillary depression**

Weight of liquid column *h* = (Volume of liquid column *h*) x (Specific weight of liquid, γ)

$$= \left(\pi r^2 h\right)\gamma$$

Vertical upward component due to surface tension =

(Component of σ in vertical upward direction) x (Circumference of water surface in glass tube over which σ acts)

$$= \left(\sigma \cos\theta\right) 2\pi r$$

γ is the specific weight of water, θ is the angle of contact between the liquid meniscus and the glass tube.

Equating the two forces, we have,

$$\left(\pi r^2 h\right)\gamma = \left(\sigma \cos\theta\right) 2\pi r$$

$\Rightarrow$ Capillary rise, $h = \dfrac{2\sigma \cos\theta}{\gamma r}$ …… (10)

The rise of liquid of specific weight γ in a capillary tube of radius r is given by

(a) $\dfrac{\sigma \cos \theta}{2r\gamma}$

(b) $\dfrac{2\sigma \cos \theta}{r}$

(c) $\dfrac{2\boldsymbol{\sigma} \cos \boldsymbol{\theta}}{\boldsymbol{r\gamma}}$

(d) $\dfrac{\gamma\sigma \cos \theta}{2r}$

where σ is the surface tension of the liquid and θ is the is the angle of contact between the liquid meniscus and the glass tube.

Examine the following four statements:

(i) Surface tension is due to cohesion only
(ii) Capillarity is due to cohesion only
(iii) Surface tension is due to both cohesion and adhesion
(iv) Capillarity is due to both cohesion and adhesion

Which of the above statements are true?

(a) (i) and (ii)
(b) (ii) and (iii)
(c) (i) and (iv)
(d) Only (iv)

Example 14. Find the pressure inside a water droplet having a diameter of 0.5 *mm* at 20°*C* if the outside pressure is 1.03 x 10^4 $N\ m^{-2}$ and the surface tension of water at that temperature is 0.0736 $N\ m^{-1}$.

Data given:

Diameter of water droplet, d = 0.5 *mm* = 0.5 x 10^{-3} *m*
Surface tension of water in contact with air at 20° *C* = 0.0736 N m^{-1}

Required:

Pressure inside the water droplet, *p*

Solution.

$p = 2\sigma/r$

where r = radius of water droplet = $d/2 = (0.5 \times 10^{-3}\ m) / 2 = 0.25 \times 10^{-3}\ m$

Hence, $p = (2 \times 0.0736\ N\ m^{-1}) / (0.25 \times 10^{-3}\ m) = 589\ N\ m^{-2}$

Example 15. A soap bubble 51 *mm* in diameter has an internal pressure in excess of outside pressure of 0.00021 $kg(f)/cm^2$, calculate the tension in the soap film.

Solution.

Diameter of soap bubble, $d = 51\ mm$
Radius of soap bubble, $r = d/2 = 51\ mm / 2 = 25.5\ mm = 25.5 \times 10^{-3}\ m$
Internal pressure, $p = 0.00021\ kg(f)/cm^2 = [(0.00021 \times 9.81) / (1/100)^2]\ N/m^2$
$= 20.601\ N/m^2$

$$p = \frac{4\sigma}{r}$$

$$\Rightarrow 20.601\ N/m^2 = \frac{4\sigma}{25.5 x 10^{-3} m}$$

$\Rightarrow \sigma = 0.1313\ N/m$

Example 16. If the pressure inside a droplet of water is 196.2 N/m^2 in excess of the external pressure, what is the diameter of the droplet? Given the value of surface tension of water in contact with air at 20°*C* is 0.07358 N/m.

Solution.

Data given:
Pressure inside water droplet, $p = 196.2\ N/m^2$
Surface tension of water in contact with air at 20°*C*, $\sigma = 0.07358\ N/m$

Required:
Diameter of water droplet, d = ?

$$\boldsymbol{p = \frac{2\sigma}{r}}$$

where r = radius of water droplet

$\rightarrow$ 196.2 N/m^2 = $\dfrac{\mathbf{2(0.07358}\,\boldsymbol{N/m}\mathbf{)}}{\boldsymbol{r}}$

$\rightarrow$ r = 0.00075 m

$\rightarrow$ diameter of water droplet, d = 2r = 2 x 0.00075 = 0.0015 m = **1.5 *mm***

Example 17. Find the excess pressure inside a cylindrical jet of diameter 4 *mm* than the outside pressure? The surface tension of water is 0.0736 $N\ m^{-1}$ at that temperature.

