MECHATRONICS

Learning Resources
Unit wise
Compiled by

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Unit I & II
INTRODUCTION

What Is Mechatronics?

The term mechatronics used for this integration of microprocessor control systems, electrical systems and mechanical systems. A mechatronics system is not just a marriage of electrical and mechanical systems and is more than just a control system; it is complete integration of all them.

Open Loop and Closed Loop System

An open – loop system the speed of rotation of the shaft might be determined solely by the initial setting of a knob which affects the voltage applied to the motor. Any changes in the supply voltage, the characteristics of the motor as a result of temperature changes, or the shaft load will change the shaft speed but not to be compensated for. There is no feedback loop.

With a closed-loop system, however, the initial setting of the control knob will be for a particular shaft speed and this will be maintained by feedback, regardless of any changes in supply voltage, motor characteristics or load. In an open-loop control system the output from the system has no effect on the input signal. In a closed-loop control system the output does have an effect on the input signal, modifying it to maintain an output signal at the required value.

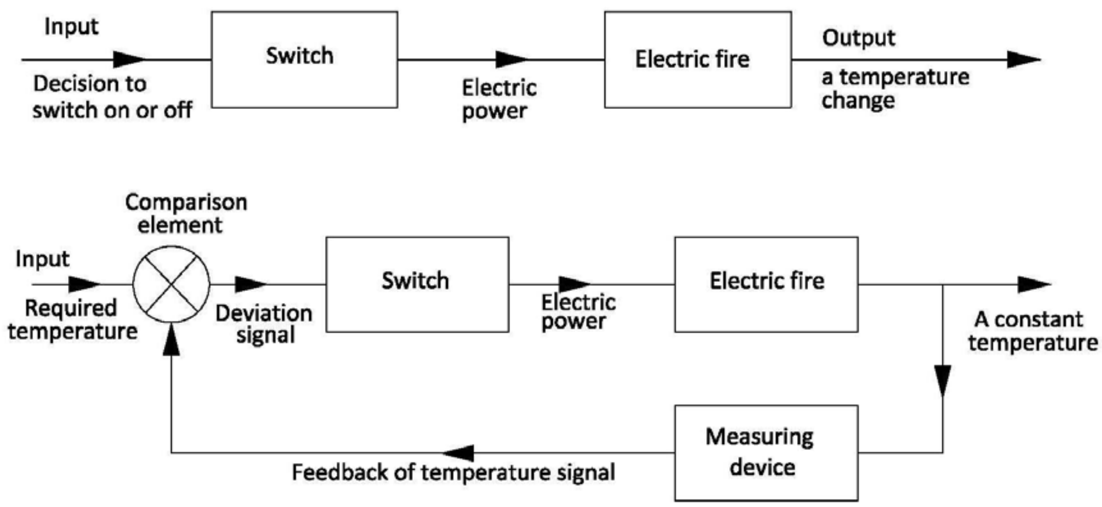


Figure 1.1

Kirchhoff's Law or Low of Governing

Law 1: The total current flowing towards a junction is equal to the total flowing from the junction, i.e. the algebraic sum of the currents at the junction is zero.

Law 2: In a closed circuit or loop, the algebraic sum of the potential differences across each part of the circuit is equal to the applied e.m.f.

A convenient way of using law 1 is called node analysis since the law is applied to each principal node of a circuit, a node being a point of connection or junction between building blocks or circuit elements and principal node being one where three or more branches of the circuit meet. A convenient way of using law 2 is called mesh analysis since the law is applied to each mesh, a mesh being a closed path or loop which contains no other loop.

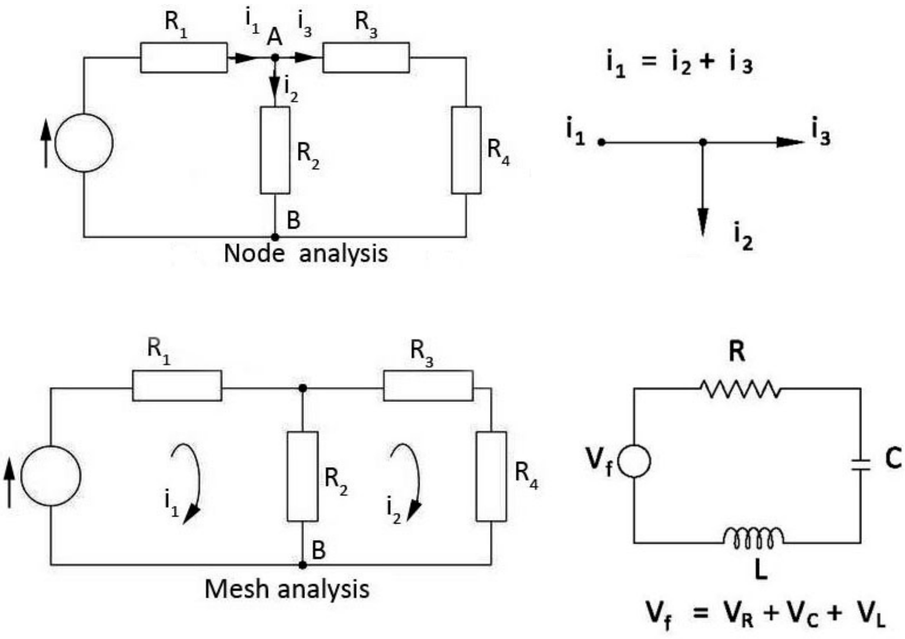


Figure 1.2

SYSTEM MODELS

MECHANICAL TRANSLATIONAL SYSTEMS

The model of mechanical translational systems can be obtained by using three basic elements Mass, Spring and Dash-Pot. These three elements represent three essential phenomena which occur in various ways in mechanical systems.

List of Symbols Used In Mechanical Translational System

x = Displacement, m.

v = $\frac{dx}{dt}$Velocity, m/sec^2

a= $\frac{dv}{dt}$= $\frac{d^2x}{dt^2}$Acceleration, m/sec^2

f = Applied force, N (Newton's)

f_m = Opposing force offered by mass of the body, N

f_k = Opposing force offered by the elasticity of the body (spring), N

f_b = Opposing force offered by the friction of the body (dash – pot), N

M = Mass, kg

K = Stiffness of spring, N/m

B = Viscous friction co – efficient, N – sec/m

Note: Lower case letters are functions of time.

Mass

Consider an ideal mass element shown in figure1.3which has negligible friction and elasticity. Let a force be applied on it. The mass will offer an opposing force which is proportional to acceleration of the body.

Let f = Applied force

f_m= Opposing force due to mass

Here **$f_m \alpha$ a**

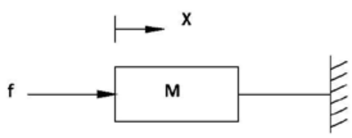


Figure 1.3

$f_m \, \alpha \frac{d^2x}{dt^2}$ or $f_m = M \frac{d^2x}{dt^2}$

By Newton's second law, $f = f_m = M\frac{d^2x}{dt^2}$............. (1)

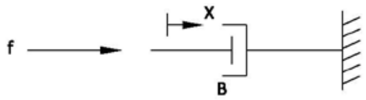


Figure 1.4

Dash-Pot

Consider an ideal frictional element dashpot shown in figure 1.4 which has negligible mass and elasticity. Let a force be applied on it. The dash-pot will offer an opposing force which is proportional to velocity of the body.

Let f = Applied force

f_b = Opposing force due to friction

Here, **$f_b \alpha v$**

$f_b \alpha \frac{dx}{dt}$ or $fb = B\frac{dx}{dt}$

By Newton's second law, $f = f_b = B\frac{dx}{dt}$ .... (2)

When the dashpot has displacement at both ends as shown in figure 1.5 the opposing force is proportional to differential velocity.

$f_b \, \alpha \frac{d(x_1 - x_2)}{dt}$; $f_b = B\frac{d(x_1 - x_2)}{dt}$

$\therefore f = f_b = B\frac{d(x_1 - x_2)}{dt}$ .... (3)

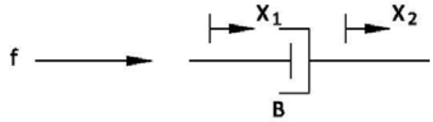


Figure 1.5

Spring

Consider an ideal elastic element spring shown in figure 1.6 which has negligible mass and friction. Let a force be applied on it. The spring will offer an opposing force which is proportional to displacement of the body.

Let f = Applied force

f_k = opposing force due to elasticity

Here $f_k \propto x or f_k = Kx$

By Newton's second law, $f = f_k = Kx$ .... (4)

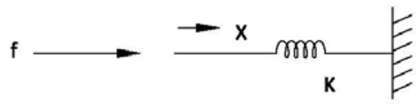


Figure 1.6

When the spring has displacement at both ends as shown in figure 1.7 the opposing force is proportional to differential displacement.

$$f_k \propto (x_1 - x_2)$$
$$f_k = K(x_1 - x_2)$$
$$\therefore f = f_k = K(x_1 - x_2) \quad ....(5)$$

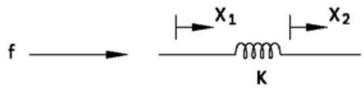


Figure 1.7

1] Problems

Write the differential equations governing the mechanical system shown in figure 1 and determine the transfer function.

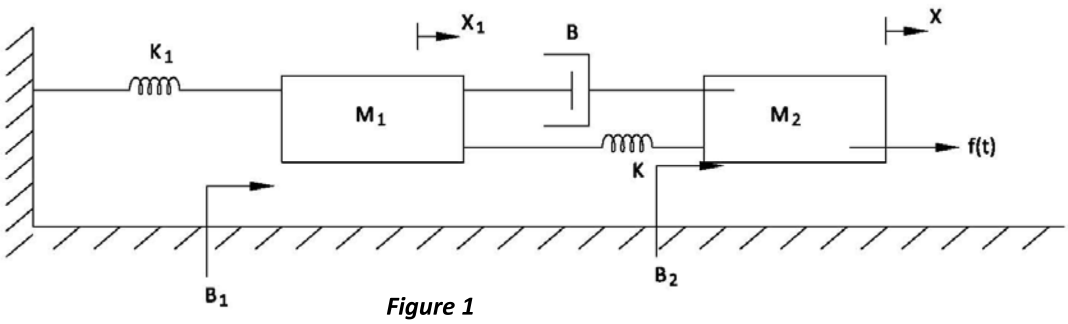


Figure 1

SOLUTION

In the given system, applied force f(t) is the input and displacement x is the output.

Let Laplace transform of f (t) =$L[f(t)] = F(s)$

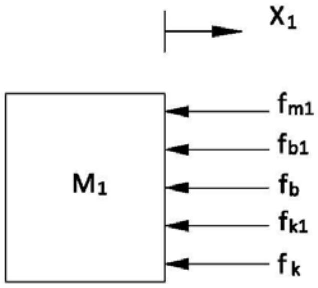
And Laplace transform of x = L [x] = X (s)

Hence the required transfer function is $\frac{X(s)}{F(s)}$

The system has two nodes and they are mass $M_1$and M_2. The free body diagram of mass $M_1$is shown in figure 2. The opposing forces acting on mass M_1 are marked as f_{m1} ,f_{b1} ,f_b ,$f_{k1}$and f_k

$f_{m1} = M_1 \frac{d^2 x_1}{dt^2}$; $f_{b1} = B_1 \frac{dx_1}{dt}$; $f_{k1} = K_1 x_1$

$$B \frac{d}{dt}(x_1 - x_2); \; f_k = K\,(x_1 - x)$$



By Newton's second law

$$f_{m1} + f_{b1} + f_b + f_{k1} + f_k = 0$$

$\therefore M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x) + K_1 x_1 + K(x_1 - x) = 0$.......... (1)

On taking Laplace transform with zero initial conditions

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs[X_1(s) - X(s)] + K_1 X_1(s) + K[X_1(s) - X(s)] = 0$$

$$X_1(s)[M_1 s^2 + (B_1 + B)s + (K_1 + K)] - X(s)[Bs + K] = 0$$

$$X_1(s)[M_1 s^2 + (B_1 + B)s + (K_1 + K)] = X(s)[Bs + K]$$

$\therefore$ X_1 (s) = X (s) $\frac{Bs+K}{M_1 s^2 + (B_1 + B)s + (K_1 + K)}$ .... (2)

Note: Laplace transform of $x_1 = L[x] = X_1(s)$

The free body diagram of mass $M_2$is shown in figure 3. The opposing forces acting on M_2 are marked as f_{m2}, f_{b2}, f_b and f_k.

$F_{m2} = M_2 \frac{d^2 x}{dt^2}$; $f_{b2} = B_2 \frac{dx}{dt}$

$$f_b = B \frac{d}{dt}(x - x_1) \quad ; \quad f_k = K(x - x_1)$$

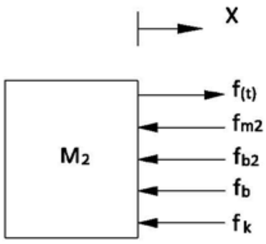


Figure 3

By Newton's second law

$$f_{m2} + f_{b2} + f_b + f_k = f(t)$$

$M_2 \frac{d^2 x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$ .... (3)

On taking Laplace transform with zero initial conditions

$$M_2 s^2 X(s) + B_2 s X(s) + Bs[X(s) - X_1(s)] + K[x(s) - X_1(s)] = F(s)$$

$X(s)[M_2 s^2 + (B_2 + B)s + K] - X_1(s)[Bs + K] = F(s)$ .... (4)

Substituting for X_1 (s) from equation (2) in equation (4) we get,

$$X(s)\,[M_2s^2 + (B_2 + B)s + K] - X(s)\frac{(Bs+K)^2}{M_1s^2 + (B_1+B)s + K_1 + K)} = F(s)$$

$$X(s)\left[\frac{[M_1s^2 + B_1 + B)s + (K_1+K)]\;[M_2s^2 + (B_2+B)s + K] - (Bs+K)^2}{M_1s^2 + (B_1+B)\,s + (K_1+K)}\right] = F(s)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{M_1s^2 + (B_1 + B)s + (K_1 + K)}{[M_1s^2 + (B_1 + B)s + (K_1 + K)]\,[M_2s^2 + (B_2 + B)s + K] - (Bs + K)^2]}$$

RESULT

The differential equations governing the system are

1. $M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x) + K_1x_1 + K(x_1 - x) = 0$
2. $M_2\frac{d^2x}{dt^2} + B_2\frac{dx}{dt} + B\frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$

The transfer function of the system is

$$\frac{X(s)}{F(s)} = \frac{M_1s^2 + (B_1 + B)s + (K_1 + K)}{[M_1s^2 + (B_1 + B)s + (K_1 + K)][M_2s^2 + (B_2 + B)s + K] - (Bs + K)^2}$$

MECHANICAL ROTATIONAL SYSTEMS

The model of rotational mechanical systems can be obtained by using three elements, moment inertia [J] of mass, das-pot with rotational frictional coefficient [B] and torsional spring with stiffness [K].

