DESIGN OF STEEL STRUCTURES

by

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BEAMS

- One of the frequently used structural members is a beam whose main function is to transfer load principally by means of flexural or bending action.

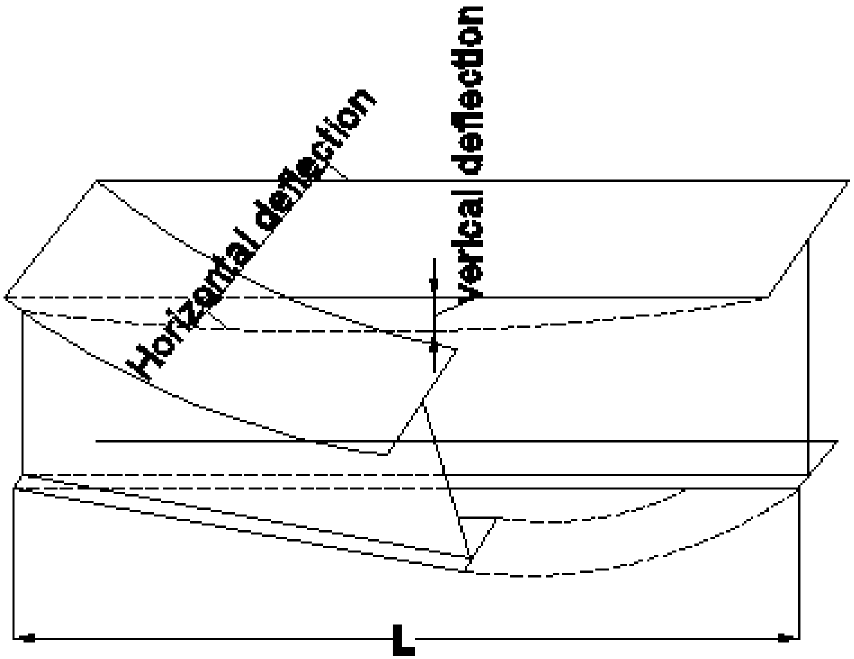
- In a structural framework, it forms the main horizontal member spanning between adjacent columns or as a secondary member transmitting floor loading to the main beams.

- Normally only bending effects are predominant in a beam except in special cases such as crane girders, where effects of torsion in addition to bending have to be specifically considered.

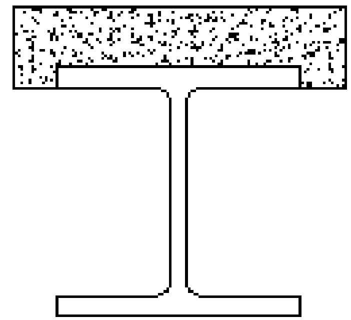
LATERALLY SUPPORTED BEAM

- When the lateral support to the compression flange is adequate, the lateral buckling of the beam is prevented and the section flexural strength of the beam can be developed.
- The strength of I-sections depends upon the width to thickness ratio of the compression flange.
- When the width to thickness ratio is sufficiently small, the beam can be fully plastified and reach the plastic moment, such sections are classified as compact sections.
- However provided the section can also sustain the moment during the additional plastic hinge rotation till the failure mechanism is formed. Such sections are referred to as plastic sections.

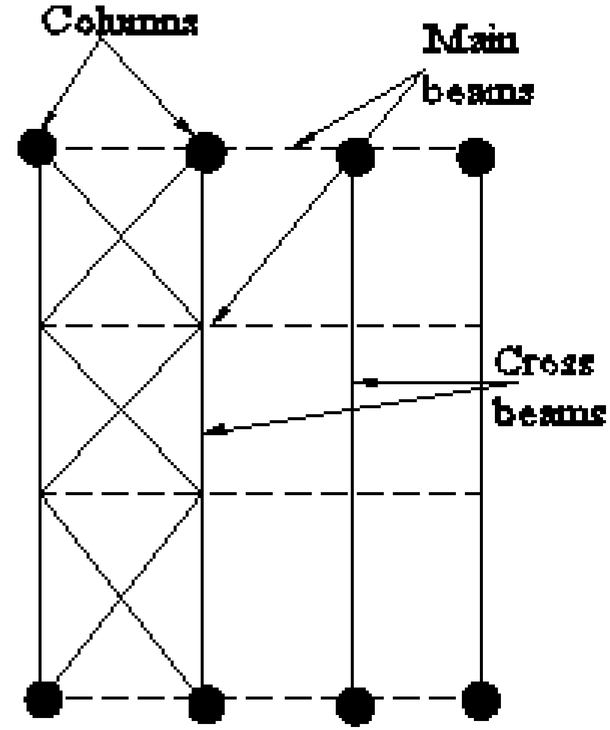
LATERALLY SUPPORTED BEAM



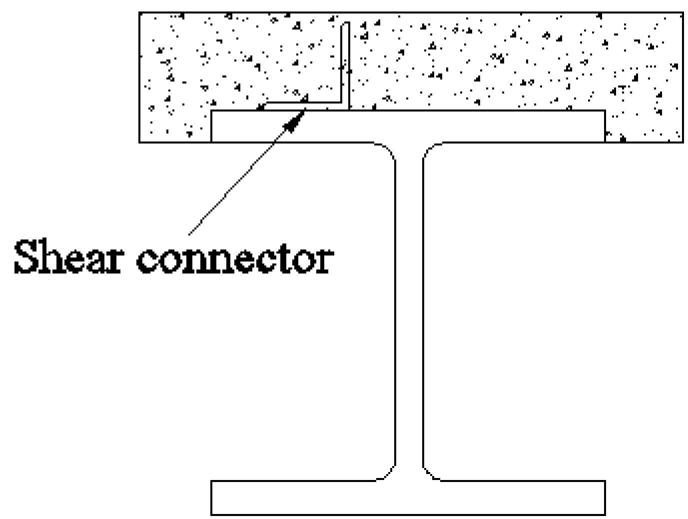
(a) Buckling of compression f



(b)



(d)