Data given:

Diameter of cylindrical jet, d = 4 *mm*
Radius of cylindrical jet, $r = d/2$ = 4/2 = 2 *mm* = 0.002 *m*
Surface tension of water (in contact with air) at 20°*C* = 0.0736 $N\ m^{-1}$

Required:

Excess pressure, p, inside the cylindrical jet than the outside pressure

$p = \sigma / r$ = 0.0736 / 0.002 = 36.8 $N\ m^{-2}$

Example 18. Compare the capillary rise of water and mercury in a glass tube of 2 *mm* diameter at 20°*C*. Given that the surface tension of water and mercury at 20°*C* are 0.0736 $N\ m^{-1}$ and 0.051 $N\ m^{-1}$ respectively. Contact angles of water and mercury are 0° and 130° respectively.
Data given:

Diameter of glass tube, d = 2 *mm* = 0.002 *m*
Radius of glass tube, r = d/2 = 0.002/2 = 0.001 *m*
Surface tension of water (in contact with air) at 20°*C*, σ_w = 0.0736 $N\ m^{-1}$
Surface tension of mercury (in contact with air) at 20°C, σ_m = 0.051 $N\ m^{-1}$
Contact angle of water = 0°
Contact angle of mercury = 130°

Required:

Capillary rise of water, h_w
Capillary rise of mercury, h_m

Solution.

Capillary rise of water, $h_w = (2\sigma_w \cos\theta) / \gamma_w r$
$= (2 \times 0.0736 \times \cos 0°) / (9810 \times 0.001) = 0.015\ m$

Capillary rise of mercury, $h_m = (2\sigma_m \cos\theta) / \gamma_m r$
$= (2 \times 0.051 \times \cos 130°) / (13.6 \times 9810 \times 0.001)$
$= -0.491 \times 10^{-3}\ m = -0.000491$ m
(*Note: the negative sign indicates capillary depression*)

Example 19. A glass tube 0.25 *mm* in diameter contains mercury column with air above mercury at 20°*C*. The surface tension of mercury in contact with air is 0.0051 *kg*(*f*)/*m*. What will be the capillary depression of mercury if angle of contact $\theta = 130°$ and specific gravity of mercury = 13.6.

Solution.

Data given:
Diameter of glass tube, $d = 0.25\ mm = 0.25 \times 10^{-3}\ m$
Surface tension of mercury in contact with air at 20°*C* = 0.0051 *kg*(*f*)/*m*
$= (0.0051 \times 9.81)\ N/m$
$= 0.050031\ N/m$
Angle of contact of mercury with glass tube, $\theta = 130°$
Specific gravity of mercury = 13.6

Required:

Capillary depression of mercury, h = ?

$$h = \frac{2\sigma\cos\theta}{\gamma r}$$

where γ = specific weight of mercury = (specific gravity of mercury) x (specific weight of water)
$= 13.6 \times 9810\ N/m^3 = 133416\ N/m^3$
r = radius of glass tube = $d/2 = (0.25 \times 10^{-3}\ m) / 2 = 0.125 \times 10^{-3}\ m$

Therefore, capillary depression, $h = \dfrac{2(0.050031 N/m)\cos 130^\circ}{(133416 N/m^3)(0.125 x 10^{-3} m)} = 0.00386\ m$

= **3.86 *mm***

Example 20. A glass tube 0.25 *mm* in diameter contains a mercury column with water above the mercury. The temperature is 20°*C* at which the surface tension of mercury in contact with water is 0.363 *N*/*m*. What will be the capillary depression of mercury? Take angle of contact θ = 130°.

Solution.