List of Symbols Used In Mechanical Rotational System

θ = Angular displacement, rad

$\frac{d\theta}{dt}$ = Angular velocity, rad/sec

$\frac{d^2\theta}{dt^2}$ = Angular acceleration, rad/sec^2

T = Applied torque, N-m

J = Moment of inertia, kg-m^2/rad

B = Rotational frictional coefficient, N-m / (rad/sec)

K = Stiffness of the spring, N-m/rad

Mass

Consider an ideal mass element shown figure 1.8which has negligible friction and elasticity. The opposing torque due to moment of inertia is proportional to the angular acceleration.

Let T = Applied torque

T_j = Opposing torque due to moment of inertia of the body.

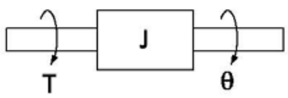


Figure 1.8

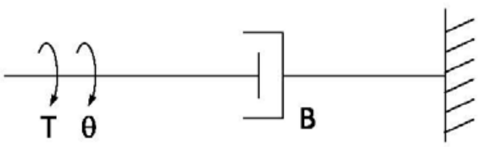
Here $T_j \propto \frac{d^2\theta}{d^t}$ or $T_j = J\frac{d^2\theta}{dt^2}$

By Newton's second law

$T = T_j = J\frac{d^2\theta}{dt^2}$ .... (1)

Dash pot

Consider an ideal frictional element dash pot shown in figure which has negligible moment of inertia and elasticity. Let a torque be applied on it. The dash pot will offer on opposing torque which is proportional to the angular velocity of the body.



Let T = Applied torque

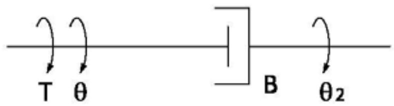
T_b= Opposing torque due to friction

$$T_b\,\alpha\frac{d\theta}{dt} or T_b = B\frac{d\theta}{dt}$$

By Newton's second law

T = $T_b = B\frac{d\theta}{dt}$ .... (2)

When the dash point has angular displacement at both ends as shown in fig. the opposing torque is proportional to the differential angular velocity.



$T_b\,\alpha\frac{d}{dt}(\theta_1-\theta_2)$ $T_b = B\frac{d}{dt}\ (\theta_1-\theta_2)$

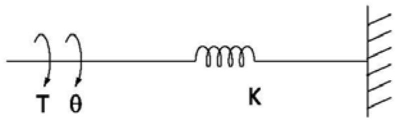
$\therefore T = T_b = B\frac{d}{dt}(\theta_1 - \theta_2)$ .... (3)

Spring

Consider an ideal elastic element, torsional spring as shown in fig. which has negligible moment of inertia and friction. Let a torque be applied on it. The torsional spring will offer an opposing torque which is proportional to angular displacement of the body.

Let T = Applied torque.

T_k = Opposing torque due to elasticity



$T_k\alpha\theta$ $T_k = K\theta$

By Newton's second law

$T = T_k = K\theta$ .... (4)

When the spring has angular displacement at both ends shown in fig. the opposing torque is proportional to differential angular displacement.

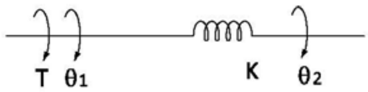


Figure 1.9

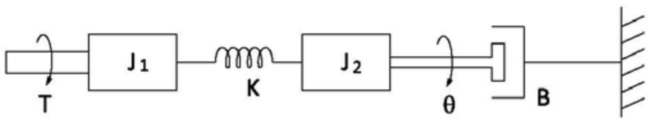
$T_k\,\alpha(\theta_1-\theta_2)$

$T_k = K\ (\theta_1-\theta_2)$

$\therefore T = T_k = K\ (\theta_1 - \theta_2)$ .... (5)

2] Problem

Write the differential equations governing the mechanical rotational system shown in figure 1. Obtain the transfer function of the system.



(Applied torque) (Output)

Figure 1

SOLUTION

In the given system, applied torque T is the input and angular displacement θ is the output.

Let Laplace transform of T = L [T] =T(s)

And Laplace transform of θ = L [θ] = θ(s)

Hence the required transfer function is$\frac{\theta(s)}{T(s)}$

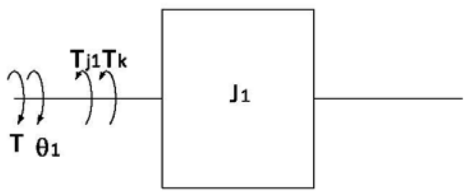


Figure 2

The system has two nodes and they are masses with moment of inertia J_1andJ_2. The differential equations governing the system are given by torque balance equations at these nodes.

Let the angular displacement of mass with moment of inertia J_1 beθ_1. The Laplace transform of θ_1 = L [θ_1] = θ_1 (s). The free body diagram of J_1 is shown fig. The opposing torques acting on J_1 are marked as

$$T_{j1} \text{ and} T_k$$

$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2}$$

$$\therefore T_k = K\,(\theta_1 - \theta)$$

By Newton’s second law,

$$T_{j1} + T_k = T$$

$$J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$$

$$J_1 \frac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta = T \quad \text{.............. (1)}$$

On taking Laplace transform with zero initial conditions,

$$J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$$

$$(J_1 s^2 + K)\theta_1(s) - K\theta(s) = T(s) \quad \text{........... (2)}$$

The free body diagram of mass with moment of inertia J_2 is shown in figure 3. The opposing torques acting on $J_2$are marked as T_{j2}, T_b and T_k.

$$T_{j2} = J_2 \frac{d^2\theta}{dt^2}\,; \qquad T_b = B\frac{d\theta}{dt}$$

and $T_k = K(\theta - \theta_1)$

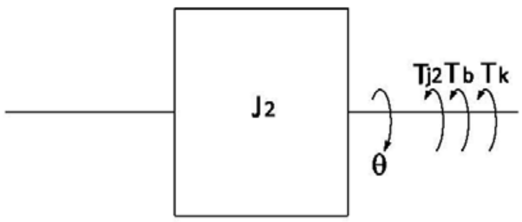


Figure 3

By Newton’s second law,

$$T_{j2} + T_b + T_k = 0$$

$$\therefore J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0 \quad \text{........... (3)}$$

On taking Laplace transform with zero initial conditions,

$$J_2 s^2 \theta(s) + Bs\theta(s) + K\theta(s) - K\theta_1(s) = 0$$

$$(J_2 s^2 + Bs + K)\theta(s) - K\theta_1(s) = 0$$

$$\theta_1(s) = \frac{(\boldsymbol{j_2 s^2 + Bs + K})}{K}\,\theta(s) \quad \text{........... (4)}$$

Substituting for θ_1(s) from equation (4) in equation (2) we get,

$$(J_1 s^2 + K)\frac{(\boldsymbol{J_2 s^2 + Bs + K})}{K}\theta(s) - K\theta(s) = T(s)$$

$$\left[\frac{(\boldsymbol{J_1 s^2 + K})\,(\boldsymbol{J_2 s^2 + Bs + K}) - \boldsymbol{K^2}}{K}\right]\theta(s) = T(s)$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{K}{(J_1 s^2 + K)\,(J_2 s^2 + Bs + K) - K^2} \quad \text{............. (5)}$$

RESULT

The differential equations governing the system are

1. $J_1 \frac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta = 0$

2. $J_2 \frac{d^2\theta_1}{dt^2} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0$

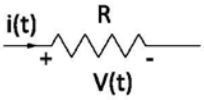
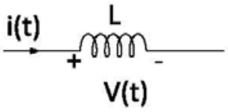
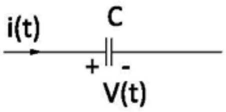
The transfer function of the system is $\frac{\theta(s)}{T(s)} = \frac{K}{(J_1s^2+K)(J_2s^2+Bs+K)-K^2}$

ELECTRICAL SYSTEMS

The models of electrical systems can be obtained by using resistor, capacitor and inductor. The current-voltage relation of resistor, inductor and capacitor are given in table 1.1. For modeling electrical network or equivalent circuit is formed by using R, L and voltage or current source.

The differential equations governing the electrical systems can be formed by writing Kirchoff's current law equations by choosing various nodes in the network or Kirchoff's voltage law equations by choosing various closed path in the network. The transfer function can be obtained by taking Laplace transform of the differential equations and rearranging them as a ratio of output to input.

Table 1.1: Current-voltage relation of R, L and C

Element	Voltage across the element	Current through the element
	$v(t) = Ri(t)$	$i(t) = \frac{v(t)}{R}$
	$v(t) = L\frac{d}{dt}i(t)$	$i(t) = \frac{1}{L}\int v(t)dt$
	$vt = \frac{1}{c}\int i(t)dt$	$i(t) = C\frac{dv(t)}{dt}$

3] Problem

Obtain the transfer function of the electrical network shown in figure 1

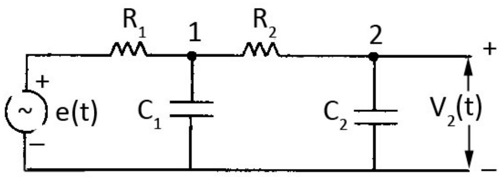


Figure 1

SOLUTION

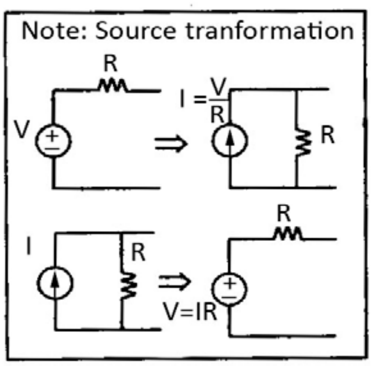
In the given network input is e(t) and output is V_2 (t)

Let Laplace transform of e(t) = L[e(t)] = E(s)

Laplace transform of $V_2(t) = L[V_2(t)] = V_2(s)$

The transfer function of the network is $\frac{V_2(s)}{E(s)}$

Transform the voltage source in series with resistance R_1 into equivalent current source as shown in figure. The network has two nodes. Let the node voltage be v_1 and v_2. The Laplace transform of node voltages v_1 and v_2 are $v_1(s)$ and $v_2(s)$ respectively. The differential equations governing the network are given by the Kirchoff's current law equations at these nodes.



At node 1, by Kirchoff's current law

$\frac{V_1}{R_1} + C_1\frac{dv1}{dt} + \frac{v_1-v_2}{R_2} = \frac{e}{R_1}$........(1)

On taking Laplace transform with zero initial conditions

$\frac{V_1}{R_1} + C_1sV_1(s) + \frac{V_1(S)}{R_2} - \frac{V_2(S)}{R_2} = \frac{E(S)}{R_1}$

$V_1(S)\left[\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right] - \frac{V_2(S)}{R_2} = \frac{E(S)}{R_1}$ .... (2)

At node 2, by Kirchoff's law

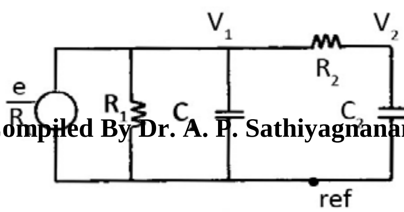


Figure 2

$$\frac{V_2 - V_1}{R_2} + C_2\frac{dV_2}{\mathrm{dt}} = 0 \qquad \text{.......... (3)}$$

On taking Laplace transform of equation (3) with zero initial condition

$$\frac{V_2(s)}{R_2} - \frac{V_1(s)}{R_2} + C_2S\ V_2(s) = 0$$

$$\frac{V_1(s)}{R_2} = \frac{V_2(s)}{R_2} + C_2S\ V_2(s) = \left[\frac{1}{R_2} + sC_2\right] V_2\ (s)$$

$$\therefore V_1(S)\ = [1 + sC_2R_2]V_2(s)$$

Substituting for $V_1(s)$ from equation (4) in equation (2)

$$(1 + sR_2C_2)\ V_2(s) = \left[\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1}$$

$$\left[\frac{(1+sR_2C_2)(R_2+R_1+sC_1R_1R_2) - R_1}{R_1R_2}\right] V_2\ (s) = \frac{E(s)}{R_1}$$

$$\therefore \frac{V_2(s)}{E(s)} = \left[\frac{R_2}{(1+sR_2C_2)(R_2+R_1+sC_1R_1R_2) - R_{1_1}}\right]$$

RESULT

The (node basis) differential equations governing the electrical network are

1. $\frac{V_1}{R_1} + C_1\frac{dv1}{dt} + \frac{v_1 - v_2}{R_2} = \frac{e}{R_1}$

2. $\frac{V_2 - V_1}{R_2} + C_2\frac{dV_2}{\mathrm{dt}} = 0$

3. $\frac{V_2(s)}{E(s)} = \left[\frac{R_2}{(1+sR_2C_2)(R_2+R_1+sC_1R_1R_2) - R_{1_1}}\right]$

<u>Thermal System</u>

List of symbols used in thermal systems

q	=	Heat flow rate, Kcal/sec
θ_1	=	Absolute temperature of emitter, °K
θ_2	=	Absolute temperature of receiver, °K
$\Delta\theta$	=	Temperature difference, °C
A	=	Area normal to heat flow, m3
K	=	Conduction or convection coefficient, Kcal/sec-°C
Kr	=	Radiation coefficient, Kcal/sec-°C
H	=	K/A = Convection coefficient, Kcal/m-sec-°C
K	=	Thermal conductivity, Kcal/m-sec °C
ΔX	=	Thickness of conductor, m
R	=	Thermal resistance, °C-sec/Kcal
C	=	Thermal capacitance, Kcal/°C

Heat flow rate

Thermal systems are those that involve the transfer of heat from one substance to another. There are three different ways of heat flow from one substance to another. They are conduction, convection and radiation.

For conduction,

Heat flow rate, q = K$\Delta\theta$ = $\frac{KA}{\Delta X}$ ………………(1)

For convection,

Heat flow rate, q = K$\Delta\theta$= HA$\Delta\theta$ …………….. (2)

For radiation,

Heat flow rate, q = Kf$(\theta_1^4 - \theta_2^4)$

If $\theta_1 >> \theta_2$ then, q = $K_r\bar{\theta}^4$ ………………. (3)

Where $\bar{\theta}^4 = (\theta_1^4 - \theta_2^4)^{\frac{1}{4}}$

Note: $\bar{\theta}^4$ is called effective temperature difference of the emitter and receiver.

Basic elements of thermal system

The models of thermal system are obtained by using thermal resistance and capacitance which are the basic elements of the thermal system.

The thermal resistance and capacitance are distributed in nature. But for simplicity in analysis lumped parameter mode is used. In lumped parameter model it is assured that the substances that are characterized by resistance to heat flow have negligible heat capacitance and the substances that are characterized by heat capacitance have negligible resistance to heat flow.

The thermal resistance, R for heat transfer between two substances is defined as the ratio of change in temperature and change in heat flow rate.