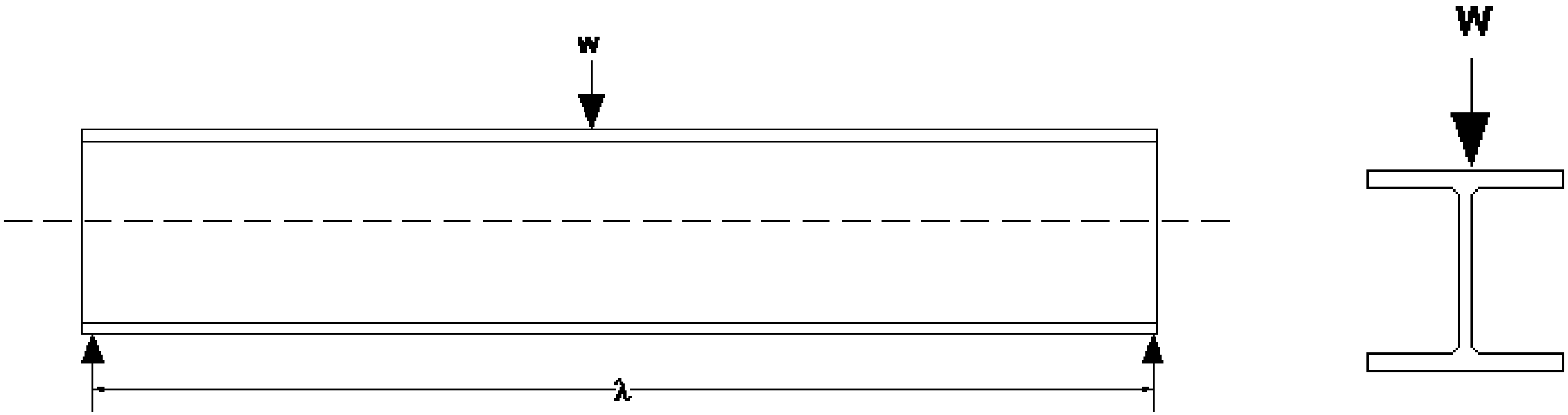


(c)

- When the compression flange width to thickness ratio is larger, the compression flange may buckle locally before the complete plastification of the section occurs and the plastic moment is reached.
- Such sections are referred to as non-compact sections.
- When the width to thickness ratio of the compression flange is sufficiently large, local buckling of compression flange may occur even before extreme fibre yields.
- Such sections are referred to as slender sections.

LATERALLY UNSUPPORTED BEAMS

- Under increasing transverse loads, a beam should attain its full plastic moment capacity.



Two important assumptions have been made therein to achieve the ideal beam behaviour.

They are:

- The compression flange of the beam is restrained from moving laterally; and
- Any form of local buckling is prevented.

1.Design a continuous beam of spans 4.9 m, 6 m, and 4.9 carrying a uniformly distributed load of **32.5 kN/m and the beam is laterally supported.**

Factored load calculation

Factored uniformly distributed load = 1.5 x 32.5 = 48.75 kN/m

The bending moment and shear force distribution are shown below

Maximum bending moment = 146.25 kN m

Maximum shear force = 146.25 + 146.25 = 292.5 kN

Plastic section modulus required

$$z_p = \frac{M x \gamma_{mo}}{f_y} = \frac{146.25 x 10^6 x\ 1.10}{250} = 643.5 x 10^3\ mm^3$$

Selection of suitable section

Choose a trial section of ISLB 350 @0.495 kN/m.

Overall depth *(h) = 350 mm*

Width of flange *(b)= 165 mm*

Thickness of flange $(t_f) = 11.4$ *mm*

Depth of web (d) = $h - 2(t_f + R) = 350 - 2(11.4 + 16) = 295.2$ *mm*

Thickness of web $(t_w) = 7.4$ *mm*

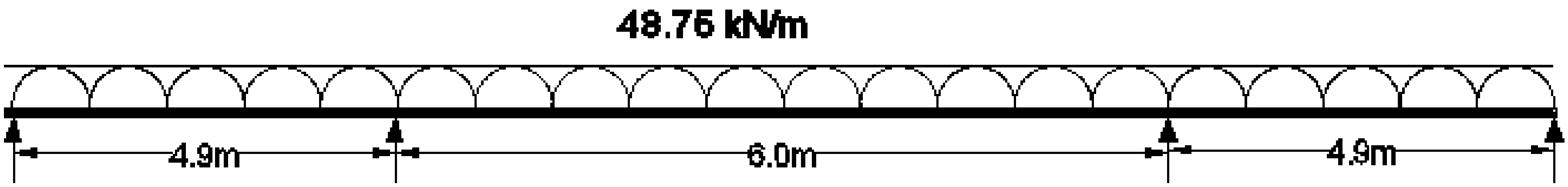
Moment of inertia about major axis *I, = 13158.3 x lo4 mm4*

Elastic section modulus $(Z_e) = 75\ 1.9 \times 10^3 mm^3$

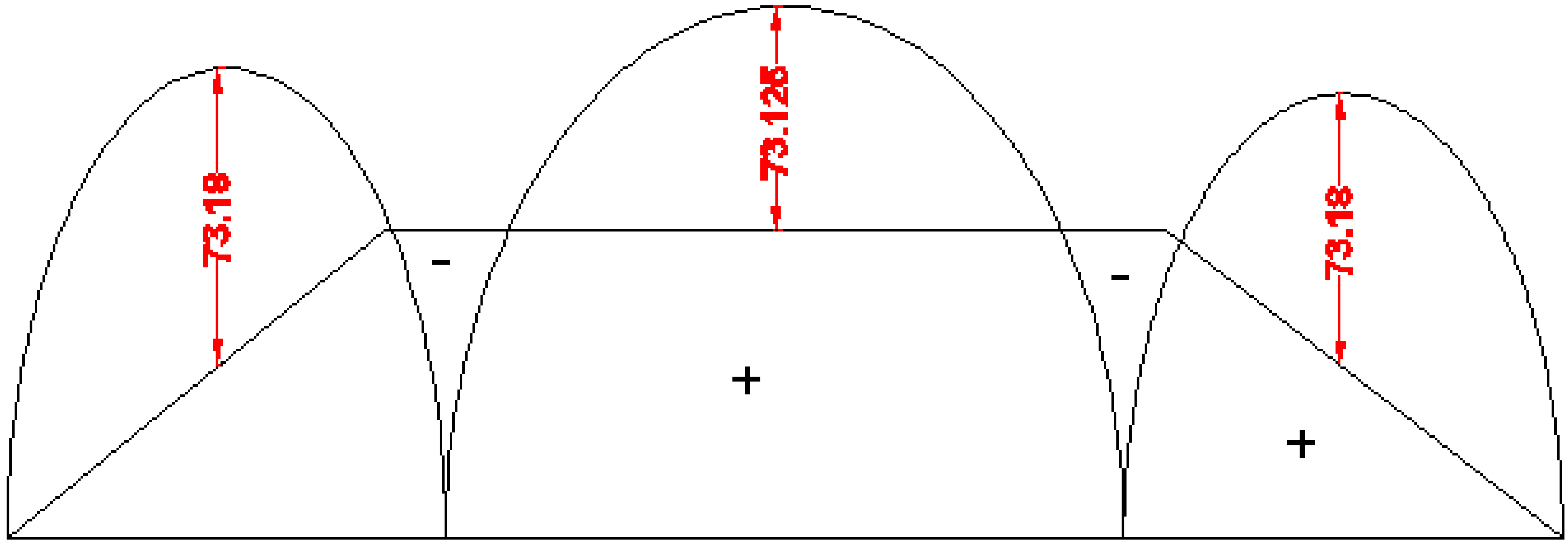
Plastic section modulus $(Z_p) = 851.11 \times 10^3\ mm^3$

