Register Number:
Name of the Candidate:

B.Sc. DEGREE EXAMINATION, May 2015

(MATHEMATICS)

(SECOND YEAR)

(PART – III)

GROUP-A: MAIN

650. ALGEBRA AND SOLID GEOMETRY

Time: Three hours   Maximum: 100 marks

Answer any FIVE questions   (5 × 20 = 100)

1. a) Solve $2x^3 - x^2 - 22x - 24 = 0$ given that two of its roots are in the ratio 3:4.
   b) Solve $2x^6 - 9x^5 + 10x^4 - 3x^3 + 10x^2 - 9x + 2 = 0$

2. a) If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two. Prove that $p^3 + 8r = 4pq$.
   b) Find the smallest number with 18 divisors.

3. a) If N=ab where a and b are prime to one another Show that $\phi(N) = \phi(a) . \phi(b)$
   b) State and prove Fermat's theorem.

4. a) Show that an injective mapping from a finite set to itself is bijective.
   b) If R is an equivalence class defined on a set S then prove that
      (i) aRb if and only if the equivalence class determined by a is same as that by b.
      (ii) S is the union of all the equivalence classes.

5. a) State and prove fundamenta theorem of homomorphisms on groups.
   b) Let G be a finite group and $\theta:G \rightarrow G$ an automorphism such that $\theta^2 = Id$ and $\theta(a) = a$ implies a=e. Show that G is abelian.

6. a) Let G be a finite group. Show that the order of any subgroup of G divides the order of G.
   b) Prove that if H is subgroup of G and N is a normal subgroup of G, then HN is a subgroup of G.

7. a) Show that the straight line whose direction cosines are given by al+bm+cn=0, fmn+ghl+hlm=0 are perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$ and parallel if $\sqrt{af}+\sqrt{hg}+\sqrt{ch}=0$.

   b) Find the equation of the plane which passes through the point (−1, 3, 2) and perpendicular to the two plane x+2y+3z=5, 3x+3y+2z=8.

8. a) Find the equation of the plane through the point (x_1, y_1, z_1) parallel to the plane ax+by+cz+d=0.

   b) Find the equation of the image of the line $\frac{x-1}{2}=\frac{y+2}{-5}=\frac{z-3}{2}$ in the plane 2x−3y+2z+3=0.

9. a) A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere OABC is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$

   b) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0$, x+2y+3z=8 and touches the plane 4x+3y=25.

10. a) Show that $33x^2+13y^2-95z^2-144yz-96zx-48xy=0$ represents a right circular cone whose axis is the line 3x=2y=z. Find its vertical angle.

   b) Prove that the equation $2x^2-6yz-12z^2+18yz+2zx+xy=0$ represents a pair of planes and find the angle between them.