

10. (a) Find the equation to the sphere through the four points  
 $(0, 1, 3)$ ,  $(1, 2, 4)$ ,  $(2, 3, 1)$ ,  
 $(3, 0, 2)$ .

- (b) Find the equations of the two tangent planes to the sphere

$$x^2 + y^2 + z^2 = 9$$

which pass through the line

$$x + z = 6,$$

$$2y - z + 3 = 0$$

Register Number :

Name of the Candidate :

**5 2 8 4**

**B.Sc. DEGREE EXAMINATION, 2012**

( PHYSICS & ELECTRONIC SCIENCE )

( FIRST YEAR )

( PART - III )

( PAPER - II )

**550. MATHEMATICS - I**

[ Common with B.Sc. (Electronic Science),  
 B.Sc. (Applied Chemistry) ]

December ]

[ Time : 3 Hours

Maximum : 75 Marks

*Answer any FIVE questions.*

*ALL questions carry EQUAL marks.*

(5 × 5 = 75)

1. (a) Sum the series

$$\frac{1}{10} + \frac{1 \cdot 4}{10 \cdot 20} + \frac{1 \cdot 4 \cdot 7}{10 \cdot 20 \cdot 30} + \dots$$

**Turn Over**

$$1 + \frac{1}{1+2} + \frac{1}{1+2+2^2} + \frac{1}{3i} + \dots$$

(b) Sum the series

2. (a) Let  $G$  be a group and if  $x \in G$ , then prove that  $xG = Gx = G$ .

(b) Prove that if  $H$  is a proper subgroup of a finite group  $G$ , then there exists  $x \in G$  such that  $x$  is not in any conjugate of  $H$ .

3. (a) If  $y = x^2 \cos x$ ,

prove that

$$x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + (x^2 + 6)y = 0$$

(b) If  $y = a \cos(\log x) + b \sin(\log x)$ ,

show that

$$x^2 \frac{d^2y}{dx^2} + (2n + 1)xy^{n+1} + (n^2 + 1)y^n = 0.$$

(b) If  $\tan(x + iy) = u + iv$ ,

prove that

$$\frac{ax}{cz} = \frac{by}{cz} = \frac{m}{n},$$

$$\frac{1}{x} = \frac{y}{m} = \frac{z}{n},$$

9. (a) Show that the lines

$$\frac{z}{\sin 2x} = \frac{y}{\sin 2y} = \frac{x}{\sin 2z}$$

are coplanar if  $a = b$  or  $b = c$  or  $c = a$ .

(b) Find the shortest distance between the lines

$$\frac{x-3}{y-4} = \frac{-1}{z+2} = \frac{2}{1}$$

$$\frac{x-1}{y+7} = \frac{1}{z+2} = \frac{3}{2}$$

6. Find the characteristic equation of the matrix

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

and hence, obtain its inverse.

7. (a) Prove that

$$2^7 \sin^8 \theta =$$

$$\cos 8\theta - 8 \cos 6\theta + 28 \cos 4\theta - 56 \cos 2\theta + 35$$

- (b) Find

8. (a) If  $\cos hu = \sec \theta$ , show that

$$u = \log \tan$$

4. (a) Prove that the radius of curvature at the point  $\theta$  on the curve

$$x = 3a \cos \theta - a \sin 3\theta$$

$$y = 3a \sin \theta - a \sin 3\theta$$

is  $3a \sin \theta$ .

- (b) Discuss the maxima and minima of the function  $x^3 y^2 (6 - x - y)$ .

5. (a) Find the rank of  $A =$

$$\begin{bmatrix} \frac{1}{2} + \frac{1}{3} & \sin \theta - \sin n\theta \\ \frac{1}{4} + \frac{1}{5} & \cos \theta - \cos n\theta \\ 2 & 13 & 10 \end{bmatrix}$$

- (b) Solve the following system of equations if they are consistent :

$$2x + 6y + 11 = 0,$$

$$6x + 20y - 6z + 3 = 0,$$

$$6y - 18x + 1 = 0.$$