Section classification

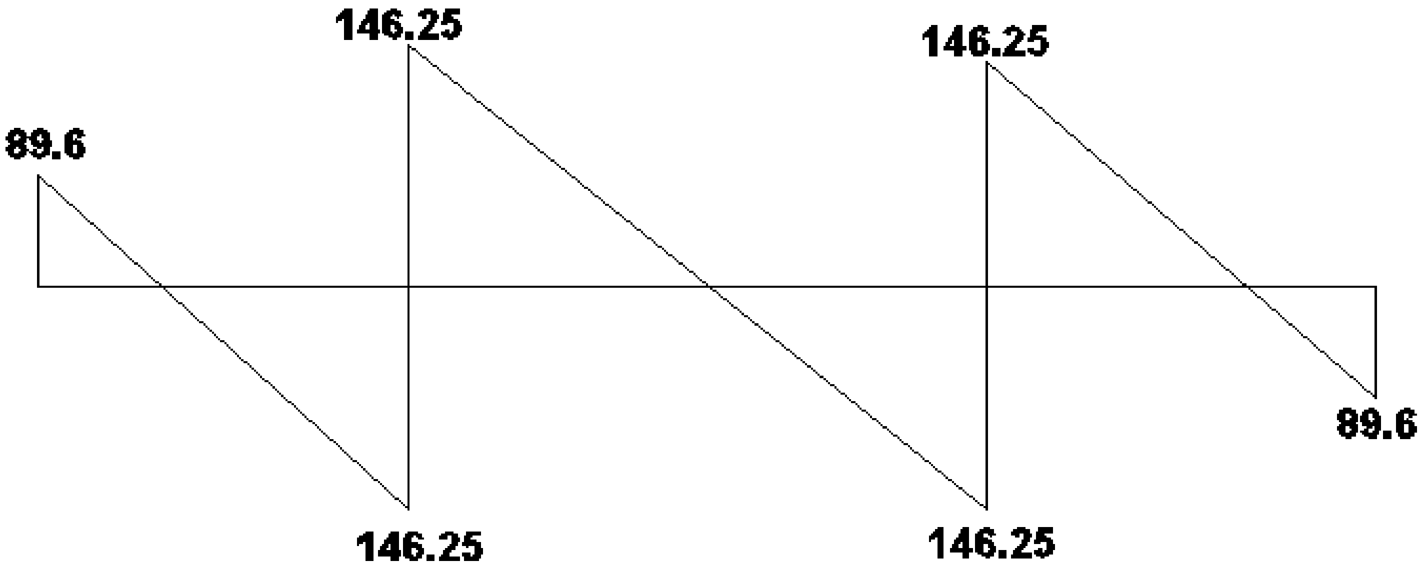
$$\frac{b}{t_f} = \frac{82.5}{11.4}$$



BEAM LOADING (a)



BENDING MOMENT DIAGRAM (b)



SF DIAGRAM (c)

$$\frac{b}{t_f} = \frac{295.2}{7.4} = 39.9 < 84$$

Hence the section is plastic.

Check for shear capacity of section

$$V_d = \frac{f_y}{m_o \text{ x } \sqrt{3}} \text{ x h x } t_w = \frac{250}{1.1 \text{ x } \sqrt{3}} \text{ x 350 x 7.4} = 340 \text{ kN}$$

$0.6\ v_d = 204$ kN < 292.5 kN

This shows a high shear condition.

Check for moment capacity of the section [Eqn 6.8(a)]

$M_{dv} = M_d - \beta\ (M_d - M_{fd}) \le 1.09 \times Z_e \times f_y$

where M_{fd} is the plastic design strength of the area of cross section excluding the shear area.

$$\beta = \left[2 \times \left(\frac{v}{v_d}\right) \times 1\right]^2 = \left[2 \times \left(\frac{292.5}{340}\right) \times 1\right]^2$$

Calculation of section modulus of flange

$$Z_{fd} = Z_p - A_w y_w$$

$$= 851.11 \times 10^3 - \left(350 \times 7.4 \times \frac{350}{4}\right)$$

$$= 624.485 \times 10^3\ \text{mm}^3$$

Therefore, $M_{fd} = \dfrac{Z_{fd}\ x\ f_y}{\gamma_{mo}}$

$$= \frac{624.485\ x 10^3 x 250}{1.10}$$

$$= 141.93\ kNm$$

Moment capacity of the section

$$M_d = \frac{Z_p\ x\ f_y}{\gamma_{mo}} = \frac{851.11\ x 10^3 x 250}{1.10}$$

$$= 193.43 \text{ kNm}$$

therefore, $M_{dv} = 193.43 - 0.52(193.43 - 141.93)$

$$= 165.65 \text{ kN m} < \frac{1.2\ x\ Z_e\ x\ f_y}{\gamma_{mo}} = \frac{1.2\ x\ 751.9\ x\ 10^3 x 250}{1.10}$$

$$= 205.06 \text{ kN m} > 146.25 \text{ kN m}$$

Hence the section is safe.

2. *Design a laterally unrestrained beam to carry a uniformly distributed load* ***of 30 kN/m. The beam is unsupported for a length of 3 m and is simply placed*** *on longitudinal beams at its ends.*

Calculation of load

Factored load = 1.5 x 30 =45 kN/m

Calculation of bending moment and shear force

$$\text{BM} = \frac{wl^2}{8} = \frac{45\ x\ 3^2}{8} = 50.625\ \text{kN.m}$$

$$\text{SF} = \frac{wl}{2} = \frac{45\ x\ 3}{2} = 67.5\text{kN}$$

Initialization of section

Assume λ = 100; $\frac{h}{t_f} = 25$ and hence from table 14, $f_{cr,b}$ = 291.31 N/mm^2

$$\lambda_{LT} = \sqrt{\frac{f_y}{f_{crb}}} = \sqrt{\frac{250}{291.31}} = 0.926$$

$$\Phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

$$= 0.5[1 + 0.21(0.926 - 0.2) + 0.9262] = 1.005$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + [\Phi^2{}_{LT} - \lambda^2{}_{LT}]^{0.5}} \leq 1.0$$

$$= \frac{1}{1.005 + [1.005^2 - 0.926^2]^{0.5}} = 0.716 \leq 1.0$$

$$f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{mo}} = \frac{0.716 \ x \ 250}{1.10} = 162.7 \text{ N/mm}^2$$

Therefore required z of section = $\frac{50.625 \ x \ 10^6}{162.7}$

$= 311.1 \text{x} 10^3 \text{mm}^3$

Choose a section of ISMB 225 @ 0.3 12 kN/m.