Data given:
Diameter of glass tube, d = 0.25 *mm* = 0.25 x 10^{-3} *m*
Surface tension of mercury in contact with water at 20°*C* = 0.363 *N*/*m*
Angle of contact, θ = 130°

Required:
Capillary depression of mercury, h = ?

$$h = \frac{2\sigma \cos\theta}{r(\gamma_1 - \gamma_2)}$$

where γ_1 = specific weight of mercury = (specific gravity of mercury) x (specific weight of water)
= 13.6 x 9810 N/m^3 = 133416 N/m^3
γ_2 = specific weight of water = 9810 N/m^3
r = radius of glass tube = d/2 = 0.25 *mm*/2 = 0.125 *mm* = 0.125 x 10^{-3} *m*

Hence, capillary depression, $h = \dfrac{2(0.363 N/m)\cos 130^\circ}{(0.125 x 10^{-3} m)(133416 N/m^3 - 9810 N/m^3)}$
= 0.0302 *m* = **30.2 *mm***

Example 21. Calculate the capillary rise in a glass tube of 3 *mm* diameter when immersed in water at 20°*C*. Take surface tension for water at 20°*C* as 0.0736 *N*/*m*. What will be the percentage increase in the capillary rise if the diameter of the glass tube is 2 *mm*?

Solution.

Data given:
Diameter of glass tube, d = 3 *mm* = 3 x 10^{-3} *m*
Surface tension of water at 20°*C* = 0.0.736 *N*/*m*

Required:
Percentage increase in capillary rise when the diameter of glass tube d is 2 *mm* = ?

Computation of capillary rise of water when diameter of glass tube, d, is 3 mm

$$h_1 = \frac{2\sigma \cos\theta}{\gamma r}$$

where γ = specific weight of water = 9810 N/m^3
r = radius of glass tube = $d/2$ = (3 x 10^{-3} m) / 2 = 1.5 x 10^{-3} m

Hence, $h_1 = \dfrac{2(0.0736 N/m)\cos 0^\circ}{(9810 N/m^3)(1.5 x 10^{-3} m)}$ = 0.01 m

Computation of capillary rise of water when diameter of glass tube, d, is 2 mm

$$h_2 = \frac{2\sigma \cos\theta}{\gamma r}$$

where γ = specific weight of water = 9810 N/m^3
r = radius of glass tube = $d/2$ = (2 x 10^{-3} m) / 2 = 1 x 10^{-3} m

Hence, $h_1 = \dfrac{2(0.0736 N/m)\cos 0^\circ}{(9810 N/m^3)(1 x 10^{-3} m)}$ = 0.015 m

Hence, the percentage increase in capillary rise when the diameter of glass tube is 2 mm is given by $\dfrac{(h_2 - h_1)}{h_1} x 100$ = 50%

Example 22. Show that for two vertical parallel plates t distance apart, held partially immersed in a liquid of surface tension σ and specific weight γ, the capillary rise h is given by $h = \dfrac{2\sigma \cos\theta}{\gamma t}$, where θ is the angle of contact.

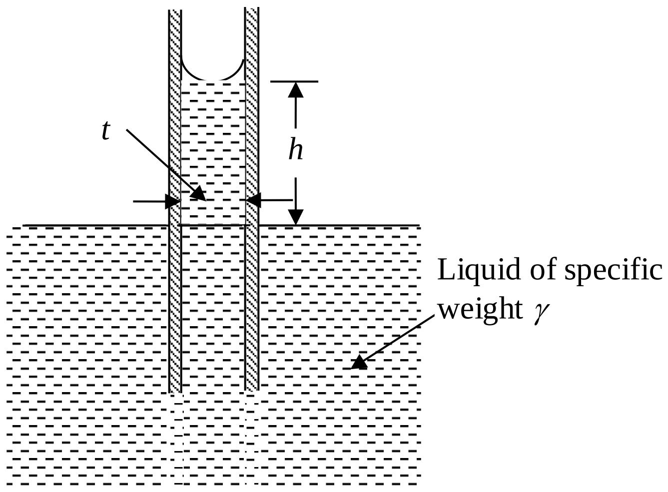
Solution.

Data given:
Distance between two parallel plates is t.
Surface tension of liquid in which the two plates are immersed is σ
Specific weight of liquid is γ
Capillary rise of liquid in the gap between the two vertical parallel plates is h
Angle of contact of liquid is θ

Required:

To show that the capillary rise is given by $\boldsymbol{h = \frac{2\sigma\cos\theta}{\gamma t}}$

Figure shows the two vertical parallel plates, separated by a distance t, immersed partially in a liquid of surface tension σ and specific weight γ.