Thermal resistance, R = $\frac{\text{Change in Temperature , °C}}{\text{Change in heat flow rate, Kcal/sec}}$

For conduction or convection,

Heat flow rate, q = K$\Delta\theta$

On differentiating we get,

dq = K d($\Delta\theta$)

$\therefore \frac{d(\Delta\theta)}{dq} = \frac{1}{K}$

But thermal resistance, R = $\frac{d(\Delta\theta)}{dq}$

$\therefore$ Thermal resistance, R = $\frac{1}{K}$ for conduction …………… (1)

= $\frac{1}{K} = \frac{1}{HA}$ for convection ………… (2)

For radiation,

Heat flow rate, q = $K_r\bar{\theta}^4$

On differentiating we get

dq = Kr 4 $\bar{\theta}^3$ d $\bar{\theta}$

$\therefore \frac{d(\Delta\theta)}{dq} = \frac{1}{K}$

But thermal resistance, R = $\frac{d\bar{\theta}}{dq}$

$\therefore$ Thermal resistance, R = $\frac{1}{K_r 4\bar{\theta}^3}$ ………… (3)

(for radiation)

Thermal capacitance, C is defined as the ratio of change in heat stored and changes in temperature

Thermal capacitance, C = $\frac{\text{Change in Temperature , °C}}{\text{Change in heat flow rate , Kcal /sec}}$

Let M = Mass of substance considered, kg

Cp = Specific heat of substance, Kcal/kg-°C

Now, Thermal capacitance, C = Mc_p ………… (4)

EXAMPLE OF THERMAL SYSTEM

Consider a simple thermal system shown in figure 1.10. Let us assume that the tank is insulated to eliminate heat loss of the surrounding air, there is no heat storage in the insulation and liquid in the tank is kept at uniform temperature by perfect mixing with the help of a stirrer. Thus, a single temperature is used to describe the temperature of the liquid in the tank and of the out flowing liquid. The transfer function of this system can be derived as shown below.

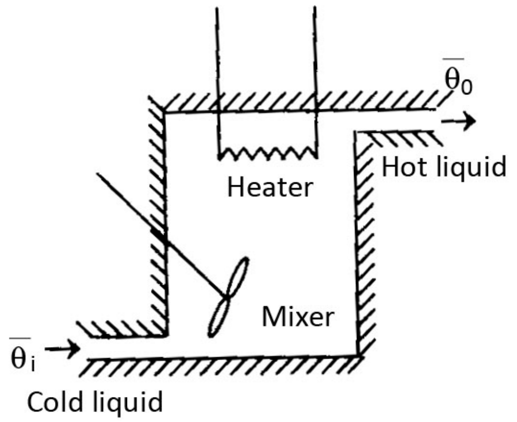


Figure 1.10 Thermal system

Let

$\bar{\theta}_i$ = Steady state temperature of inflowing liquid, °C

$\bar{\theta}_0$ = Steady state temperature of out flowing liquid,

G = Steady state liquid flow rate, kg/sec

M = Mass of liquid in tank, kg.

c = Specific heat of liquid, Kcal/kg °C

R = Thermal resistance, °C – sec/Kcal.

C = Thermal capacitance, Kcal/°C

Q = Steady state heat input rate. Kcal/sec

Let us assume that the temperature of inflowing liquid is kept constant. Let the heat input rate to the system supplied by the heater is suddenly changed from $\bar{Q}$ to $\bar{Q}$ +q_i. Due to this, the heat output flow rate will gradually change from $\bar{Q}$ to $\bar{Q}$ + q_0. The temperature of the out flowing liquid will also be changed from $\bar{\theta}_0$ to $\bar{\theta}_0 + \theta$.

For this system the equation for q_0, C and R are obtained as follows,

Change in output heat flow rate, q_0

= Liquid flow rate, G × Specific heat of liquid,c× Changing temperature, θ

$$= Gc\theta \qquad \text{………… (1)}$$

Thermal capacitance, C = Mass, M × specific heat of liquid, c

$$= Mc \qquad \text{………… (2)}$$

Thermal resistance, R $= \frac{\text{Change in Temperature , } \theta}{\text{Change in heat flow rate , } q_0}$

$$= \frac{\theta}{q_0} \qquad \text{………… (3)}$$

On substituting for q_0 from equation (1) in equation (3) we get,

$$R = \frac{\theta}{q_0} = \frac{1}{G_c} \qquad \text{………… (4)}$$

In this system, the rate of change of temperature is directly proportional to change in heat input rate.

$$\therefore \frac{d\theta}{dt} \alpha q_i - q_0$$

The constant of proportionality in the capacitance C of the system.

$$\therefore C \frac{d\theta}{dt} \alpha\ qi - q_0 \qquad \text{..........} \quad (5)$$

Equation (5) is the differential equation governing the system. Since equation (5) is of first order equation, the system is first order system.

From equation (1.20), $R = \frac{\theta}{q_0}$, $\quad \therefore\ q_0 = \frac{\theta}{R}$ .... (6)

On substituting for q_0 from equation (6) in equation (5) we get,

$$C \frac{d\theta}{dt} = qi - \frac{\theta}{R}$$

$$C \frac{d\theta}{dt} = qi - \frac{Rq_i - \theta}{R}$$

$$RC \frac{d\theta}{dt} = Rqi - \theta$$

$$RC \frac{d\theta}{dt} + \theta = Rq_i \qquad \text{..........} \quad (7)$$

Let, $L[\theta] = \theta(s)$; $L\left[\frac{d\theta}{dt}\right] = s\theta(s)$ and $L[q_i] = Q_i(s)$

On taking Laplace transform of equation (7) we get,

$$RC\, s\, \theta(s) + \theta(s) = R\, Q_i(s)$$

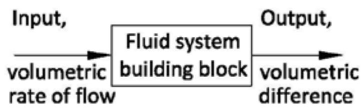
$$\theta(s)\ [sRc + 1\} = R\, Q_i(s)$$

$\frac{\theta(s)}{Q_1(s)}$ is the required transfer function of the system

$$\therefore \frac{\theta(s)}{Q_1(s)} = \frac{R}{sRC+1} = \frac{R}{RC\left(s+\frac{1}{RC}\right)} = \frac{\frac{1}{C}}{s+\frac{1}{RC}} \qquad \text{.........} \quad (8)$$

FLUID SYSTEM BUILDING BLOCKS

In fluid flow systems there are three basic building block which can be considered to be the equivalent of electrical resistance, capacitance and inductance. For such systems (figure 8.19) the input, the equivalent of the electrical current, is the volumetric rate of flow q, and the output, the equivalent of electrical potential difference, is pressure difference ($p_1 - p_2$). Fluid systems can be considered to fall into two categories: Hydraulic, where the fluid is a liquid and is dement to be incompressible; and pneumatic, where it is a gas which can be compressed and consequently shows a density changes.



ELECTRICAL EQUIVALENT FOR MECHANICAL SYSTEMS

Transfer function of armature controlled dc motor

The speed of dc motor is directly proportional to armature voltage and inversely proportional to flux is field winding. In armature controlled dc motor the desired speed is obtained by varying the armature voltage. This speed control system is an electro-mechanical control system. The electrical system consists of the armature and the field circuit but for analysis purpose, only the armature circuit is considered because the field is excited by a constant voltage. The mechanical system consists of the rotating part of the motor and load connected to the shaft of the motor. The armature controlled dc motor speed control system is shown in figure.

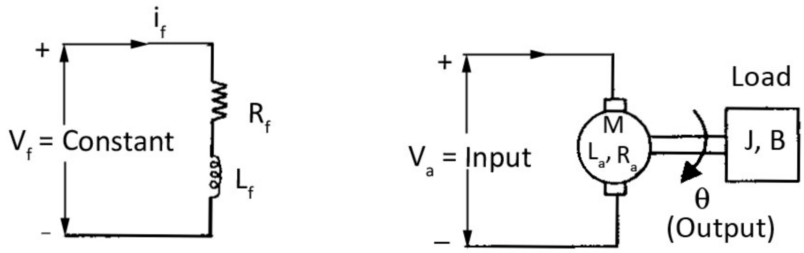


Figure 1.12Armature controlled DC motor

Let

R_a = Armature resistance, Ω

L_a = Armature inductance, H

i_a = Armature current, A

v_a = Armature voltage, V

e_b = Back e.m.f, V

Kt = Torque constant, N-m/A

T = Torque developed by motor, N-m

θ = Angular displacement of shaft, rad.

J = Moment of inertia of motor and load, kg-m^2/rad.

B = Frictional coefficient of motor and load, N-m(rad./sec)

K_b = Backe.m.f constant, V/(rad./sec)

The equivalent circuit of armature is shown in figure 1.13

By Kirchoff's voltage law, we can write

$$I_a R_a + L_a \frac{di_a}{dt} + e_b = v_a \quad \text{............(1)}$$

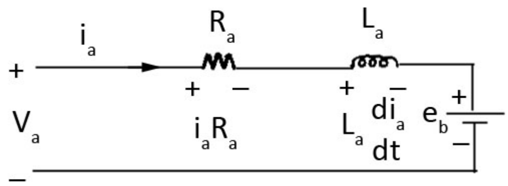


Figure 1.13 Equivalent circuit of armature

Torque of dc motor is proportional to the product of flux and current. Since flux is constant in this system, the torque is proportional to i_a alone.

$$T \propto i_a$$

$$\therefore \text{ Torque, } T = K_t i_a \quad \text{......... (2)}$$

The mechanical system of the motor is shown in figure 1.14. The differential equation governing the mechanical system of motor is given by

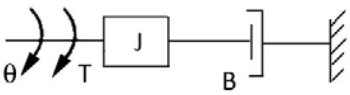


Figure 1.14

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{........... (3)}$$

The back e.m.f for dc machine is proportional to speed (angular velocity) of shaft

$$\therefore e_b \propto \frac{d\theta}{dt} \quad ; \quad \text{Back e.m.f, } e_b = K_b\frac{d\theta}{dt} \quad \text{...........(4)}$$

The Laplace transform of various times domain signals involved in this system are shown below.

$L[V_a] = V_a(s)$ $\quad$ $L[e_b] = E_b(s)$ $\quad$ $L[T] = T(s)$

$L[i_a] = I_a(s)$ $\quad$ $L[\theta] = \theta(s)$

The differential equations governing the armature controlled dc motor speed control system are

$$i_a R_a + L_a\frac{di_a}{dt} + e_b = v_a \qquad J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T$$

$$T = K_t i_a \qquad e_b = K_b\frac{d\theta}{dt}$$

On taking Laplace transform of the system differential equations with zero initial conditions we get

$I_a(s)\ R_a + L_a s I_a(s) + E_b(s) = V_a(s)$ ………….. (5)

$T(s) = KtIa(s)$ ………….. (6)

$Js^2\theta(s) + B\,s\theta(s) = T(s)$ ………….. (7)

$E_b(s) = K_b s\theta(s)$ ………….. (8)

On equating equations (6) and (7) we get,

$$K_t I_a(s) = (Js^2 + Bs)\,\theta(s)$$

$$I_a(s) = \frac{(Js^2+Bs)}{K_t}\theta(s) \qquad \text{………….. (9)}$$

Equation (5) can be written as

$$(R_a + sL_a)\ I_a(s) + E_b(s) = V_a(s) \qquad \text{………….. (10)}$$

Substituting for $E_b(s)$ and $I_a(s)$ from equation (8) and (9) respectively in equation (10),

$$(R_a + sL_a)\ \frac{(Js^2+Bs)}{K_t}\theta(s) + K_b s\theta(s) = V_a(s)$$

$$\left[\frac{(R_a+sL_a)(Js^2+Bs)K_b K_t s}{K_t}\right]\theta(s) = V_a(s)$$

The required transfer function is $\theta(s)/V_a(s)$

$$\therefore \frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a+sL_a)\ (Js^2+Bs)+K_b K_t s} \qquad \text{………….. (11)}$$

$$= \frac{K_t}{R_a J s^2 + R_a Bs + L_a Js^3 + L_a Bs^2 + K_b K_t s}$$

$$= \frac{K_t}{[JL_a s^2 + (JR_a + BL_a)s + (BR_a + K_b K_t)]}$$

$$= \frac{K_t/JL_a}{a\left[s^2 + \left(\frac{JR_a+BL_a}{JL_a}\right)s + \left(\frac{BR_a+K_b K_t}{JL_a}\right)\right]} \qquad \text{………….. (12)}$$

The transfer function of armature controlled dc motor can be expressed in another standard from as shown below

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a+sL_a)\ (Js^2+Bs)+K_b K_t s}$$

$$= \frac{K_t}{R_a\left(\frac{sL_a}{R_a}+1\right) Bs\ \left(\frac{Js^2}{Bs}\right)+K_b K_t s}$$

$$= \frac{K_t/R_a B}{s\left[(1+sT_a)(1+sT_m)+\left(\frac{K_b+K_t}{R_a B}\right)\right]} \qquad \text{………….. (13)}$$

Where, La/Ra = Ra = Electrical time constant

And J/B = Tm = Mechanical time constant

Transfer function of field controlled dc motor

The speed of dc motor is directly proportional to armature voltage and inversely proportional to flux. In field controlled dc motor the armature voltage is kept constant and the speed is varied by varying the flux of the machine. Since flux is directly proportional to field current, the flux is varied by varying field current. The speed control system is an electromechanical control system. The electrical system consists of armature and field circuit but for analysis purpose, only field circuit is considered because the armature is excited by a constant voltage. The mechanical system consists of the rotating part of the motor and the load connected to the shaft of the motor. The field controlled dc motor speed control system is shown in figure 1.15.