Overall depth ***(D) = 225 mm***

Width of flange ***(b_f) = 110 mm***

Thickness of flange *(t_f) = 11.8 mm*

Thickness of web *(t_w) = 6.5 mm*

Depth of web (d) = ***D - 2(t_f + R) = 225 - 2(11.8 + 12) = 177.4 mm***

Moment of inertia about major axis $I_{zz} = 3440 \times 10^4\ mm^4$

Moment of Inertia about minor axis $I_{yy} = 218 \times 10^4\ mm^4$

Elastic section modulus $(Z_{ez}) = 305.9 \times 10^3\ mm^3$

Plastic section modulus $(Z_{ey}) = 348.27 \times 10^3\ mm^3$

Minimum radius of gyration $(r_y) = 18.6\ mm$

Section classification

Outstand of compression flange = (1 10/2)/11.8 = 4.66 < 9.4

Web with neutral axis at mid depth = 177.4/6.5 = 27.3 < 84

Therefore the section is plastic.

Calculation of lateral-torsional buckling moment

$$M_{cr} = \sqrt{\frac{\pi^2 E I_y}{(KL)^2}\left(G I_t + \frac{\pi^2 E I_w}{(KL)^2}\right)}$$ (from clause 8.2.2.1)/p-54

$$G = \frac{E}{2(1+\mu)} = \frac{2\ x\ 10^5}{2(1+0.3)} = 76.923 \text{ x } 10^3 \text{ N/mm}^2$$

$$I_t = \sum \frac{b_i {t_i}^3}{3} = \frac{2x110x11.8^3}{3} + \frac{(225-2x11.8)x6.5^3}{3}$$

$= 138.926 \times 10^3 mm^3$

$I_w = (1-\beta_f)\, \beta_f\, I_y\, h^2_f$

$$\beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} = 0.5$$

$h_f = 225-11.8 = 213.2$ mm

$I_w = (1-0.5) \times 0.5 \times 218\times10^4 \times 213.2^2 = 24.77\times 10^9 mm^6$

$$M_{cr} = \sqrt{\frac{\pi^2 x 2 x 10^5 x 218 x 10^4}{3000^2}\left(76.923x10^3 + 138.926x10^3\right) + \frac{\pi^2 x 2 x 10^5 x 24.77 x 10^9}{2}}$$

$= 87.79$kNm

$$\lambda_{LT} = \sqrt{\frac{Z_p f_y}{M_{cr}}} = \sqrt{\frac{348.27x10^3x250}{87.79x10^6}} = 0.9959$$

$$\Phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda^2{}_{LT}]$$

$$\alpha_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda^2{}_{LT}]$$

$$\alpha_{LT} = 0.21$$

$$\Phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda^2{}_{LT}]$$

$$\chi_{LT} = \frac{1}{[\Phi^2{}_{LT} + \lambda^2{}_{LT}]^{0.5}} = 0.6685 \leq 1.0$$

$$f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{mo}} = \frac{0.6685\text{x}250}{1.10} = 151.93\text{N/mm}^2$$

$$M_d = Z_p \mathrm{f_{bd}} = 348.27\mathrm{x}10^3\mathrm{x}151.93 = 52.91\mathrm{kNm}$$
$$> 50.625\mathrm{kNm}$$

Calculation of shear capacity of section

$$\mathrm{V_d} = \frac{f_y}{\gamma_{mo}\sqrt{3}} \mathrm{x\ D\ xt_w} = \frac{250}{1.10 \mathrm{\ x} \sqrt{3}} \mathrm{x}\ 225 \mathrm{\ x}\ 6.5$$

=191.kN

0.6 $\boldsymbol{V_d}$***= 115 kN*** > ***67.5 kN***

Calculation of deflection

$$\delta_b = \frac{5wl^4}{384EI}, \ w = 30kN/m$$

$$\delta_b = \frac{5\ x\ 30\ x\ 3000^4}{384\ x\ 2\ x\ 10^5 x\ 3440 x 10^4} = 4.6mm$$

Allowable deflection = $\frac{l}{300} = \frac{3000}{300} = 10\text{mm}$

Hence the section is safe against deflection.

Check for web buckling:

Assuming that longitudinal beams are of the same size,

$$A_b = (b_1 + n_1)t_w = 4.6mm$$

$$b_1 = \frac{(b_f - t_w)}{2} = \frac{110 - 6.5}{2} = 51.75mm$$

$$n_1 = \frac{D}{2} = \frac{225}{2} = 112.5mm$$

A_b = (51.75+112.5) x 6.5 = 1067.6mm^2

r_{min} = $\sqrt{\frac{I}{A}} = \sqrt{\frac{1184}{336.4}} = 1.88mm$

λ = $\frac{l_{eff}}{r_{min}} = \frac{0.7x177.4}{1.88} = 66.18$

therefore, f_{cd} = 158.36N/mm^2(from table 9c of the code)

Strength of the section against web buckling = 158.36 x 1067.6

= 169.07 **kN** > **67.5 kN**

Check for web bearing:

$F_w = (b_1 + n_2)t_w f_y/\gamma_{mo}$

$b_1 = 51.75$ mm

$n_2 = 2.5(t_f + R) = 2.5(11.8+12) = 59.5$mm

$F_w = (51.75+59.5) \times 6.5 \times 250/(1.10 \times 10^3) = 164.35$kN > 67.5 kN

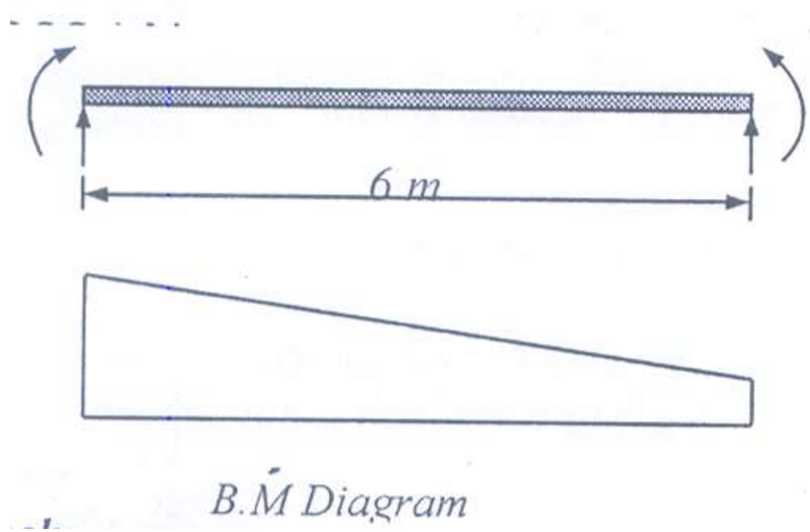
Hence the section is safe against web bearing.

PROBLEMS

3.A simply supported beam of span 6m is subjected to end moments of 202 kN.m (clockwise) and 112 kN.m (anticlockwise) under factored applied loading. Check whether ISMB-450 is safe with regard to lateral buckling.

Design check

For the end conditions given,it Is assumed that the beam is simply supported in a vertical plane,and at the ends the beam is fully restrained against lateral Deflection and twist with



no rotational restraints in plan at its ends.