Let us consider the conditions of equilibrium of two vertical parallel plates separated by distance t, held partially immersed in a liquid of specific weight γ. It is supposed that the level of liquid has risen by h above the general level of liquid surface when two vertical parallel plates separated by a small distance t is partially immersed in the liquid. For the equilibrium of vertical forces acting on the mass of liquid lying above the general level of liquid, the weight of liquid column h must be balanced by the force, at the surface of the liquid, due to surface tension σ. Considering unit length of the liquid column h perpendicular to the plane of paper, equating the two forces, we have,

Weight of liquid column h = Force due to surface tension
Weight of liquid column h = (specific weight of liquid, γ) x (volume of liquid column h)
= γ (distance between the two vertical parallel plates x height of liquid column, h x unit length perpendicular to the plane of paper)
= $\boldsymbol{\gamma(tx1xh)}$

Force due to surface tension = (vertical upward component of surface tension) x (length of liquid film on which surface tension acts)
= $(\sigma\cos\theta)\,(2t+2)$

Hence, we have,

$$\boldsymbol{\gamma(tx1xh) = (2t+2)\sigma\cos\theta}$$

$$\rightarrow \boldsymbol{h = \frac{2\sigma\cos\theta}{\gamma t}}$$

Example 23. Calculate the maximum capillary rise of water at 20°*C* to be expected between two vertical clean glass plates spaced 1 *mm* apart. Take σ = 0.07358 *N*/*m*.

Solution.

Data given:

Spacing between two vertical glass plates, t = 1 *mm* = 1 x 10^{-3} *m*
Surface tension of water, σ = 0.07358 *N*/*m*

Required:

Capillary rise of water, h = ?

$$\boldsymbol{h} = \frac{2\sigma\cos\theta}{\gamma t} = \frac{2(0.07358 N/m)\cos 0^\circ}{(9810 N/m^3)(1 x 10^{-3} m)} = 0.015\ m = \mathbf{15\ mm}$$

Example 24. A capillary tube having inside diameter 5 *mm* is dipped in water at 20°*C*. Determine the height of water which will rise in the tube. Take σ = 0.07358 *N*/*m* and angle of contact θ = 60°. Specific weight of water at 20°*C* = 9790 *N*/m^3.

Solution.

Data given:
Diameter of capillary tube, d = 5 *mm* = 5 x 10^{-3} *m*
Surface tension of water, σ = 0.07358 *N*/*m*
Angle of contact θ = 60°
Specific weight of water at 20°*C* = 9790 *N*/m^3

Required:
Capillary rise of water in the tube, h = ?

$$h = \frac{2\sigma\cos\theta}{\gamma r} = \frac{2(0.07358 N/m)\cos 60^\circ}{(9790 N/m^3)\left(\frac{5x10^{-3} m}{2}\right)} = 0.003\ m = \mathbf{3\ mm}$$

Example 15. By how much does the pressure in a cylindrical jet of water 4 *mm* in diameter exceed the pressure of the surrounding atmosphere if surface tension of water is 0.07358 *N*/*m*?

Solution.

Data given:

Diameter of cylindrical jet of water, d = 4 *mm* = 4 x 10^{-3} *m*
Surface tension of water, σ = 0.07358 *N*/*m*

Required:

Inside pressure in the cylindrical jet of water, p = ?

$$p = \frac{\sigma}{r} = \frac{0.07358 N/m}{\left(\frac{4x10^{-3} m}{2}\right)} = 36.75\ N/m^2$$

Example 25. Calculate the capillary effect in *mm* in a glass tube of 4 *mm* in diameter when immersed in (i) water and (ii) mercury, both at 20°*C*. The values of surface tension of water and mercury at 20°*C* in contact with air are respectively 0.07358 *N*/*m* and 0.51812 *N*/*m*. Take angle of contact for water, θ = 0° and for mercury, θ = 130°.

Solution.