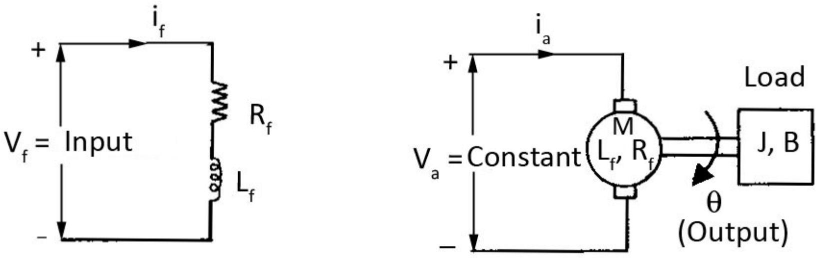


Figure 1.15 Field controlled DC motor

Let

R_f	= Field resistance, Ω
Lf	= Field inductance, H
if	= Field current, A
vf	= Field voltage, V
T	= Torque developed by motor, N-m
Ktf	= Torque constant, N-m/A
J	= Moment of inertia of motor and load, kg=m^3/rad.
B	= Frictional coefficient of motor and load, N-m/(rad./sec)

The equivalent circuit of field is shown in figure 1.16

By Kirchoff's voltage lad, we can write

$$R_f\, i_f + L_f\frac{di_f}{dt} = v_f \qquad \text{........ (1)}$$

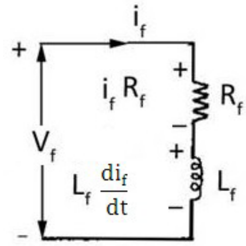


Figure 1.16 Equivalent circuit

The torque of dc motor is proportional to product of flux and armature current. Since armature current is constant in this system, the torque is proportional to flux alone, but flux is preoperational to field current.

$$T \propto e_f, \ \therefore \text{ Torque, } T = K_{if}\, i_f \qquad \text{..... (2)}$$

The mechanical system of the motor is shown in figure 1.17. The differential equation governing the mechanical system of the motor is given by

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \qquad \text{....... (3)}$$

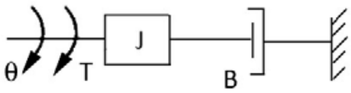


Figure 1.17

The Laplace transform of various time domain signals involved in this system are shown below

$L[i_f] = I_f(s)$; $\quad L[T] = T(s)$; $\quad L[v_f] = V_f(s)$; $\quad L[\theta] = \theta(s)$

The differential equations governing the field controlled dc motor are

$$R_f\, i_f + L_f\frac{di_f}{dt} = v_f; \quad T = K_{tf}\, i_f\,; \qquad J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T$$

On taking Laplace transform of the system differential equation, we get

$$R_f\, I_f(s) + L_f\, s\, I_f(s) = V_f(s) \qquad \text{....... (4)}$$

$$T(s) = K_{tf}\, I_f(s) \qquad \text{....... (5)}$$

$$Js^2\theta(s) + B_s\theta(s) = T(s) \qquad \text{....... (6)}$$

Equation (5) and (6) we get,

$$K_{tf}\, I_f(s) = Js^2\theta(s) + B_s\theta(s)$$

$$If(s) = s\frac{(Js+B)}{K_{tf}}\theta(s) \qquad \text{....... (7)}$$

The equation (4) can be written as

$(R_f + sL_f)\ I_f(s) + E_b(s) = V_f(s)$ .... (8)

On Substituting for $I_b(s)$ from equation (7) in equation (8) we get,

$$(R_f + sL_f)\ s\ \frac{(Js+B)}{K_{tf}}\theta(s) = V_f(s)$$

$$\frac{\theta(s)}{V_f(s)} = \frac{K_{tf}}{s(R_f+sL_f)\ (B+sJ)}$$

$$= \frac{K_{tf}}{sR_f\left(1+\frac{sL_f}{R_f}\right)(B+sJ)}$$

$$= \frac{K_m}{s(1+sT_f)(1+sT_m)} \qquad \text{....... (9)}$$

Where, Motor gain constant, $K_m = K_{tf}/R_fB$

Field time constant, $T_f = L_f/R_f$

Mechanical time constant, $T_m = J/B$

Electrical analogous of mechanical translational systems

Systems remain analogous as long as the differential equations governing the systems or transfer functions are of identical from. The electric analogue of any other kind of system is of great importance since it is easier to construct electrical models and analyse them.

The three basic elements mass, dash-pot and spring that are used in modeling mechanical translational systems are analogous to resistance, inductance and capacitance of electrical systems. The input force in mechanical system is analogous to either voltage source or current source in electrical systems. The output velocity (first derivative of displacement) in mechanical system is analogous to either current or voltage in an element in electrical system. Since the electrical systems has two types of inputs either voltage or current source, there are two types of analogies: force-voltage analogy and force-current analogy.

Force-voltage analogy

The force balance equations of mechanical elements and their analogous electrical elements in force-voltage analogy are shown in table 1.2.

The following points serve as guide lines to obtain electrical analogous of mechanical systems based on force-current analogy.

1. In electrical systems element in parallel will have same voltage, likewise in mechanical systems, the elements have same force are said to be in parallel.
2. The elements have same velocity in mechanical system should have analogous same voltage in electrical analogous system.
3. Each node (meeting point of elements) in the mechanical system corresponds to a node in electrical system. A mass is considered as a node.
4. The number of nodes in electrical analogous is same as that of the number of nodes (masses) in mechanical system. Hence the number of node voltages and system equations will be same as that of the number of velocities of (nodes) masses in mechanical system.
5. The mechanical driving sources (forces) and passive elements connected to the node (mass) in mechanical system should be represented by analogous elements connected to a node in electrical system.

Table 1.2

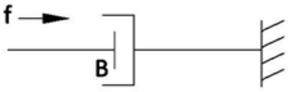
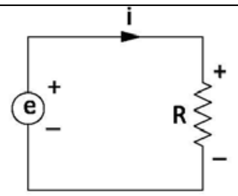
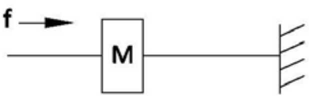
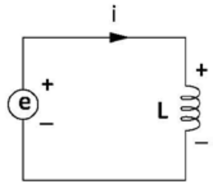
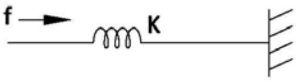
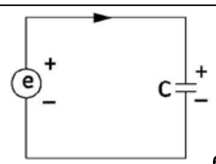
Mechanical system	**Electrical system**
Input: Force : Force Output : Velocity	Input : Voltage source Output : Current through the element
$\mid\!\rightarrow$ x $\mid\!\rightarrow$ v = $\frac{dx}{dt}$  f = B$\frac{dx}{dt}$ = Bv	 e = Rj
$\mid\!\rightarrow$ x $\mid\!\rightarrow$ a = $\frac{d^2x}{dt^2}$= $\frac{dv}{dt}$  f = $\frac{d^2x}{dt^2}$= M $\frac{dv}{dt}$	 e = L $\frac{di}{dt}$
$\mid\!\rightarrow$ x = $\int vdt$ $\mid\!\rightarrow$ v  f = Kx = K$\int vdt$	 e = $\frac{1}{C}\int idt$

Table 1.3 shows the list of analogous quantities in force-voltage analogy

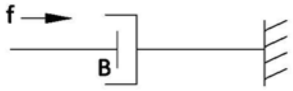
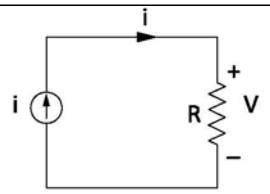
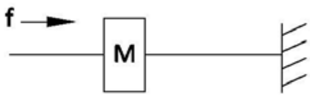
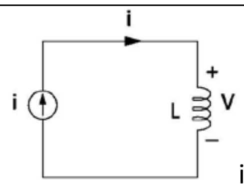
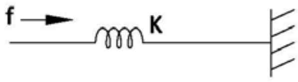
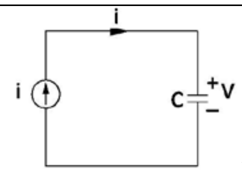
Item	Mechanical systems	Electrical system (mesh basis system)
Independent variable (input)	Force, f	Voltage, e
Dependent variable (output)	Velocity, v	Current, i
	Displacement, x	Charge, q
Dissipative element	Frictional coefficient of dashpot, B	Resistance, R
Storage element	Mass, M	Inductance, L
	Stiffness of spring, K	Inverse of capacitance, I/C
Physical law	Newton's second law $\sum F= 0$	Kirchoff's voltage law $\sum V= 0$
Changing the level of independent variable	Lever $\frac{f_1}{f_2}= \frac{l_1}{l_2}$	Transformer $\frac{e_1}{e_2}= \frac{N_1}{N_2}$

6. The mechanical driving sources (force) and passive elements connected to the node (mass) in mechanical system should be represented by analogous elements in a closed loop in analogous electrical system.
7. The element connected between two (nodes) masses in mechanical system is represented as a common element between two meshes in electrical analogous system.

FORCE- CURRENT ANALOGY

The force balance equations of mechanical elements and their analogous electrical elements in force-current analogy are shown in table 1.4. The table 1.5 shows the list of analogous quantities in force-current analogy

Table 1.4

Mechanical system	Electrical system
Input: Force : Force Output : Velocity	Input : Current source Output : Voltage across the element
$\vert\rightarrow$ x $\vert\rightarrow$ v = $\frac{dx}{dt}$  f = B$\frac{dx}{dt}$ = Bv	 I = $\frac{1}{R}$ V
$\vert\rightarrow$ x $\vert\rightarrow$ a = $\frac{d^2x}{dt^2}$= $\frac{dv}{dt}$  f = $\frac{d^2x}{dt^2}$= M $\frac{dv}{dt}$	 i = $\frac{1}{L}\int vdt$
$\vert\rightarrow$ x = $\int vdt$ $\vert\rightarrow$ v  f = Kx = K$\int vdt$	 i = C $\frac{dv}{dt}$

The following points serve as guide lines to obtain electrical analogous of mechanical systems based on force-current analogy.

1. In electrical systems element in parallel will have same voltage, likewise in mechanical systems, the elements have same force are said to be in parallel.
2. The elements have same velocity in mechanical system should have analogous same voltage in electrical analogous system.
3. Each node (meeting point of elements) in the mechanical system corresponds to a node in electrical system. A mass is considered as a node.

4. The number of nodes in electrical analogous is same as that of the number of nodes (masses) in mechanical system. Hence the number of node voltages and system equations will be same as that of the number of velocities of (nodes) masses in mechanical system.
5. The mechanical driving sources (forces) and passive elements connected to the node (mass) in mechanical system should be represented by analogous elements connected to a node in electrical system.
6. The element connected between two nodes (masses) in mechanical system is represented as a common element between two nodes in electrical analogous elements connected to a node in electrical system.

Table 1.5

Item	**Mechanical systems**	**Electrical system (node basis system)**
Independent variable (input)	Force, f	Current, i
Dependent variable (output)	Velocity, v	Voltage, V
	Displacement, x	Flux, ϕ
Dissipative element	Frictional coefficient of dashpot, B	Conductance G = 1/R
Storage element	Mass, M	Capacitance, C
	Stiffness of spring, K	Inverse of capacitance, I/L
Physical law	Newton's second law $\sum F = 0$	Kirchoff's voltage law $\sum i = 0$
Changing the level of independent variable	Lever $\frac{f_1}{f_2} = \frac{l_1}{l_2}$	Transformer $\frac{i_1}{i_2} = \frac{N_2}{N_1}$

4] Problems

Write the differential equations governing the mechanical system shown in figure 1. Draw the force-voltage and force-current electrical analogous circuits and verify by writing mesh and node equations.

SOLUTION

The given mechanical system has to nodes (masses). The differential equations governing the mechanical system are given by force balance equations at these nodes. Let the displacements of masses M_1 and M_2 be x_1 and x_2 respectively. The corresponding velocities by v_1 and v_2.

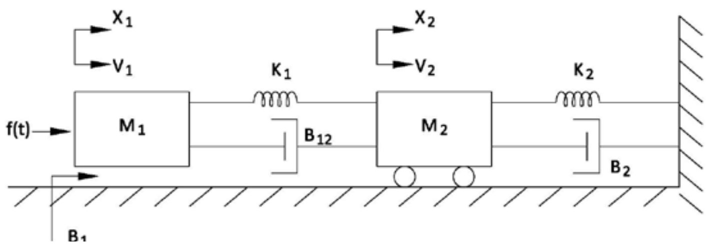


Figure 1

The free body diagram of M1 is shown in figure 2. The opposing forces are marked as f_{m1}, f_{b1}, f_{b12} and f_{k1}

$f_{ml} = M_1 = \frac{d^2x_1}{dt^2}$; $f_{b1} = B_1\frac{dx_1}{dt}$; $f_{b12} = B_{12}\frac{d}{dt}(x_1 - x_2)$ and

$f_{k1} = K_1(x_1 - x_2)$

By Newton's second law, $f_{m1} + f_{b1} + f_{b12} + f_{k1} = f(t)$

$$M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B_{12}\frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) = f(t) \quad \text{.......... (1)}$$

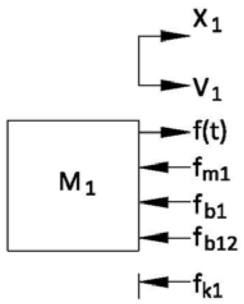


Figure 2

The free body diagram of M_2 is shown in figure 3

The opposing forces are marked as f_{m2}, f_{b2}, f=, f_{k1} and f_{k2}.

$f_{m2} = M_2\frac{d^2x_2}{dt^2}$; $f_{b2} = B_2\frac{dx_2}{dt}$;

$f_{b12} = B_{12}\frac{d}{dt}(x_2 - x_1)$; $f_{k1} = K_1 (x_2 - x_1)$ and $f_{k2} = K_2 x_2$

By Newton's second law, $f_{m2} + f_{b2} + f_{k2} + f_{b12} + f_{k1} = 0$

$M_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + K_2x_2 + B_{12}\frac{d}{dt}(x_2 - x_1) + K_1(x_2 - x_1) = 0$

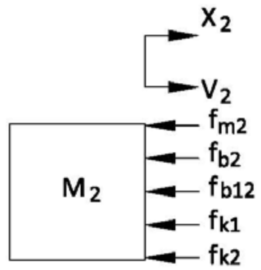


Figure 3

On replacing the displacement by velocity in the differential equations (1) and (2) of the mechanical systems we get,

$$\left(i.e, \frac{d^2x}{dt^2} = \frac{dv}{dt}; \qquad \frac{dx}{dt} = v \ \ and\ x = \int v dt\right)$$

$M_1\frac{dv_1}{dt} + B_1v_1 + B_{12}(v_1 - v_2) + K_1\int(v_1 - v_2)dt = f(t)$

$M_2\frac{dv_2}{dt} + B_2v_2 + K_2\int v_2 dt + B_{12}(v_2 - v_1) + K_1\int(v_2 - v_1)dt = 0$

Force-voltage analogous circuit

The given mechanical system has two nodes (masses). Hence the force-voltage analogous electrical circuit will have two meshes.

The force applied to mass, M_1 is represented by a voltage source in first mesh. The elements M_1, B_1, K_1 and B_{12} are connected to first node. Hence they are represented by analogous elements in mesh 1 forming a closed path. The elements K_1, B_{12}, M_2, K_2 and B_2 are connected to second node. Hence they are represented by analogous element in mesh 2 forming a closed path.

The elements K_1 and B_{12} are common between node 1 and 2 and so they are represented by analogous element as common elements between two meshes. The force-voltage electrical analogous circuit is show in figure 1.8.4.

The electrical analogous elements for the elements of mechanical system are given below.