Section classification of ISMB 450

The properties of the section are:

Depth, h = 450mm

Width, b = 150 mm

Web thickness,t_w = 9.4 mm

Flange thickness, t_f = 17.4 mm

I_y = 834 X 10^4 mm^4

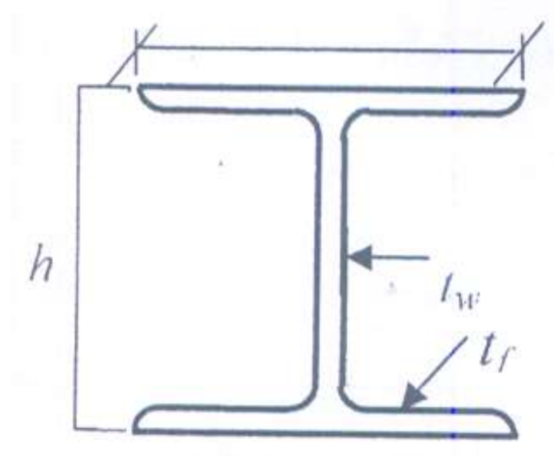
Depth between fillets,d = 379.2 mm

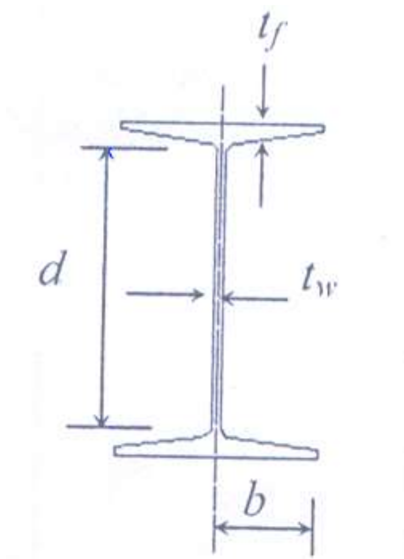
Radius of gyration about minor axis,

r_y = 30.1 mm

Plastic modulus about major axis,

z_p = 1533.36 X 10^3 mm^3





Rolled Steel Beams

Assume f_y = 250 N/mm^2 , E = 200000 N/mm^2 , γ_m = 1.10

Type of section

Flange criterion:

b= B/2 = 150/2 = 75mm

b/t_f = 75.0 /17.4 = 4.31

$b/t_f = 9.4\varepsilon$ where $\varepsilon = \sqrt{250/f_y}$

Hence , O.K

Web criterion:

d/t_w = 379.2/9.4 = 40.3

$d/t_w < 84\ \varepsilon$

Hence, O.K

Since, $b/t_f = 9.4\varepsilon < d/t_w < 84\ \varepsilon$,the section is classified as '*plastic*'

Table 3.1(section 3.7.2 of I.S 800)

Check for lateral torsional buckling :

Check for slenderness ratio:

Effective length criteria:

With ends of compression flanges fully restrained for torsion at support but both the flanges are not restrained against warping, Effective length of simply supported beam, L_{LT} = 1.0 L Where L is the span of the beam. (*Table 8.3 of I.S.800*)

Hence, L_{LT} = 1.0 x 6.0 M = 6000mm , L_{LT}/ r = 6000/30.1 =199.33

Since the moment is varying from 155 k-Nm to 86 k-Nm,there will be moment gradient .So for calculation f_{bd} , critical moment ,M_{cr} is to be calculated

Now, critical moment

$$M_{cr} = C_1 \frac{\pi^2 EI}{(KL)^2} \{[(\frac{K}{K_w})^2 \frac{I_w}{I_y} + (\frac{GI_t(KL)^2}{\pi^2 EI_y} + (C_2 y_g - C_3 y_1)^2]^{0.5} - (C_2 y_g - C_3 y_t)\}$$

Where ,

C_1, C_2 , C_3= factors depending upon the loading and end restraint conditions

K,K_w = effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports, Here,both K and K_w can be taken as 1.0 and

y_g = y distance between the point of application of the load and the shear centre of the cross-section and is positive when the load is acting towards the shear centre from the point of application

$y_j = y_s - 0.5 \int_A (z^2 - y^2) y \, dA / I_z$

y_s = coordinate of the shear centre with respect to centroid, positive when the shear centre is on the compression side of the centroid.

Here,for plane and equal flange I section,

$y_g = 0.5 \times h = 0.5 \times 0.45 = 0.225$ M =225mm.

$y_j = 1.0(2\beta_f - 1) h_y / 2.0$ (when $\beta_f \leq 0.5$)

h_y = distance between shear centre of the two flanges of the cross-section) = $h - t_f$

Here, $\beta_f = 0.5$ and $h_y = h - t_f = 450 - 17.4 = 432.6$mm

Hence, $y_j = 1.0 \times (2.0 \times 0.5 - 1) 432.6 / 2.0 = 0$ and $y_s = 0$

$I_t = \sum b_i t_i^3$,for open section

$= 2 \times 150 \times 17.4^3 + (450 - 2 \times 17.4) \times 9.4^3$

The warping constant, I_w is given by,

$I_w = (1- \beta_f)\ \beta_f\ I_y\ h_y^2$ for I sections mono-symmetric about weak axis,

$=(1-0.5) \times 0.5 \times 834 \times 10^4 \times 432.6^2 = 39019265.46 \times 10^4 mm^6$

Modulus of rigidity ,$G = 0.769 \times 10^5\ N/mm^2$

Here, $\psi = 86/155 = 0.555$ and $K = 1.0$ for which,

$C_1 = 1.283$, $C_2 = 0$ and $C_3 = 0.993$

Hence, critical moment

$$M_{cr} = C_1 \frac{\pi^2 EI}{(KL)^2} \{[(\frac{K}{K_w})^2 \frac{I_w}{I_y} + (\frac{G I_t (KL)^2}{\pi^2 E I_y} + (C_2 y_g - C_3 y_1)^2]^{0.5} - (C_2 y_g - C_3 y_t)\}$$

$$=1.283\ \frac{\pi^2 x 200000 x 834 x 10^4}{(1.0\ x\ 6000)^2} \{[\left(\frac{1}{1}\right)^2 \frac{39019265 x 10^4}{834 x 10^4} + \frac{0.769 x 10^5 x 192.527 x 10^4 x 6000^2}{\pi^2 x 200000 x 834 x 10^4}]^{0.5}$$

$= 357142.72 \times 10^3$ N-mm.