Case (i) To determine the capillary rise of water in glass tube

Data given:

Diameter of glass tube, d = 4 *mm* = 4 x 10^{-3} *m*
Surface tension of water at 20°*C* in contact with air, σ = 0.07358 *N*/*m*
Angle of contact for water, θ = 0°

Capillary rise, $\boldsymbol{h = \dfrac{2\sigma\cos\theta}{\gamma r}}$

where γ = specific weight of water = 9810 N/m^3
r = radius of glass tube = $d/2$ = (4 x 10^{-3} m) / 2 = 2 x 10^{-3} m

$\therefore$ Capillary rise of water in glass tube, h = $\dfrac{\mathbf{2(0.07358}\,N/m)\cos 0^\circ}{\mathbf{(9810}\,N/m^3)(2x10^{-3}m)}$
= 0.0075 m = **7.5 *mm***

Case (ii) To determine the capillary depression of mercury in glass tube

Diameter of glass tube, d = 4 mm = 4 x 10^{-3} m
Surface tension of mercury at 20°C in contact with air, σ = 0.51812 N/m
Angle of contact for water, θ = 130°

Capillary depression of mercury in glass tube, $\boldsymbol{h = \dfrac{2\sigma\cos\theta}{\gamma r}}$

where γ = specific weight of mercury = (specific gravity of mercury) x (specific weight of water)
= 13.6 x 9810 N/m^3 = 133416 N/m^3
r = radius of glass tube = $d/2$ = (4 x 10^{-3} m) / 2 = 2 x 10^{-3} m

$\therefore$ Capillary depression of mercury in glass tube, h = $\dfrac{\mathbf{2(0.51812}\,N/m)\cos 130^\circ}{\mathbf{(133416}\,N/m^3)(2x10^{-3}m)}$
= 0.00249 m = **2.49 *mm***

Example 26. The inside diameters of the two arms of a U – tube are 1.0 mm and 1.5 mm respectively. If the U-tube is partially filled with water having surface tension of 0.0736 $N\,m^{-1}$ and zero contact angle, what will be the difference in the level of menisci between the two arms as shown in figure below.

Data given:

Diameter of left arm of U – tube, d_{left} = 1.5 mm
Diameter of right arm of U – tube, d_{right} = 1.0 mm
Surface tension of water in contact with air, σ = 0.0736 $N\,m^{-1}$
Contact angle for air-water-glass interface, θ = 0°
Radius of left arm of U – tube, r_{left} = 1.5/2 = 0.75 mm = 0.75 x 10^{-3} m
Radius of right arm of U – tube, r_{right} = 1.0/2 = 0.5 mm = 0.5 x 10^{-3} m

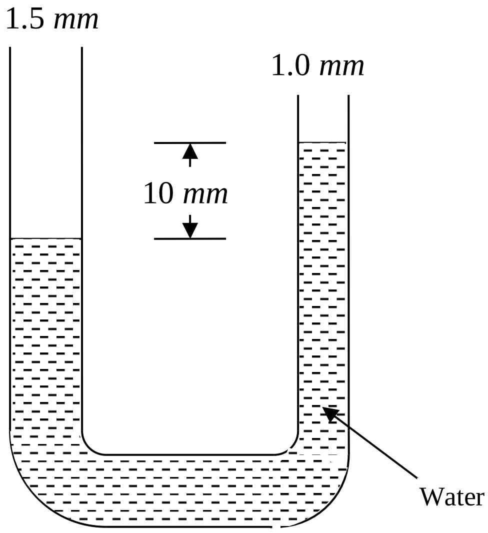


Figure U – tube containing water

Solution.

Capillary rise h in a tube of radius r is given by

$h = (2\sigma\cos\theta) / \gamma r$

Capillary rise in tube of radius $r = 0.75 \times 10^{-3}$ m is

$h_1 = (2 \times 0.0736 \times \cos 0°) / (9810 \times 0.75 \times 10^{-3}) = 0.02$ $m = 20$ mm

Capillary rise in tube of radius $r = 0.5 \times 10^{-3}$ m is

$h_2 = (2 \times 0.0736 \times \cos 0°) / (9810 \times 0.5 \times 10^{-3}) = 0.03$ $m = 30$ mm

In the U – tube, there will be a net capillary rise in the right limb of radius 0.5 *mm*. The net capillary rise is given by $(h_2 – h_1)$ = 30 *mm* – 20 *mm* = 10 *mm*.