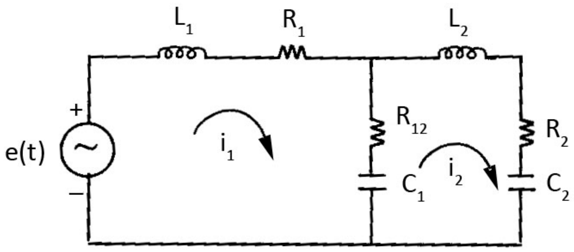


Figure 4 Force-voltage electrical analogous circuit

$f(t) \rightarrow e(t)$	$M_1 \rightarrow L_1$	$B_1 \rightarrow R_1$	$K_1 \rightarrow I/C_1$
$v_1 \rightarrow i_1$	$M_2 \rightarrow L_2$	$B_2 \rightarrow R_2$	$K_2 \rightarrow 1/C_2$
$v_2 \rightarrow i_2$		$B_{12} \rightarrow R_{12}$	

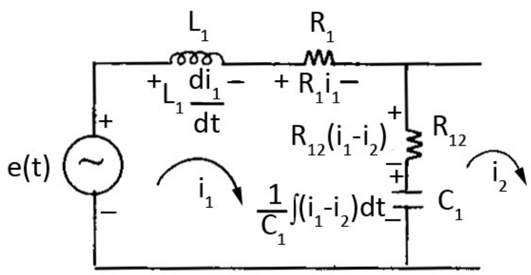


Figure 5

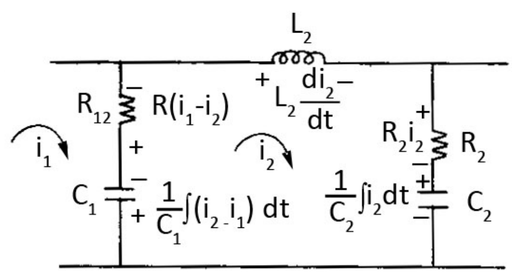


Figure 6

The mesh basis equations using Kirchoff's voltage law for the circuit shown in figure 4 are given below.

$L_1\frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt = e(t)$ .... (5)

$L_2\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2 dt + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_2 - i_1)\,dt = 0$ .... (6)

It is observed that the mesh basis equations (5) and (6) are similar to the differential equations (3) and (4) governing the mechanical system

Force-current analogous circuit

The given mechanical system has two nodes (masses). Hence the force-current analogous electrical circuit will have two nodes.

The force applied to mass M_1 is represented as a current source connected to node 1 in analogous electrical circuit. The elements M_1, B_1, K_1 and B_{12}, are connected to first node. Hence they are represented by analogous elements connected to node 1 in analogous electrical circuit. The elements K_1, B_{12}, M_2, K_2 and B_2 are connected to second node. Hence they are represented by analogous elements as elements connected to node 2 in analogous electrical circuit.

The elements K_1 and B_{12} are common between node 1 and 2 and so they are represented by analogous elements as common element between two nodes in analogous circuit. The force-current electrical analogous circuit is shown in figure 7.

The electrical analogous elements for the elements of mechanical system are given below

$f(t) \rightarrow I(t)$	$M_1 \rightarrow C_1$	$B_1 \rightarrow I/R_1$	$K_1 \rightarrow I/L_1$
$v_1 \rightarrow v_1$	$M_2 \rightarrow C_2$	$B_2 \rightarrow I/R_2$	$K_2 \rightarrow 1/L_2$
$v_2 \rightarrow v_2$		$B_{12} \rightarrow I/R_{12}$	

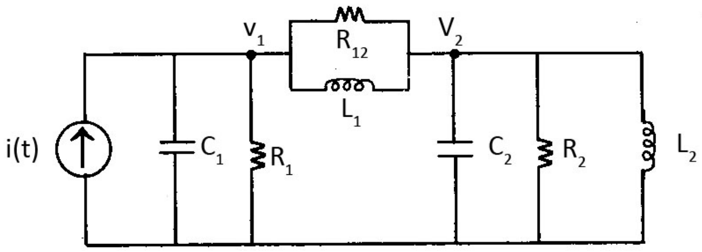


Figure 7 Force-current electrical analogous circuit

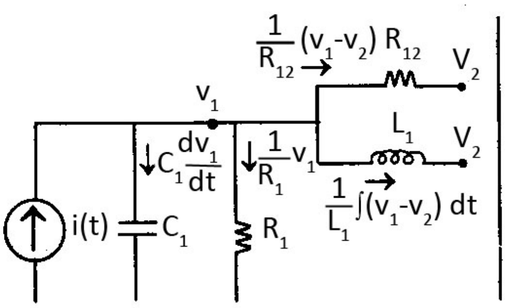


Figure.8

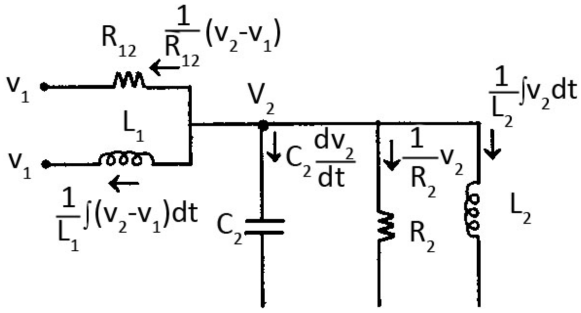


Figure 9

The node basis equations using Krichoff's current law for the circuit shown in figure 7 are given below.

$$C_1\frac{dv_1}{dt} + \frac{1}{R_1}v_1 + \frac{1}{R_2}\ (v_1 - v_2) + \frac{1}{L_1}\int (v_1 - v_2)\, dt = i(t) \qquad \text{.........}\ (5)$$

$$C_2\frac{dv_2}{dt} + + \frac{1}{R_2}\, v_2 + \frac{1}{L_2}\int v_2 dt + \frac{1}{R_2}\ (v_2 - v_1) + \frac{1}{L_1}\int (v_2 - v_1)\ dt = 0 \quad \text{......}\ (6)$$

It is observed that the node basis equations (7) and (8) are similar to the differential equations (3) and (4) governing the mechanical system.

PROPERTIES OF SIGNAL FLOW GRAPH

The basic properties of signal flow graph are the following:

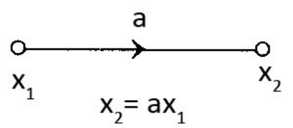
i) The algebraic equations which are used to construct signal flow graph must be in the form of cost and effect relationship
ii) Signal flow graph is applicable to linear systems
iii) A node in the signal flow graph represents the variable or signal
iv) A node adds the signals of all incoming branches and transmits the sum to all outgoing branches
v) A mixed node which has both incoming and outgoing signals can be treated as an output node by adding an outgoing branch of unity transmittance.
vi) The signals travel along branches only in the marked direction and when it travels it gets multiplied by the gain or transmittance of the branch.
vii) The signal flow graph of system is not unique. By rearranging the system equations different types of signal flow graphs can be drawn for a given system

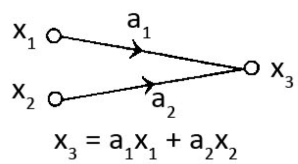
SIGNAL FLOW GRAPH ALGEBRA

Signal flow graph for a system can be reduced to obtain the transfer function of the system using the following rules. The guideline in developing the rules for signal flow graph algebra is that the signal at a node is given by same of all incoming signals.

Rule 1: Incoming signal to a node through a branch is given by the product of a signal at previous node and the gain of the branch.

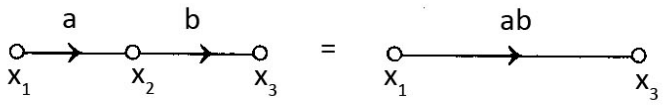
Examples





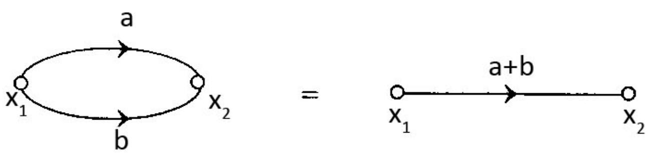
Rule 2: Cascaded branches can be combined to give a signal branch whose transmittance is equal to the product of individual branch transmittance.

Examples



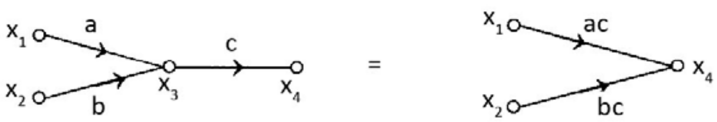
Rule 3: Parallel branches may be represented by single branch whose transmittance is the sum of individual branch transmittances.

Examples



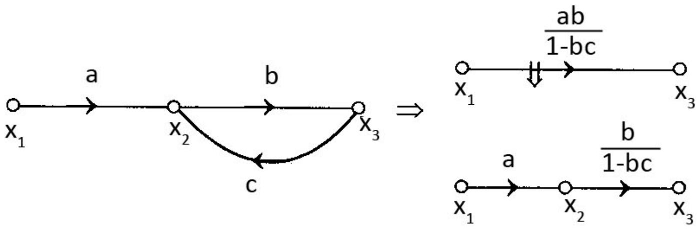
Rule 4: A mixed node can be eliminated by multiplying the transmittance of outgoing branch (from the mixed node) to the transmittance of all incoming branches to the mixed node.

Examples



Rule 5: A loop may be eliminated by writing equations at the input and output node and rearranging the equations to find the ratio of output to input. This ratio gives the gain of resultant branch.

Examples



Proof:

$x_2 = ax_1 + cx_3$

$x_3 = bx_2$

Put $x_2 = ax_1 + cx_3$ in the equation for x_3

$\therefore x_3 = b\,(ax_1 + cx_3)$

$x_3 = abx_1 + cx_3$

$x_3 - bc\,x_3 = abx_1$

$x_3(1\text{-}bc) = ab\,x_1$

$\frac{x_3}{x_1} = \frac{ab}{1-bc}$

<u>SINGLE FLOW GRAPH REDUCTION</u>

The single flow graph of a system can be reduced either by using the rules of the single flow graph algebra (i.e.) by writing equations at every node and then rearranging these equations to get the ratio of output and input (transfer function)

The signal flow graph reduction by above method will be time consuming and tedious. S. J. Mason has developed a simple procedure to determine the transfer function of the system represented as a signal flow graph. He has developed a formula called by his name Mason's gain formula which can be directly used to find the transfer function of the system.

Mason's Gain Formula

The Mason's gain formula is used to determine the transfer function of the system from the signal flow graph of the system.

Let R(s) = Input to the system

and C(s) = Output of the system

Transfer function of the system, T(s) = $\frac{C(s)}{R(s)}$

Manson's gain formula states the overall gain of the system [transfer function] as follows,

Overall gain, T = $\frac{1}{\Delta}\Sigma_k P_k \Delta_k$

Where, T = T(s) = Transfer function of the system.

$P_k =$ Forward path gain of K^{th} forward path

$\Delta =$ 1– (Sum of individual loop gains) + $\begin{pmatrix}\text{Sumof gain products of all possible}\\ \text{combinations of two non} - \text{touching loops}\end{pmatrix}$

- $\begin{pmatrix}\text{Sum of gain products ofa ll possible}\\ \text{combinations of three non} - \text{touching loops}\end{pmatrix}$+ ....

$\Delta_k =$ ΔFor that part of the graph which is not touching K^{th} forward path

5] Problem

Construct a signal flow graph for armature controlled dc motor

SOLUTION

The differential equations governing the armature controlled dc motor are

$V_a = i_a R_a + L_a \frac{di_a}{dt} + e_b$ ....(1)

$T = K_t i_a$ .... (2)

$T = J\frac{d\omega}{dt} + B\omega$ .... (3)

$e_b = K_b \omega$ .... (4)

$\omega = \frac{d\theta}{dt}$ .... (5)

On taking Laplace transform of equations (1) to (5) we get,

$V_a(s) = I_a(s)R_a + L_a s I_a(s) + E_b(s)$ .... (6)

T(s) $= K_t I_a(s)$ .... (7)

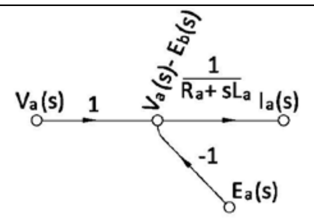
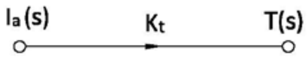
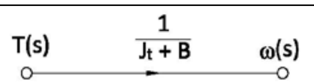
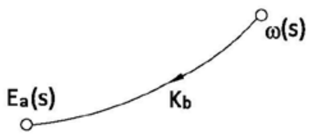
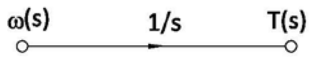
T(s) $= Js\omega(s) + B\omega(s)$ .... (8)

$E_b(s) = K_b \omega(s)$ .... (9)

$\omega(s) = s\theta\,(s)$ ....(10)

The input and output variables of armature controlled dc motor are armature voltage V_a(s) and angular displacementθ_s respectively. The variables $I_a(s), T(s), E_b(s) and \omega(s)$are intermediate variables.

The equations (6) to (10) are rearranged and the individual signal flow graph are shown in figure 1 to figure5

$V_a(s) - E_b(s) = I_a(s)\,[R_a + sL_a]$ $\therefore I_a(s) = \frac{1}{R_a + sL_a}[V_a(s) - E_b(s)]$	 Figure 1
$T(s) = K_t I_a(s)$	 Figure 2
T(s) $= \omega(s)[J_s + B]$ $\therefore \omega(s) = \frac{1}{J_s + B}T(s)$	 Figure 3
$E_b(s) = K_b \omega(s)$	 Figure 4
$\omega(s) = s\theta(s)$ $\therefore \theta(s) = \frac{1}{s}\omega(s)$	 Figure5

The overall single flow graph of armature controlled dc motor is obtained by interconnecting the individual signal flow graphs shown in figure 1 to figure 5. The overall signal flow graph is shown in figure 6.

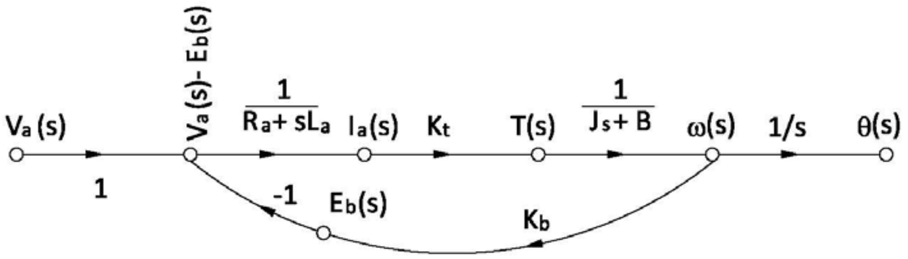


Figure 6 Signal flow graph of armature controlled DC motor

6] Problems

Find the overall transfer function of the syxtem whose signal flow graph is shown in figure.