Calculation of f_{bd}:

Now $\lambda_{LT} = \sqrt{\beta_b z_p f_y / M_{cr}} = \sqrt{1.0 \times 1533.36 \times 10^3 \times 250 / 357142.72 \times 10^3}$

$= 1.036$ *(clause 8.2.2 of I.S 800)*

For which, $\Phi_{LT} = 0.5 \times [1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$

$= 0.5 \times [1 + 0.21(1.036 - 0.2) + 1.036^2] = 1.124$

For which , $\chi_{LT} = \dfrac{1}{\{\Phi_{LT}[\Phi^2_{LT} - \lambda^2_{LT}]^{0.5}\}}$

$= \dfrac{1}{\{1.124[1.124^2 - 1.036^2]^{0.5}\}}$

$f_{bd} = \chi_{LT} f_y / \gamma_{mo} = 0.641 \times 250 / 1.10 = 145.68 \text{ N/mm}^2$

Hence, $M_d = \beta_b z_p f_{bd}$ = 1.0 x 1533.36 x 145.68 / 1000

= 223379.88/1000 ~ 223.38 kN-m.

Max. Bending moment M_{max} = 202 kN-m

Hence, $M_d > M_{max}$ = (223.38 > 202)

Therefore,ISMB 450 is adequate against lateral torsional buckling for the applied bending moments.

(ii) If the ISMB 450 is subjected to a central load producing a maximum factored moment of 202kN.m ,check whether the beam is still safe

For this problem with zero bending moments at the supports and central max bending moment being 202kN-m.

For the valur of K = 1.0, C_1 = 1.365; C_2 =0 .553 and C_3 = 1.780

$$M_{cr}=C_1\frac{\pi^2EI}{(KL)^2}\{[(\frac{K}{K_w})^2\frac{I_w}{I_y}+(\frac{GI_t(KL)^2}{\pi^2EI_y}+(C_2y_g-C_3y_1)^2]^{0.5}-$$

$$(C_2y_g-C_3y_t)\}$$

$$= 1.365\frac{\pi^2x2x10^4x834x10^4}{(1.0\ x\ 6000)^2}\{[\left(\frac{1}{1}\right)^2\frac{39019x10^9}{834x10^4}+\frac{0.769x10^5x192.527x10^4x6000^2}{\pi^2x2x10^5x834x10^4}]^{0.5}$$

$$-0.553x225\}$$

$=310158.31x10^3$N-mm

Calculation of f_{bd}:

Now, $\lambda_{LT}=\sqrt{\beta_b z_p f_y/M_{cr}} = \sqrt{1.0x1533.36x10^3x250/310158.31x10^3}$

= 1.112 *(clause 8.2.2 of I.S 800)*

For which, $\Phi_{LT}= 0.5\ x\ [1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^2]$

$= 0.5\ x\ [1 + 0.21(1.112-0.2)+1.112^2] = 1.214$

For which , $\chi_{LT} = \dfrac{1}{\{\Phi_{LT}[\Phi^2_{LT} - \lambda^2_{LT}]^{0.5}\}}$

$$= \frac{1}{\{1.214[1.214^2 - 1.112^2]^{0.5}\}}$$

$f_{bd} = \chi_{LT}\, f_y / \gamma_{mo}$ = 0.588 x 250 /1.10 = 133.64 N/mm^2

Hence, $M_d = \beta_b z_p f_{bd}$ = 1.0 x 1533.36 x 133.64 / 1000

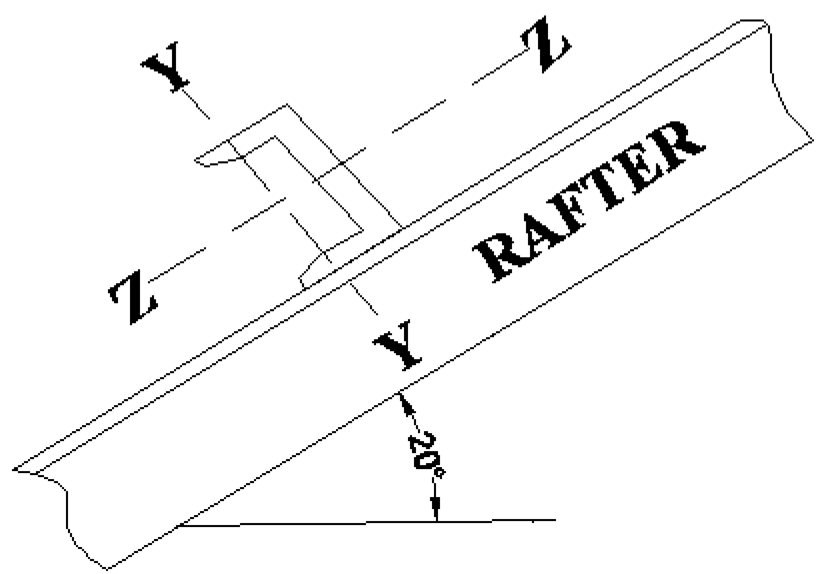
= 204918.23/1000 ~ 204.92 kN-m.

4. Design a purlin on a sloping roof truss with the dead load of 0.15 kN/ m2 (cladding and insulation), a live load of 2 kN/m2 and wind load of 0.5 kN/m^2(suction). The purlins are 2 m centre to centre and of span **4 m, simply supported on** a rafter at a slope of 20 degrees (see Fig).

(a) Provide channel section purlin

(b) Provide channel purlin with a sag rod at mid span

(c) Provide angle purlin



Solution:

Load calculation

Dead load = 0.15 x 2 = 0.3 kN/m

Live load = 2 x 2 = 4 kN/m

Wind load = 0.5 x 2 = 1 kN/m (suction)

wd, = 0.3 X COS 20" = 0.282 kN/m

Wi, = 4 x cos 20" = 3.76 kN/m

W,, = -1 kN/m

W_{iy} = 4 x sin 20" = 1.37 kN/m

w_{dy} = 0.3 X Sin 20" = 0.103 kN/m

Note that W_{wy} *is zero as wind pressure is perpendicular to the surface on which it* acts, i.e., normal to the rafter.

Factored load combination:

Z-direction:

WL + DL + LL = (1.2 x 1 .0) + (1.2 x 0.282) + (1.2 x 3.76) = 6.0552 kN/m

DL + LL = (1.5 x 0.282) + (1.5 x 3.76) = 6.063 kN/m

Y-direction:

DL + LL = (1.5 x 0.103) + (1.5 x 1.37) = 2.21 kN/m

Bending moment and shear force calculation:

$M_z = 6.063 \text{ x} 4^2/8 = 12.126$ kN m

$M_y = 2.21 \text{ x } 4^2/8 = 4.42$ kN m

$F_z = 6.063 \text{ x } 4/2 = 12.126$ kN

$F_y = 2.21 \text{ x } 4/2 = 4.42$ kN

(a) Channel section purlin

Assume an ISMC 200 channel.

Plastic section modulus required

$$= \frac{M_z \ x \ \gamma_{mo}}{f_y} + 2.5 \ x \ \frac{d}{b} \ x \ \frac{M_y \ x \ \gamma_{mo}}{f_y}$$

$$= \frac{12.126x10^6x1.10}{250} + 2.5\,x\,\frac{200}{75}\,x\,\frac{4.42\,x10^6x1.10}{250}$$

$= 183 \text{ x } 10^3 \text{ mm}^3$

Choose a channel section ISMC 200 @ 0.22 kN/m with plastic section modulus of

$Z_{pz} = 211.25 \text{ x } l0^3 \text{ mm}^3$ and $Z_{py} = 40.716 \text{ x } l0^3 \text{ mm}^3$.