Example 27. Air is forced through a tube of internal diameter of 1.5 *mm* immersed at a depth of 1.5 *cm* in a mineral oil having specific gravity 0.85. Calculate the unit surface energy of the oil if the maximum bubble pressure is 150 $N\ m^{-2}$

Data given:

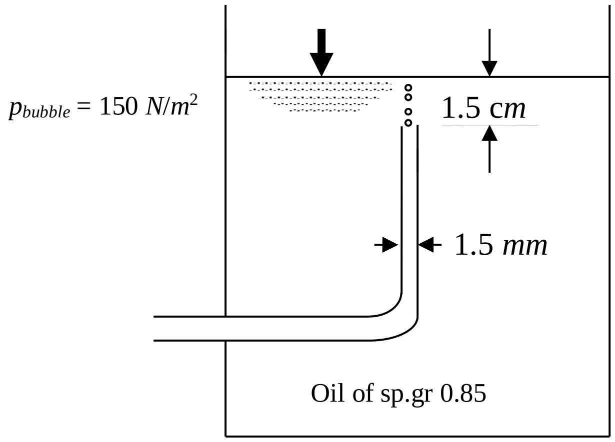


Figure Air forced through a tube immersed in mineral oil

Diameter of tube, $d = 1.5\ mm = 0.0015\ m$
Radius of tube, $r = d/2 = 0.0015/2 = 0.00075\ m$
Depth of immersion of top of tube through which air bubbles under pressure are released into the mineral oil, $h = 1.5\ cm = 0.015\ m$
Specific gravity of oil, $S = 0.85$
Specific weight of oil, $\gamma_{oil} = S\gamma_w = 0.85 \times 9810\ N\ m^{-3} = 8338.5\ N\ m^{-3}$
Maximum bubble pressure, $p_{bubble} = 150\ N\ m^{-2}$

Required:

Unit surface energy of oil, σ

Solution.

Pressure at the top of tube, where the bubbles are released into the mineral oil, due to the depth of oil of 1.5 *cm*,

$p_{oil} = \gamma_{oil}.h = (8338.5\ N\ m^{-3}) \times (0.015\ m) = 125.1\ N\ m^{-2}$

This pressure acts vertically downward on the top of tube.

Pressure of air bubbles released through the top of tube, $p_{bubble} = 150\ N\ m^{-2}$

As the bubbles are released up through the top of the tube, this pressure acts vertically upward.

Hence, effective pressure due to surface tension in each air bubble is given by

$p = p_{bubble} - p_{oil} = 150\ N\ m^{-2} - 125.1\ N\ m^{-2} = 24.9\ N\ m^{-2}$

we have, $p = 4\sigma / r$
$\Rightarrow 24.9\ N\ m^{-2} = 4\sigma / 0.00075\ m$
$\Rightarrow \sigma = 4.67 \times 10^{-3}\ N\ m^{-1}$

Example 28. Name the characteristic fluid properties to which the following phenomena are attributable: (i) rise of sap in a tree, (ii) spherical shape of a drop of liquid, (iii) cavitation, (iv) flow of jet of oil in an unbroken stream (iv) water hammer

Solution.

S. No.	*Phenomenon*	*Attributable Property*
(i)	rise of sap in a tree	Capillarity
(ii)	spherical shape of a drop of liquid	Surface tension
(iii)	cavitation	Vapour pressure
(iv)	flow of jet of oil in an unbroken stream	Viscosity
(v)	water hammer	Compressibility

Example 29. Air is introduced through a nozzle into a tank containing water to form a stream of air bubbles. The bubbles have a diameter of 2 mm. Calculate by how much the pressure of air at the nozzle must exceed that of surrounding water. Assume that surface tension of water in contact with air is 72.7×10^{-3} N m^{-1}.

Solution.

The air bubbles of diameter 2 mm are formed inside water in the tank. Each air bubble is surrounded by water medium. The pressure inside each air bubble is in excess of the pressure outside the air bubble. The excess pressure intensity inside each air bubble is given by

$$p = \frac{2\sigma}{r}$$

$$= \frac{2 \times \left(72.7 \times 10^{-3}\ Nm^{-1}\right)}{\left(\dfrac{2 \times 10^{-3}\ m}{2}\right)} = 145.4\ N\ m^{-2}$$