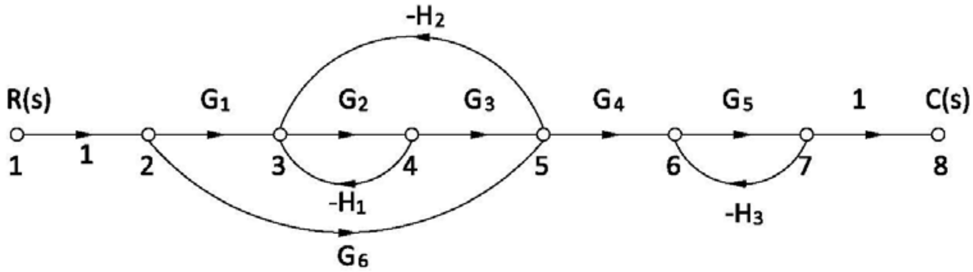


Figure 1

SOLUTION

I) Forward path gains

There are two forward paths, $\therefore K = 2$
Let forward path gains be P_1 and P_2

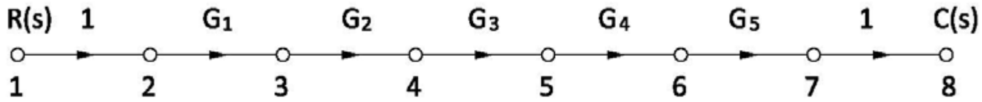


Figure 2 Forward path – I

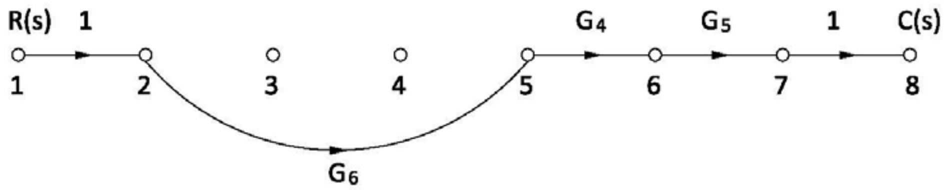
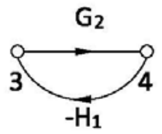


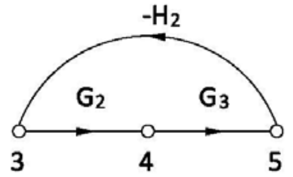
Figure3 Forward patch – 2

Gain of forward path – 1, $P_1 = G_1 \; G_2 \; G_3 \; G_4 \; G_5$
Gain of forward path – 2, $= G_4 \; G_5 \; G_6$

II) Individual Loop Gain

There are three idividual loops. Let individual loop gains be P_{11}, P_{21} and P_{13}





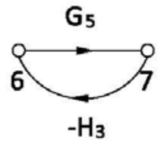


Figure 4 Loop –I ***Figure5 Loop – 2*** ***Figure 6 Loop – 3***

Loop gain of indicidual loop – 1, $P_{11} = - G_2 H_1$

Loop gain of individual loop – 2, $P_{21} = - G_2 G_3 H_2$

Loop gain of individual loop – 3, $P_{31} = - G_5 H_3$

III) Gain Products Of Two Non Touching Loops

There are two combinations of two non-touching loops. Let the gain products of two non touching loops be P_{12} and P_{22}.

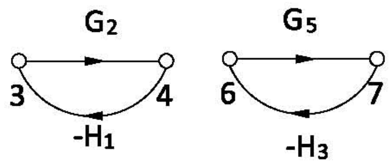


Figure 7 Firstcombinations of two non-touching loops

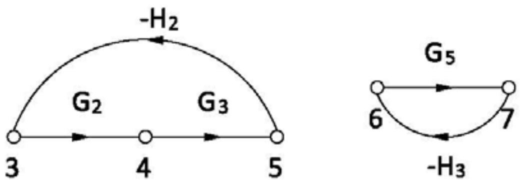


Figure 8 Second combination of two non touching loops

Gain product of first combination of two non touching loops

$$P_{12} = P_{11} P_{31}$$
$$= (-G_2H_1) \; (-G_5 H_5)$$
$$= G_2 \; G_5 \; H_1 \; H_3$$

Gain product of second combination of two non touching loops

$$P_{22} = P_{21} \; P_{31}$$
$$= (-G_2 G_3 H_2) \; (-G_5 H_3)$$
$$= G_2 \; G_3 \; G_5 \; H_2 \; H_3$$

IV) Calculation Of Δ And Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$
$$= 1 - (-G_2 H_1 - G_2 G_3 H_2 - G_5 H_3) + (G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3)$$
$$= 1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_4$$

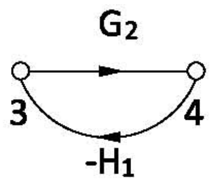


Figure 9

Δ_1= 1, Since there is no part of graph which is not touching with first forward path.

The part of the graph which is non touching with second forward path is shown in figure 9

$$\Delta_2 = 1 - P_{11} = 1 - (-G_2 H_1)$$
$$= 1 + G_2 H_1$$

V) Tranfer Function, T

By Mason's gain formaula the transfer function $T = \frac{1}{\Delta}\sum_k P_K\Delta_K$

$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

(Here K = 2, since we have only to forwar patch)

$$\therefore\ T = \frac{G_1\, G_2\, G_3\, G_4 G_5 + G_4 G_5\, G_6\, (1+G_2\, H_1)}{1+ G_2\, H_1 + G_2\, G_3 H_2 + G_5 H_3 + G_2 G_5\, H_1 H_3 + G_2\, G_3 G_5\, H_2 H_3}$$

$$= \frac{G_1\, G_2\, G_3\, G_4 G_5 + G_4 G_5\, G_6 + G_2\, G_4\, G_5 G_6\, H_1}{1+ G_2\, H_1 + G_2\, G_3 H_2 + G_5 H_3 + G_2 G_5\, H_1 H_3 + G_2\, G_3 G_5\, H_2 H_3}$$

$$= \frac{G_2\, G_4\, G_5\, [G_1 G_3 + G_6 / G_2 + G_6 H_1\,]}{1+ G_2\, H_1 + G_2\, G_3 H_2 + G_5 H_3 + G_2 G_5\, H_1 H_3 + G_2\, G_3 G_5\, H_2 H_3}$$

BLOCK DIAGRAM

A control system may consist of a number of components. It control engineering to show the function performed by each component, we commonly use a diagram called the block diagram. A black diagram of a system is a pictorial representation of the functions performed by each component and of the flow signals. Such a diagram depicts the interrelationships that exist among the various components. The elements of a block diagram are block, branch point and summing point.

Block

In a block diagram all system variable are linked to each other through functional blocks. The functional block or simply block is a symbol for the mathematical operation on the input signal to the block that procures the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals. Figure 1shows the functional block of the block diagram.

***Figure 1** Functionalblock*

The arrowhead pointing towards the block indicates the input, and arrowhead leading away from the block represents the output. Such arrows are referred to as signals. The output signal from the block is given by the product of input signal and transfer function in the block.

Summing point

Summing points are used to add two or more signals in the system. Referring to figure 2, a circle with a cross is the symbol that indicates a summing operation.

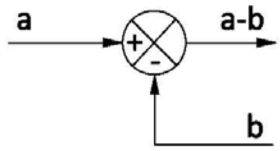


***Figure 2** Summing point*

The plus or minus sign at each arrowhead indicates whether the signal isto be added or subtracted. It is important that the quantities being added or subtracted have the same dimensions and the same units.

Branch point

A branch point is a point from which the signal from a block goes concurrently to other blocks or summing points.

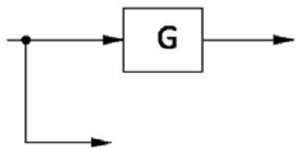


***Figure 3** Branch point*

Constructing block diagram for control systems

A control system can be represented diagrammatically by block diagram. The differential equations governing the system are used to construct the block diagram. By taking Laplace transform the differential equations are converted to algebraic equations. The equations will have variables and constants. From the working knowledge of the system the input and output variables are identified and the block diagram for each equation can be draw. Each equation gives one section of block diagram. The output of one section will be input for another section. The various sections are interconnected to obtain the overall block diagram of the system.

7] Problems

Construct the block diagram of armature controlled dc motor.

SOLUTION

The differential equations governing the armature controlled dc motor are

$$V_a = i_a R_a + L_a \frac{di_a}{dt} + e_m \quad \text{.......... (1)}$$

$$T = K_t i_a \quad \text{......... (2)}$$

$$T = J\frac{d\omega}{dt} + B\omega \quad \text{.......... (3)}$$

$$e_b = K_b \omega \quad \text{.......... (4)}$$

$$\omega = \frac{d\theta}{dt} \quad \text{.......... (5)}$$

On taking Laplace transform of equation (1) we get,

$$V_a(s) = I_a(s)R_a + L_a s I_a(s) + E_b(s) \quad \text{......... (6)}$$

In equation (6), V_a(s) and $E_b(s)$ are inputs and $I_a(s)$ is the out put. Hence the equation (6) is rearranged and the block diagram for this equation is shown in figure 1

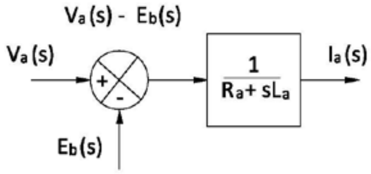


Figure 1

$$V_a(s) - E_b(s) = I_a(s)[R_a + sL_a]$$

$$\therefore I_a(s) = \frac{1}{R_a + sL_a}[V_a(s) - E_b(s)]$$

On taking Laplace transform of equation (2)we get,

$$T(s) = K_t I_a(s) \quad \text{......... (7)}$$

In equation (7), $I_a(s)$ is theinput and T(s) is the output. The block diagram for this equation is shown figure.

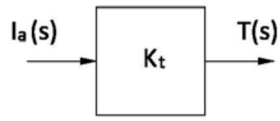


Figure 2

On taking Laplace transform of equation (3) we get,

$$T(s) = Js\omega(s) + B\omega(s) \quad \text{........ (8)}$$

In equation (8), T(s) is the input and $\omega(s)$ is the output. Hence the equation (8) is rearranged and the block diagram for this equation is shown in figure.

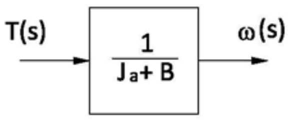


Figure 3

$$T(s) = (Js + B)\omega(s)$$

$$\therefore \omega(s) = \frac{1}{Js + B}T(s)$$

On taking Laplace transform of equation (4) we get,

$$E_b(s) = K_b \omega(s) \quad \text{........ (9)}$$

In equation (9) $\omega(s)$ is the input and E_b(s) is the output. The block diagram for this equation is shown in figure.

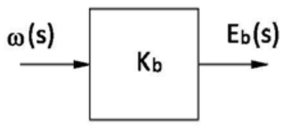


Figure 4

On taking Laplace transform of equation (5) we get,

$$\omega(s) = s\theta(s) \quad \text{...... (10)}$$

In equation (10), $\omega(s)$ is the input and θ(s) is the output. Hence equation (10) is rearranged and the block diagram for this equation is shown in figure.

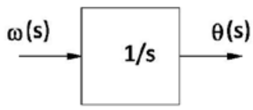


Figure 5

$$\theta(s) = \frac{1}{s}\omega(s)$$

The overall block diagram of armature controlled dc motor is obtained by connecting the various sections shown in figure 1 to figure 5. The overall block diagram is shown in figure 6.

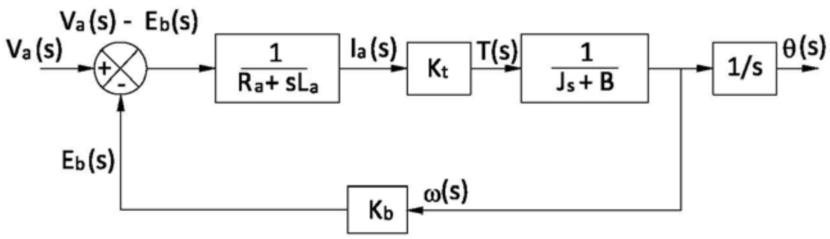


Figure 6 *Block diagram of armature controlled dc motor*

<u>Block Diagram Reduction</u>

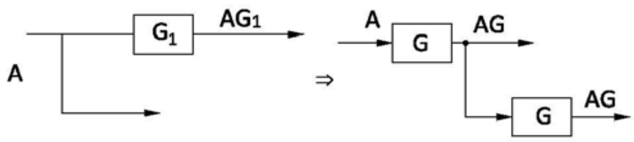
The block diagram can be reduced to find the overall transfer function of the system. The following rules can be used for block diagram reduction. The rules are framed such that any modification made on the diagram does not alter the input output relation.

RULES OF BLOCK DIAGRAM ALGEBRA

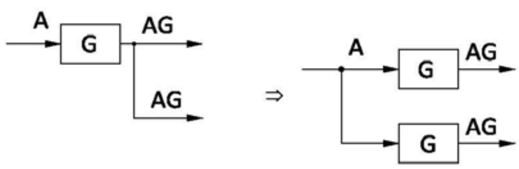
1. Combining the blocks in cascade

2. Combining parallel blocks (or combining feed forward paths)

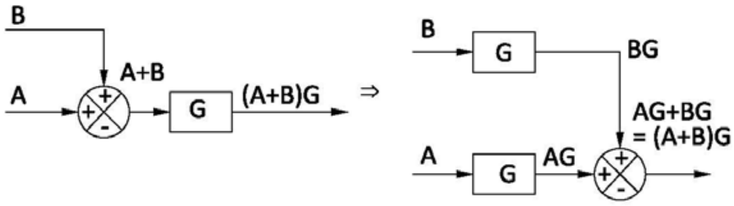
3. Moving the branch point ahead of the block



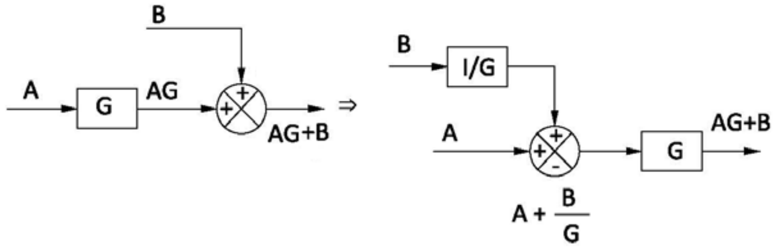
4. Moving the branch point before the block



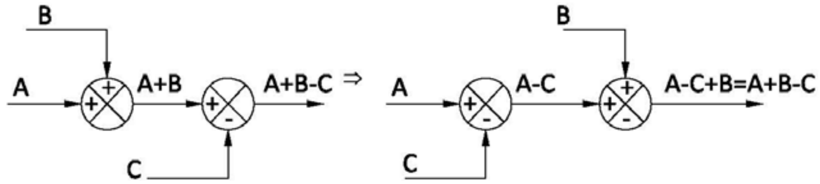
5. Moving the summing point a head of the block



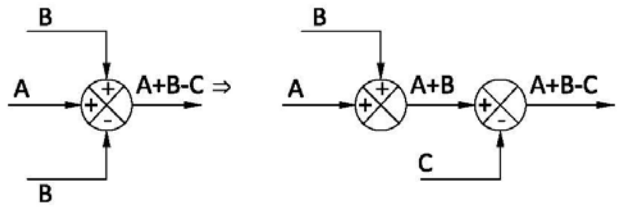
6. Moving the summing point before the block