Section Properties:

Cross sectional area $A = 2821\ mm^2$

Depth of the section $h = 200\ mm$

Width of flange $b = 75\ mm$

Thickness of flange $t_f = 11.4\ mm$

Thickness of web $t_w = 6.1\ mm$

Depth of web $d = h - 2(9+ R) = 200 x 2 (11.4 + 11) = 155.2$ *mm*

Elastic section modulus $Z_{ez} = 181.7 x l0^3 mm^3$

Elastic section modulus $Z_{ey} = 26.3 \, x \, l0^3 \, mm^3$

Plastic section modulus $Z_{pz} = 2\ 11.25 \, x \, 10^3 \, mm^3$

Plastic section modulus $Z_{py} = 40.716 \text{ x } l0^3 \text{ mm}^3$

Moment of inertia $I_{zz} = 1830 x l0^4 mm^4$

Moment of inertia $I_y = 14\ 1 x 10^4 mm^4$

Section classification:

$$\frac{t}{b_f} = \frac{75}{11.4} = 6.58 < 9.4$$

$$\frac{d}{t_w} = \frac{155.2}{6.1} = 25.44 < 42$$

Hence the section is plastic.

Calculation ***of shear capacity of the section*** Z-direction

$$v_d = \frac{f_y}{\gamma_{mo} x \sqrt{3}} x\ h\ x\ t_w = \frac{250}{1.1\ x \sqrt{3}}\ x\ 200\ x\ 6.1 = 160.18\ kN$$

$0.6V_d$ = 96 kN > 12.126 kN

Y-direction

Shear capacity = $\frac{250}{11.1\ x \sqrt{3}}\ x\ 2\ x\ 75\ x\ \frac{11.4}{10^3} = 224.4\ kN > 4.42 kN.$

Note that in purlin design, the shear capacity is usually high relative to the shear force.

Design capacity of the section

$$M_{dz} = \frac{z_{pz}\ x\ f_y}{\gamma_{mo}} = \frac{211.25\ x\ 10^3}{1.1\ x\ 10^6} = 48kN.m$$

$$\leq \frac{z_{pz}\ x\ f_y}{\gamma_{mo}} = \frac{1.8x\ 181.7\ x\ 10^3 x250}{1.1\ x\ 10^6} = 49.55kN.m$$

Hence, M_{dz} = 48 kN.m > 12.126 kN.m

$$M_{dy} = \frac{z_{py}\ x\ f_y}{\gamma_{mo}} = \frac{40.716\ x\ 10^3 x250}{1.1\ 0} = 9.25kN.m$$

$$\leq \frac{r_f\ x\ ze_y x f_y}{\gamma_{mo}} = \frac{1.5x\ 26.3\ x\ 10^3 x250}{1.1\ x\ 10^6} = 8.96kN.m$$

Since the ratio z_p / z_e is greater than 1.2,the constant in the preceding equation is replaced by the ratio of γ_f =1.5 ,Hence

M_{dy} = 8.96 kN.m > 4.42 kN.m

Overall member strength (local capacity)

To ascertain the overall member strength, the following interaction equation should be satisfied.

$$\frac{M_z}{M_{dz}} + \frac{M_y}{M_{dy}} \leq 1$$

$$\frac{12.126}{48} + \frac{4.42}{8.96} = 0.75 \leq 1$$

Hence,the overall member strength is satisfactory

Check for deflection

$$\delta = \frac{5wl^4}{384EI} = \frac{5\ x\ 3.76\ x4000^4}{384x2x10^5x1830x10^4}$$

Allowable deflection $= \frac{l}{180} = \frac{4000}{180} = 22.22mm$

(Table 6 of I.S 800)

Hence, the section is safe.

Check for wind suction:

The effect of wind suction has not been considered till now; it can become critical in some situations. It has to be combined with dead load

Factored wind load W_z = 0.9 x 0.282 – 1.5 x 1 =-1.246kN/m

W_y = 0.9 x 0.103 =-0.0927kN/m

Buckling resistance of section

Equivalent length l_e = 4 m

Moment = M_z = w1^2/8 = -1.246 x 4^2/8 = -2.492 kN m

My = 0.0927 x 42/8 = 0.1854 kN m

The value of M_z *is much lower than the value* 12.126 kN m *earlier, but the negative* sign indicates that the lower flange of the channel is in compression and this flange is unrestrained. Hence the buckling resistance of the channel must be found.

$$M_{cr} = \sqrt{\frac{\pi^2 EIy}{(KL^2)}} \left(GI_t + \frac{\pi^2 EIw}{(KL^2)}\right)$$

$$G = \frac{E}{2(1+\mu)}) = \frac{2X10^5}{2(1+0.3)} = 76.923X10^3 N/mm^2$$

$$I_t = \sum \frac{b_i {t_i}^3}{3} = \left[\frac{2X75x11.4^3}{3} + \frac{(200-11.4)x6.1^3}{3}\right] = 88346.77mm^4$$

$$I_w = (1-\beta_f)\ \beta_f\ I_y\ h^2_f$$

$$h_f = 200-11.4 = 188.6mm$$

$$\beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} = 0.5$$

$$I_w = (1-0.5) \times 0.5 \times 141 \times 10^4 \times 188.6^2$$

$$= 1.2538 \times 10^{10} mm^6$$

$$M_{cr} = \sqrt{\left[\frac{\pi^2 x 2 x 10^5 x 141 x 10^4}{4000^2}(76.923x10^4 x 88346.7 + \frac{\pi^2 x 2 x 10^5 x 1.2538 x 10^{10}}{4000^2}\right]}$$

$$= 38.09 \text{ kN m}$$

$$\lambda_{LT} = \sqrt{\frac{\beta_b z_p f_y}{M_{cr}}}$$

$$= \sqrt{\frac{1.0\ x\ 211.25x10^3 x250}{38.09x10^6}} = 1.1775$$

$$\Phi_{LT} = 0.5[1+\alpha_{LT}\ (\lambda_{LT} - 0.2) + \lambda^2_{LT}]$$

$$= 0.5[1+0.21(1.1775-0.2)+1.1775^2]$$

$$= 1.296$$

$$\chi_{LT} = \sqrt{\frac{1.0}{\Phi_{LT} + [\Phi^2_{LT} - \lambda^2_{LT}]^{0.5}}} \leq 1.0$$

$$= \sqrt{\frac{1.0}{1.296 + [1.296 - \lambda^2_{LT}]^{0.5}}} \leq 1.0$$

$$=0.544 \leq 1$$

$$f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{m0}} = \frac{0.544 \text{ X } 250}{1.10} = 123.71 \text{ N/mm}^2$$

$$M_{dz} = z_p f_{bd}$$

$$= 211.25 \times 10^3 \times 123.71$$

$$= 26.13 \text{ kNm} > 2.492 \text{ kNm}$$

The buckling resistanve M_{dy} of the section need not be found out,because the purlin is restrained by the cladding in the z-plane and hence instability is not considered for a moment about the minor axis .

