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Register Number:
Name of the Candidate:

## B.Sc. DEGREE EXAMINATION, May 2015

### (MATHEMATICS)

### (THIRD YEAR)

### (PART – III)

### 730. ANALYSIS-III

Time: Three hours | Maximum: 100 marks

**Answer any FIVE questions** **(5 × 20 = 100)**

1. A function is defined in the range (0, 2π) by the relations $f(x)=\begin{cases} x & for\, 0<x<\pi \\ 2\pi - x & for\, \pi<x<2\pi \end{cases}$ Express f(x) as Fourier series in the interval (0, 2π).

   Deduce that $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}.......\frac{\pi^2}{8}$

2. If f(x) is defined within the range (0, π) by the relations $f(x)=\begin{cases} x & for\, 0<x<\frac{\pi}{2} \\ (\pi - x) & for\, \frac{\pi}{2}<x<\pi \end{cases}$ show that $f(x)=\frac{4}{\pi}\left(\frac{\sin x}{1^2}-\frac{\sin 3x}{3^2}+\frac{\sin 5x}{5^2}-.....\right)$

3. a) Find $L^{-1}\left[\frac{1+2s}{(s+2)^2(s-1)^2}\right]$

   b) Show that $L[f''(t)]+sf(0)=s^2L[f(t)-f'(0)]$.

4. Solve the differential equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$ given that $y=\frac{dy}{dt}=0$ when t=0.

5. a) Show that $F_s\left\{\frac{1}{\sqrt{x}}\right\}=\frac{1}{\sqrt{s}}$

   b) Find $F_C\left\{\frac{1}{1+x^2}\right\}$

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6. Find a Fourier cosine transform for f(x) defined by $f(x)=\begin{cases}1 & for|x|<1\\ 0 & for|x|>1\end{cases}$ hence deduce that (i) $\int_0^\infty \frac{\sin t}{t}dt=\frac{\pi}{2}$ (ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$

7. a) Eliminate the constants from $y_n=(A+Bn)2^n$ and derive the corresponding difference equation of the lowest order.

   b) Solve the equation $y_{n+3}-3y_{n+1}+2y_n=0$ given $y_1=0$, $y_2=8$ and $y_3=-2$.

8. Solve the equation $y_{n+2}+y_{n+1}-56y_n=2^n(n^2-3)$.

9. a) Prove that $\Gamma(\frac{1}{2})=\sqrt{\pi}$

   b) Evaluate the intergral $\int_0^1 (x\log x)^3 dx$, using gamma function

10. Using beta and gamma functions, evaluate the integrals

    a) $\int_0^a x^4\sqrt{a^2-x^2}\,dx$

    b) $\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\,d\theta$

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