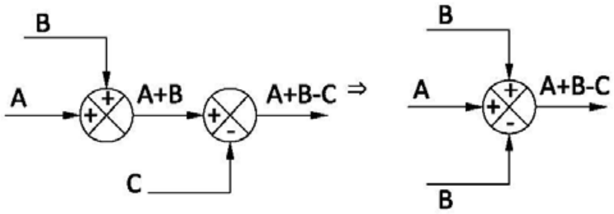
7. Interchanging summing point



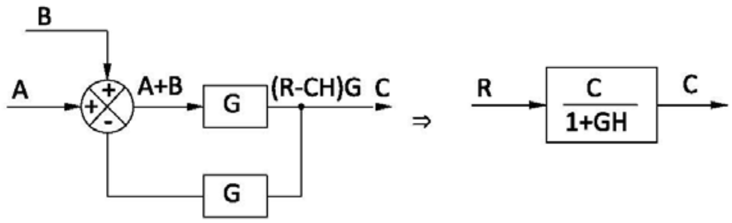
8. Splitting summing points



9. Combining summing points



10. Elimination of feed back loop



Proof: C = (R – CH) G

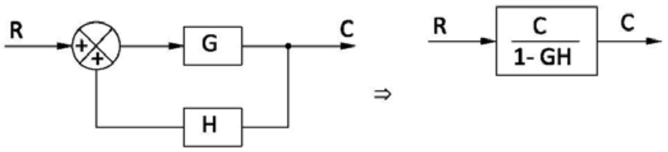
$C = RG - CHG$

$C + CHG = RG$

$C(1 + HG) = RG$

$\frac{C}{R} = \frac{G}{1+GH}$

Also



8] Problem

Reduce the block diagram shown in figure 1 and find C/R

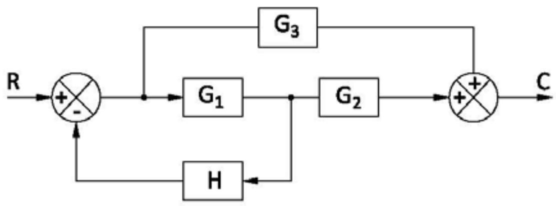
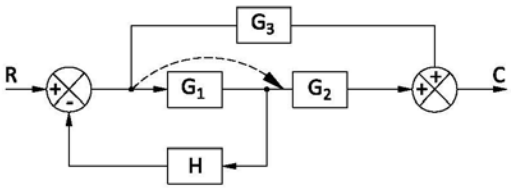


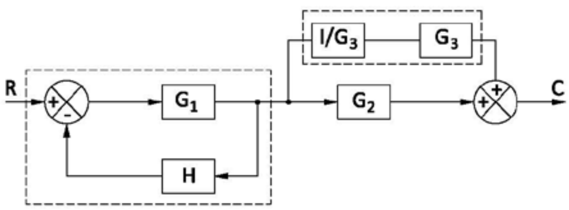
Figure1

SOLUTION

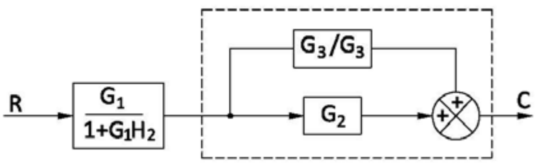
Step 1: Move the branch point after the block



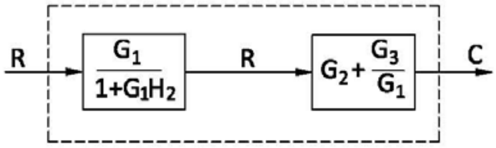
Step 2: Eliminate the feedback path and combining blocks in cascade



Step 3: Combining parallel blocks



Step 4: combining blocks in cascade



RESULT

The overall transfer function of the system, $\frac{C}{R} = \frac{G_1G_2+G_3}{1+G_1H}$

ROUTH HURWITZ CRITERION

The Routh stability criterion is based on ordering the coefficients of the characteristic equation, $a_0s^n + a_2s^{n-1} + a_2s^{n-2} + \ldots\ldots.. + a_{n-1}s + a_n = 0$,where $a_0 > 0$ into a schedule, called the Routh array as shown below.

$$
\begin{array}{lcl}
s^n & : & a_0 a_2 a_4 a_6 a_8 \ldots. \\
s^{n-1} & : & a_1 a_3 a_5 a_7 a_9 \ldots. \\
s^{n-2} & : & b_0 b_1 b_2 b_3 b_4 \ldots. \\
s^{n-3} & : & c_0 c_1 c_2 c_3 c_4 \ldots. \\
 & \vdots & \\
s^1 & : & g_o \\
s_0 & : & h_0
\end{array}
$$

The Routh stability criterion can be stated as follows.

"The necessary and sufficient condition for stability is that all of the elements in the first column of the Routh array be positive. If this condition is not met, the system is unstable and the number of single changes in the elements of the first column of the Routh array corresponds to the number of roots of the characteristic equation in the right half of the s-plane"

Note: If the order of sign of first column elements is +, +, - , + and +. Then + to − is considered as one sign change and – to + as another sign change.

CONSTRUCTION OF ROUTH ARRY

Let the characteristic polynomial be

$$a_0s^n + a_1s^{n-1} + a_2s^{n-2} + a_3s^{n-3} + \ldots a_{n-1}s^1 + a_ns^0$$

The coefficients of the polynomial are arranged in two rows as shown below.

$$
\begin{array}{lcl}
s^n & : & a_0 a_2 a_4 a_6 \ldots. \\
s^{n-1} & : & a_1 a_3 a_5 a_7 \ldots.
\end{array}
$$

If n is even then s^n row is formed by coefficient of even order terms and s^{n-1} row is formed by coefficients of odd order terms (i.e., coefficients of odd powers of s).

If n is odd, then s^n row is formed by coefficients of odd order term and s^{n-1} row is formed by coefficients of even order terms (i.e., coefficients of even powers of s)

The other rows of routh array uptos^0 row can be formed by the following procedure. Each row of Routh array is constructed by using the elements of previous two rows.

Consider two consecutive rows of Routh array as shown below.

s^{n-1} : $x_0 x_1 x_2 \quad x_3 x_4 \quad \ldots\ldots$

s^{n-x-1} : $y_0 y_1 y_2 y_3 y_4 \ldots\ldots$

Let the next row be,

s^{n-x-2} : $z_o z_1 z_2 z_3 \ldots\ldots..$

The elements of s^{n-x-2} row are given by,

$$z_0 = \frac{(-1)\begin{vmatrix} x_0 & x_1 \\ y_0 & y_1 \end{vmatrix}}{y_0} = \frac{y_0 x_1 - y_1 x_0}{y_0}$$

$$z_1 = \frac{(-1)\begin{vmatrix} x_0 & x_2 \\ y_0 & y_2 \end{vmatrix}}{y_0} = \frac{y_0 x_2 - y_2 x_0}{y_0}$$

$$z_2 = \frac{(-1)\begin{vmatrix} x_0 & x_3 \\ y_0 & y_3 \end{vmatrix}}{y_0} = \frac{y_0 x_3 - y_3 x_0}{y_0}$$

$$z_3 = \frac{(-1)\begin{vmatrix} x_0 & x_4 \\ y_0 & y_4 \end{vmatrix}}{y_0} = \frac{y_0 x_4 - y_4 x_0}{y_0}$$

The elements $z_{0,}z_{1,}$ z_2, $z_{3,}$ …… are computed until an element equals to zero or for all possible computations as shown above.

I the process of constructing Routh array the missing terms are considered as zeros. Also, all the elements of any row can be multiplied or divided by a positive constant to simplify the computational work.

In the construction of Routh array one may come across the following three cases.

Case 1: Normal Routh array (Non-zero elements in the first column ofrouth array)

Case 2: A row of all zeros

Case 3: A first element of a row is zero but some or other elements are not zero.

<u>Case 1: Normal Routh array (Non-zero elements in the first column of routh array)</u>

9] Problem

Using Routh criterion, determine the stability of the system represented by the characteristic equation, $S^4 + 8s^3 + 18s^2 + 16s + 5 = 0$. Comment on the location of the roots of characteristic equation.

SOLUTION

The characteristic equation of the system is $S^4 + 8s^3 + 8s^2 + 16s + 5 = 0$

The given characteristic equation is 4th order equation and so it has 4 roots. Since the highest power of s is even number, form the first row of routh array using the coefficients of even powers of s and from the second row using the coefficient of odd powers of s.

s^4	:	1	18	5	…. Row – 1
s^3	:	8	16		…. Row – 2

The elements of s^3 row can be divided by 8 to simplify the computations.

s^4	:	1	18	5	…. Row – 1
s^3	:	1	2		…. Row – 2
s^2	:	16	5		…. Row – 3
s^1	:	1.7			…. Row – 4
s^0	:	5			…. Row – 5
s^3	:	8	16		…. Row – 2

Column – 1

$s^2 : \frac{1\times18-2\times1}{1} \; \frac{1\times5-0\times1}{1}$

$s^2 : 16 \qquad 5$

$s^1 : \frac{16\times2-5\times1}{16}$

$s^1 : 1.6875 \approx 1.7$

$s^0 : \frac{1.7\times5-0\times16}{1.7}$

$s^0 : 5$

On examining the elements of first column of routh array it is observed that all the elements are positive and there is no sign change. Hence all the roots are lying on the left half or s-plane and the system is stable.

RESULT

1. Stable system
2. All the four roots are lying on the left half of s-plane.

Case 2: A row of all zeros

10] Problem

Construct Routh array and determine the stability of the system whose characteristic equation is $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$. Also determine the number of roots lying on right half of s-plane, left half of s-plane and on imaginary axis.

SOLUTION

The characteristic equation of the system is $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$.

The given characteristic polynomial is 6^{th} order equation and so it has 6 roots. Since the highest power of s is even number from the first row of routh array using the coefficients of even powers of s and form the second row using the coefficients of odd powers of s.

s^6	:	1	8	20	16	…. Row – 1
s^5	:	2	12	16		…. Row – 2

The elements of s^5 row can be divided by 2 to simplify the computations.

s^6	:	1	8	20	16	.... Row – 1
s^5	:	1	6	8		.... Row – 2
s^4	:	1	6	8		.... Row – 3
s^3	:	0	0			.... Row – 4
s^3	:	1	3			.... Row – 4
s^2	:	3	8			.... Row – 5
s^1	:	0.33				.... Row – 6
s^0	:	8	8			.... Row – 7

Column – 1

s^4 : $\frac{1\times8-6\times1}{1}$ $\frac{1\times20-8\times1}{1}$ $\frac{1\times16-0\times1}{1}$

s^4 : 2 12 16

divide by 2

s^4 : 1 6 8

s^1 : $\frac{1\times6-6\times1}{1}$ $\frac{1\times8-8\times1}{1}$

s^1 : 0 0

The auxiliary equation is A = s^4 + 6s2 + 8. On differentiating A with respect to s we get

: $\frac{dA}{ds}$ = 4s3 + 12s

The coefficients of $\frac{dA}{ds}$ are used to form s^3 row

s^3 : 4 12

divide by 4

s^3 : 1 3

s^2 : $\frac{1\times6-3\times1}{1}$ $\frac{1\times8-0\times1}{1}$ s^2 : 3 8
s^1 : $\frac{3\times3-8\times1}{3}$ s^1 : 0.33
S^0 : $\frac{0.33\times8-0\times3}{0.33}$ S^0 : 8

On examining the elements of 1st column of routh array it is observed that there is no sign change. The rows with all zeros indicate the possibility of roots on imaginary axis. Hence the system is limitedly or marginally stable.

The auxiliary polynomial is

$$S^4 + 6s^2 + 8 = 0$$

Let $s^2 = x$

$$\therefore x^2 + 6x + 8 = 0$$

The roots of quadratic are, $x = \frac{-6\pm\sqrt{6^2-4\times8}}{2}$

$= -3 \pm 1 = 2$ or -4

The roots of auxiliary polynomial is, $s = \pm\sqrt{x} = \pm\sqrt{-2}$ and $\pm\sqrt{-4}$

$= +j\sqrt{x}$, $-j\sqrt{-2}$ $+j2$ and $-j2$

Theroots of auxiliary polynomial are also roots of characteristic equation. Hence 4 roots are lying on imaginary axis and the remaining two roots are lying on the left half of S - plane.

RESULT

1. The system is limitedly or marginally stable
2. Four roots are lying on imaginary axis and the remaining two roots are lying on the left half of s-plane.

Case 3: A first element of a row is zero but some or other elements are not zero

11] Problem

Construct Routh array and determine the stability of the system represented by the characteristic equation $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$. Comment of the location of the roots of characteristic equation.

SOLUTION

The characteristic equation of the system is $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$

The given characteristic polynomial is 5th order equation and so it has 5 roots. Since the highest power of s is odd number, form the first row of routh array using the coefficients of odd powers of s and from the second row using the coefficients of even powers of s.

s^5	:	1	2	3	…. Row – 1
s^4	:	1	2	5	…. Row – 2
s^3	:	ϵ	-2		…. Row – 3
s^2	:	$\frac{2\epsilon+2}{\epsilon}$	5		…. Row – 4
s^1	:	$\frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2}$			…. Row – 5
s^0	:	5			…. Row – 6

On letting $\epsilon \rightarrow 0$, we get

s^5	:	1	2	3	…. Row – 1
s^4	:	1	2	5	…. Row – 2
s^3	:	0	-2		…. Row – 3
s^2	:	∞	5		…. Row – 4
s^1	:	-2			…. Row – 5
s^0	:	5			…. Row – 6

Column - 1

s^2 : $\frac{1\times2-2\times1}{1}$ $\frac{1\times3-5\times1}{1}$

s^3 : 0 -2

Replace: 0 by ϵ

s^3 : ϵ -2

s^2 : $\frac{\epsilon\times2-(-2\times1)}{\epsilon}$ $\frac{\epsilon\times5-0\times1}{\epsilon}$

s^2 : $\frac{2\epsilon+2}{\epsilon}$ 5

s^1 : $\frac{\frac{2\epsilon+2}{\epsilon}\times(-2)-(-5\times\epsilon)}{\frac{2\epsilon+2}{\epsilon}}$

s^1 : $\frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2}$

S^0 : 5

On observing the elements of first column of routh array, it is found that three are two sign changes. Hence two roots are lying on the right half of s-plane and the system is unstable. The remaining three roots are lying on the lift half of s-plane.

RESULT

1. The system is unstable.
2. Two roots are lying on the right half of s-plane and three roots are lying on the left of s-plane.

12] Problem

By routh stability criterion determine the stability of the system represented by the characteristic equation $9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$. Comment on the location of roots of characteristic equation.

SOLUTION

The characteristic polynomial of the system is $9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$

On examining the coefficients of the characteristic polynomial, it is found that some of the coefficients are negative and so some roots will lie on the right of s-plane. Hence the system is unstable. The routh array can be constructed to find the number of roots lying on right half of s-plane.