Overall member strength

To ascertain the overall member buckling strength,the following interaction should be satisfied .

$$\frac{M_z}{M_{dz}} + \frac{M_y}{M_{dy}} \leq 1$$

$$\frac{2.492}{26.13} + \frac{0.1854}{8.96} = 0.097 < 1$$

Hence the overall member strength is satisfactory.

- It has to be noted that the maximum buckling moment occurs at the centre of the beam and the maximum shear force at the supports.
- Hence it is not necessary to check the moment capacity in the presence of shear force.
- Also purlins are not normally checked for web bearing and crippling

as the applied concentrated loads are low (note the low value of Shear force)

(*b*) *Channel section purlin with one sag rod at mid span*

Since the channel section purlin is provided with a sag rod at mid – span,the bending moment in the y- direction will be reduced considerably .

$M_y = 2.21 \times 4^2 / 32 = 1.105$kN m

$M_z = 12.126$ kN m

Required section modulus = $(M_z \times \gamma_{m0}/f_y) + 2.5(d/b)(M_y \times \gamma_{m0}/f_y)$

Assuming ISMC 100 with d = 100mm and b = 50 mm,

Required Z = $(12.126 \times 10^6 \times 1.1/250)$

$= 77.66 \times 10^3 mm^3$

Provide ISMC 150 with following section properties

Depth of section h = 150mm; r_y = 22 mm

Width of flange b = 75 mm

Thickness of flange t_f = 9.0 mm

Thickness of web t_w = 5.7 mm

Elastic section modulus $z_{ez} = 105\text{x}10^3 mm^3$

Elastic section modulus $z_{ey} = 19.5 \text{ x } 10^3 mm^3$

Plastic section modulus

$$z_{pz} = 119.5 \text{ x } 10^3 mm^3 > 77.66 \text{ x } 10^3 mm^3$$

Moment of inertia $I_{pz} = 788 \text{ x } 10^4 mm^3$

Section classification

$$b/t_f = 75/9.0 = 8.33 < 9.4$$

d/t_w =[150-2(9.0 + 10)]/5.7 = 19.65 < 42

Hence the section is plastic.Shear capacity is not being checked since the shear foece is small and hence the section will be adequate.

Design capacity of the section

$M_{dz} = (z_{pz} \text{ x } f_y/\gamma_{m0})$

$= (119.83 \text{ x } 10^3 \text{ x } 250/1.1 \text{ x } 10^6) = 27.23 \text{kN m}$

$\leq (1.2 \text{ x } z_{ez} f_y/\gamma_{m0}) = [(1.2 \text{ x } 105 \text{ x } 10^3 \text{ x } 250)/(1.1 \text{ x } 10^6)]$

$= 28.63$

$Z_{py} = 2t_f\, b^2{}_f/4 + (h- 2t_f)\, t^2{}_w /4 = 2 \text{ x } 9.0 \text{ x } 75^2 /4 + (150\text{-}2 \text{ x } 9.0)$

$5.7^2/4 = 26384.6 \text{ mm}^3$

$M_{dy} = (z_{py}f_y/\gamma_{m0})$

$= (26384.6 \text{ x } 250/1.1 \text{ x } 10^6) = 6.0 \text{ kN m}$

$\leq (1.5 \text{ x } z_{ey}f_y/\gamma_{m0}) = 1.5 \text{ x } 19.5 \text{x } 10^3 \text{ x } 250)/(1.1 \text{ x } 10^6)]$

$= 6.6 \text{ kN m}$

Hence the section is safe.

Overall member strength

For overall member srength ,the following interaction equation must be satisfied.

$(M_z/M_{dz}) + (M_y/M_{dy}) \leq 1.0$

$(12.126/27.23) + (1.105/6.0) = 0.629 < 1.0$

Hence the member strength is satisfactory.

Check for deflection

$$\delta = (5wl^4/ 384EI) = (5 \times 3.76 \times 4000^4) /(384 \times 2 \times 10^5 \times 788 \times 10^4)$$

$$= 7.95 \text{ mm} < 22.22 \text{ mm}$$

Hence the section is safe.

Check for wind suction

From part (a) , $M_z = 2.492$ kN m

$M_y = 0.0927 \times 4^2/32 = 0.0464$ kN m

$f_{cr} =[1473.5/ (KL/r_y)/(h/t_f)]^2\}^{0.5}$

$KL/r_y = 4000/22 = 181.8$

$h/t_f = 150/9.0 = 16.67$

Thus, $f_{cr} = (1473.5/11.8)^2 \ \{1+(1/20) [181.8/16.67]^2\}^{0.5}$

$=173.1\ N/mm^2$

$f_{bd} = 120.0\ N/mm^2$ (from table 13a of the code)

$M_{dz} = Z_{pz}\, f_{bd} = 119.82 \times 10^3 \times 120.0/10^6 = 14.38\ kN\ m$

Overall buckling strength

For overall buckling strength,the following interaction equation should be satisfied.

$(M_z / M_{dz}) + (M_y / M_{dy}) = (2.492/14.38) + (0.0464/6.0)$

$= 0.18 < 10$

Hence the overall buckling strength is satisfactory.

Hence by using one sag rod ,it was possible to reduce the section from ISMC 200 to ISMC 150 (about 25% reduction in weight).

(c) *Angle Section Purlin (as per BS 5950-1:2000)*

From part (a) M_z = 12.126 kN m; W_p = (1.0 +0.282+3.76) x 4

=20.168 kN

Moment at working load = 12.126 /1.5 = 8.084 kN m

Let us assume that bending about z-z axis resists the vertical loads and the horizontal component is resisted by the sheeting.

Design strength f_y = 250 Mpa

Applied moment = moment capacity of single angle

$8.084 \times 10^6 = 250 \times Z_{ez}$

Required $Z_{ez} = 8.084 \times 10^6 / 250 = 32.33 \times 10^3$ mm^3

Provide ISA 150 x 75 x 10 angle @ 0.17 kN/m,

With $Z_{ez} = 51.9 \times 10^3$ mm^3 > $20.168 \times 4 \times 10^6 / 1800 = 10^3$ mm^3

$= 44.817 \times 10^3 mm^3$

d/t = 150/10 = 15.0 > 10.5 but < 15.7

The section is *semi – compact.*

Leg length perpendicular to plane of cladding

= 4000/45 = 88.88 mm < 150 mm

Leg length parallel to plane of cladding

= 4000/60 = 66.66 mm < 75 mm

Deflection need not be checked in this case.

Thank You