

MASTER OF PHILOSOPHY

MATHEMATICS

(FT/PT)

(For the Students admitted in the Academic Year 2021-2022)

PART I

CORE COURSE I

ALGEBRA AND ANALYSIS

UNIT-I: RINGS, IDEALS AND MODULES

Rings and ring homomorphisms - Ideals, Quotient rings - Zero-divisors, Nilpotent elements, Units - Prime ideals and maximal ideals - Nilradical and Jacobson radical - Operations on ideals - Extension and contraction - Exercises - Modules and module homomorphisms - Submodules and quotient modules - Operations on submodules - Direct sum and product - Finitely generated modules - Exact sequences - Tensor product of modules - Restriction and extension of scalars - Exactness properties of the tensor product - Algebras - Tensor product of algebras - Exercises.

UNIT-II: RINGS, MODULES OF FRACTIONS AND PRIMARY DECOMPOSITION

Local properties - Extended and contracted ideals in rings of fractions Exercises - Primary Decomposition - Exercises.

UNIT-III: CHAIN CONDITIONS, NOETHERIAN RINGS AND ARTIN RINGS

Chain conditions - Exercises - Primary decomposition in Noetherian rings - Exercises - Artin Rings - Exercises.

UNIT-IV: ABSTRACT INTEGRATION AND L^p - SPACES

The concept of measurability - simple functions - Elementary properties of measures - Integration of positive functions - Integration of complex functions - The role played by sets of measure zero - Convex functions and inequalities - L^p - spaces

UNIT-V: FOURIER TRANSFORMS AND HOLOMORPHIC FOURIER TRANSFORMS

Formal properties - The Inversion Theorem - the Plancherel Theorem - The Banach algebra L^1 - Introduction - Two Theorems of Paley and Wiener - Quasi - analytic classes - The Denjoy - Carleman theorem.

TEXT BOOKS:

1. M.F. Atiyah, I.G. Macdonald, Introduction to Commutative Algebra, Addison - Wesley Publishing Company, 1969.

Unit - I Chapter - 1 (pp 1 - 10), Chapter - 2 (pp 17 - 31)
Unit - II Chapter - 3 (pp 36 - 43), Chapter - 4 (pp 50 - 55)
Unit - III Chapter - 6 (pp 74 - 78), Chapter - 7 (pp 80 - 84)
Chapter - 8 (pp 89 - 91)

2. Walter Rudin, Real and Complex Analysis, 3rd Edition, McGraw Hill International, 1986.

Unit - IV Chapter - 1 (pp 5 - 31), Chapter - 3 (pp 61 - 69),
Unit - V Chapter - 9 (pp 178 - 193), Chapter - 19 (pp 371 - 383)

PART I
CORE COURSE II
TOPOLOGY AND DIFFERENTIAL EQUATIONS

UNIT-I: FUNDAMENTAL GROUP AND COVERING SPACES

Homotopy - Fundamental group - Covering spaces.

UNIT-II: SIMPLICIAL COMPLEXES

Geometry of Simplicial Complexes - Bary Centric subdivisions - Simplicial approximation Theorem - Fundamental Group of a simplicial Complex.

UNIT-III: LINEAR SYSTEMS

Uncoupled Linear System - Diagonalization - Exponentials operators - The Fundamental Theorem for linear system - Linear System in \mathbb{R}^2 - Complex Eigen Values - Multiple Eigen Values - Non Homogeneous Linear System.

UNIT-IV: NON LINEAR SYSTEMS: LOCAL THEORY

Some preliminary concepts & definitions - The Fundamental Existence - Uniqueness Theorem - Dependence on Initial Conditions and Parameters - The Maximum Interval of Existence - The Flow Defined by a Differential Equation.

UNIT-V: NON LINEAR SYSTEMS

Linearization - The Stable Manifold Theorem - Dynamical Systems and Global Existence Theorems - Limit Sets and Attractors

TEXT BOOK(S):

1. **I.M. Singer, J.A. Thorpe**, Lecture Notes on Elementary Topology and Geometry, Springer - Verlag, New York, 1967.

Unit - I Chapter - 3 (pp 49 - 77)

Unit - II Chapter - 4 (pp 78 - 108)

2. **L. Perko**, Differential Equation and Dynamical System, Third Edition, Springer - Verlag, New York, 2006.

Unit - III Chapter - 1 (Sections 1.1 to 1.7 and 1.10) (pp 1 - 39, 60 - 63)

Unit - IV Chapter - 2 (Sections 2.1 to 2.5) (pp 65 - 101)

Unit - V Chapter - 2 (Sections 2.6 and 2.7) (pp 101 - 118)

Chapter - 3 (Sections 3.1 and 3.2) (pp 181 - 199)