$9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$

The given characteristic polynomial is 5th order equation and so it has 5 roots. Since the highest power of s is odd number, form the first row of routh array using the coefficients of odd powers of s and form the second row using coefficients of even powers of s.

s^5	:	9	10	-9	.... Row – 1
s^4	:	-20	-1	-10	.... Row – 2
s^3	:	9.55	-13.5		.... Row – 3
s^2	:	-29.3	-10		.... Row – 4
s^1	:	-16.8			.... Row – 5
s^0	:	-10			.... Row – 6

Column - 1

$$s^3 : \quad \frac{-20\times10-(-1)\times9}{-20} \quad \frac{-20\times(-9)-(-10)\times9}{1}$$

$$s^3 : \quad 9.55 \qquad -13.5$$

$$s^2 : \quad \frac{9.55\times(-1)-(-13.5)\times(-20)}{9.55} \quad \frac{9.55\times(-10)}{9.55}$$

$$s^2 : \quad -29.3 \qquad -10$$

s^1 : $\frac{-29.3\times(-13.5)-(-10)\times 9.55}{-29.3}$ s^1 : -16.8
s^0 : $\frac{-16.8\times(-10)}{-16.8}$ s^1 : 10

By examining the elements of 1st column of routh array it is observed that there are three sign changes and so three roots are lying on the right half of s-plane and the remaining two rrots are lying on the left half of s-plane.

RESULT

1. The system is unstable
2. Three roots are lying on the right half and two roots are lying on the left half of s-plane.

FREQUENCY RESPONSE PLOTS

Frequency response analysis of control systems can be carried either analytically or graphically. The various graphical techniques available for frequency response analysis are

1. Bode plot
2. Polar plot (or Nyquist plot)
3. Nichols plot

The Bode plot, Polar plot and Nichols plot are usually drawn for open loop systems. From the open loop response plot the performance and stability of closed loop system are estimated. The M and N circles and Nichols chart are used to graphically determine the frequency response of unity feedback closed loop system from the knowledge of open loop response.

The frequency response plots are used to determine the frequency domain specifications, to study the stability of the systems and to adjust the gain of the system to satisfy the desired specifications.

POLAR PLOT

The polar plot of a sinusoidal transfer function $G(J\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle of$G(j\omega)$on polar coordinates as ω is varied from zero to infinity. Thus the polar plat is the locus of vectors $|G(J\omega)| < G(j\omega) as \omega$ is varied from zero to infinity. The polar plat is also called Nyquist plot.

The polar plot is usually plotted on a polar graph sheet. The polar graph sheet has concentric circles and radial lines. The circles represent the magnitude and the radial lines represent the phase angles. Each point on the p9olar graph has a magnitude of a point is given by the value of the circle passing through that point and the phase angle is given by the radial line passing through that point. In polar graph sheet a positive phase angle is measured in anticlockwise from the reference axis (0°) and a negative angle is measured clockwise from the reference axis (0°).

Alternatively, if $G(j\omega)$ can be expressed in rectangular coordinates as,

$G(j\omega)$ = $G_R(j\omega) + jG_I(j\omega)$

Where $G_R(jw) = Reaal part of G(j\omega)$

and$G_I(j\omega) = Imaginary part of G(j\omega)$,

then the polar plot can be plotted in ordinary graph sheet between

$G_R(j\omega) and G_I(j\omega)as\omega$ is varied from 0 to∞.

To plot the polar plot, first compute the magnitude and phase of G (jω) for various values of ω and tabulate them. Usually the choices of frequencies are corner frequencies and frequencies around corner frequencies. Choose proper scale for the magnitude circles. Fix all the points on polar graph sheet and join the points by smooth curve. Write the frequency corresponding to each point of the plot.

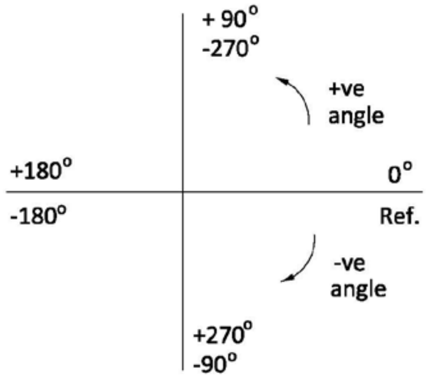
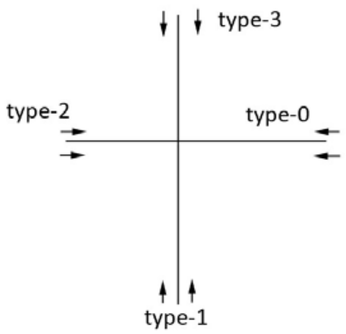
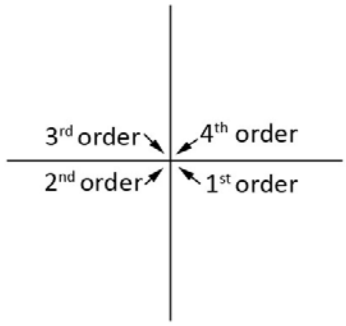
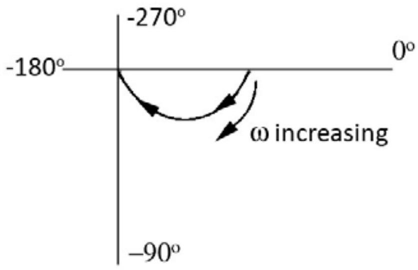
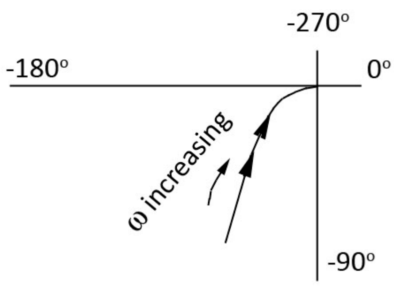


Figure 2.1

To plot the polar plot on ordinary graph sheet, compute the magnitude and phase for various values ofω. Then convert the polar coordinates to rectangular coordinates using P → R conversion (polar to rectangular conversion) in the calculator. Sketch the polar plot using rectangular coordinates.

For minimum phase transfer function with only poles, the type number of the system determines at what quadrant the polar plot starts and the order of the system determines at what quadrant the polar plot ends.

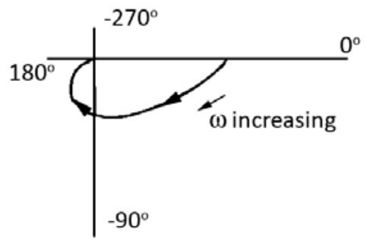
(note: The minimum phase systems are systems with all poles and zeros on the left half of s-plane)

 Figure 2.2 Start of polar plot	 Figure 2.3 End of polar plot
Typical sketches of polar plot Type: 0, Order : 1 G(s) $= \frac{1}{1+sT}$ G(jω) $= \frac{1}{1+j\omega T}$	
Type: 1 Order:2 G(s) $= \frac{1}{s(1+sT)}$ G(jω) $= \frac{1}{j\omega\,(1+j\omega T\,)}$	

Type: 0 Order:2

G(s) $= \frac{1}{(1+sT_1)(1+sT_2)}$

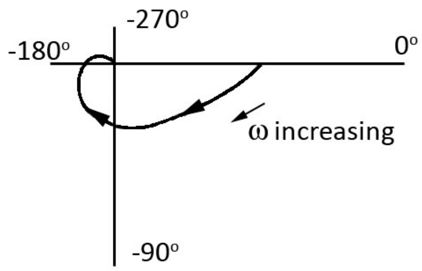
G(jω) $= \frac{1}{(1+j\omega\, T_1)(1+j\omega\, T_2)}$



Type: 0,Order:3

G(s) $= \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$

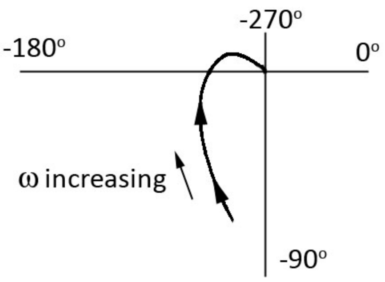
G(jω) $= \frac{1}{(1+j\omega\, T_1)(1+j\omega\, T_2)(1+j\omega\, T_3)}$



Type: 1, Order:3

G(s) $= \frac{1}{s(1+sT_1)(1+sT_2)}$

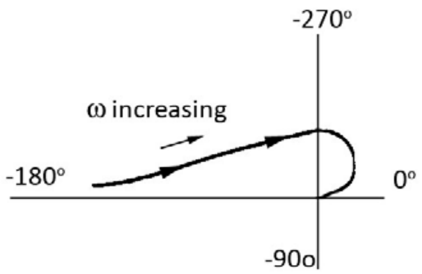
G(jω) $= \frac{1}{j\omega\,(1+j\omega\, T_1)(1+j\omega\, T_2)}$



Type: 2, Order:4

G(s) $= \frac{1}{s^2(1+sT_1)(1+sT_2)}$

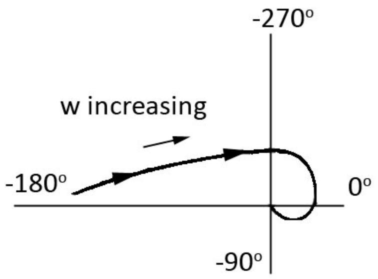
G(jω) $= \frac{1}{(j\omega\,)^2(1+j\omega\, T_1)(1+j\omega\, T_2)}$



Type: 0,Order:3

G(s) $= \frac{1}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$

G(jω) $= \frac{1}{(j\omega\,)^2(1+j\omega\, T_1)(1+j\omega\, T_2)(1+j\omega\, T_3)}$



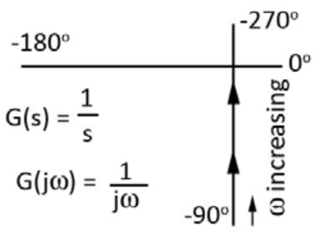


Figure 2.4

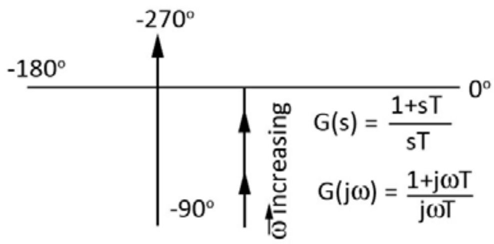


Figure 2.5

DETERMINATION OF GAIN MARGIN AND PHASE MARGINE FROM POLAR PLOT

The gain margin is defined as the inverse of the magnitude of $G(j\omega)$ at phase crossover frequency. The phase crossover frequency is the frequency at which the phase of $G(j\omega)$ is 180°

Let the polar plot cut the 180°axis at point B and the magnitude circle passing through the point B beG_B. Now the Gain margin, $K_g = \frac{1}{G_B}$.If the point B lies within unity circle then the Gain margin is positive otherwise negative. (If the polar plat is drawn in ordinary graph sheet using rectangular coordinates then the point B is the cutting point of $G(j\omega)$ locus with negative real axis and $K_g = 1/|G_B|$ where G_B is the magnitude corresponding to point B).

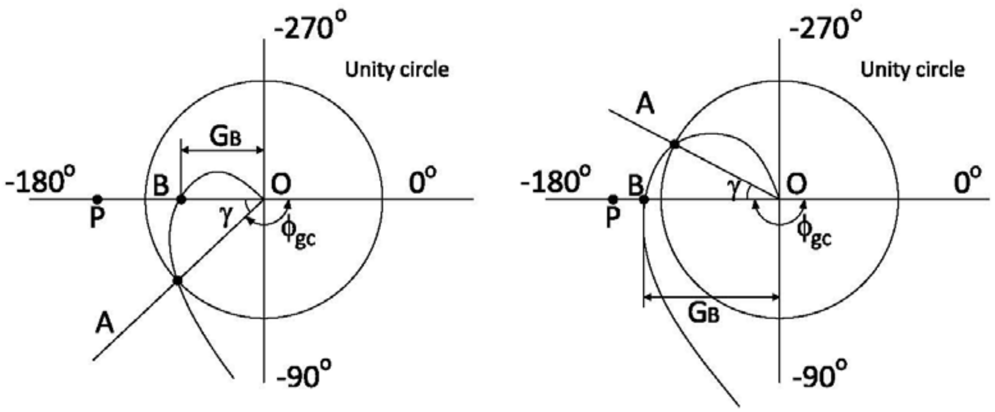


Figure 2.6 Polar plot showing positive gain margin and phase margin.

Figure 2.7 Polar plot showing negative gain margin and phase margin.

Gain margin, $K_g = \frac{1}{G_B}$

Phase margin, $\gamma = 180° + \phi_{gc}$

Gain margin, $K_g = \frac{1}{G_B}$

Phase margin, $\gamma = 180° + \phi_{gc}$

The phase margin is defined as, phase margin, $\gamma = 180 + \ \phi_{gc} where \phi_{gc}$ is the phase angle of $G(j\omega)$ at gain crossover frequency. The gain crossover frequency is the frequency at which the magnitude of $G(j\omega)$ is unity.

Let the polar plot cut the unity circle at point A as shown in fig. Now the phase marginγ is given by $\angle$AOP, ie. If $\angle$AOP is below –180°axis then the phase margin is positive and if it is above- 180°axis then the phase margin is negative.

13] Problem

The open loop transfer function of a unity feedback system is given by G(s) = 1/s (1+s) (1+2s). Sketch the polar plot and determine the gain margin and phase margin.

SOLUTION

Give that, G(s) = 1/s (1+s) (1 + 2s)

Put s - jω

$\therefore$ G (jω) = $\frac{1}{j\omega\,(1+j\omega)(1+j2\omega)}$

The corner frequencies are ω_{cl} = ½ = 0.5 rad/sec and ω_{c2} = 1 rad/sec. The magnitude and phase angle of G(jω) are calculated for the corner frequencies and for frequencies around corner frequencies and tabulated in table 1. Using polar to rectangular conversion, the polar coordinates and tabulated in table 2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in figure 1.

$$G(j\omega) = \frac{1}{j\omega\,(1+j\omega)(1+j2\omega)}$$

$$= \frac{1}{\omega\angle 90^\circ \sqrt{1+j\,\omega^2 tan^{-1}\omega}\sqrt{1+4\omega^2}\angle tan^{-1}2\omega}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} \angle -90^\circ - Tan^{-1}\omega - tan^{-1}2\omega$$

$$\therefore |G(j\omega)| = \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} = \frac{1}{\omega\sqrt{1+4\omega^2+\omega^4}}$$

$$= \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1} 2\omega$$

Table 1: Magnitude and phase of G(jω) at various frequencies

ω Rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
|G(jω)|	2.2	1.8	1.5	1.2	0.9	0.7	0.3
∠G(jω) deg	-144	-150	-156	-162	-171	-179.5 ≈ -180	-198

Table 2: Real and imaginary part of g(jω) at various frequencies

ω Rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$G_R(j\omega)$	-1.78	-1.56	-1.37	-1.14	-0.89	-0.7	-0.29
$G_I(j\omega)$	-1.29	-0.9	-0.61	-0.37	-0.14	0	0.09

RESULT

Gain margin, K_g = 1.4286

Phase margin, γ= +12°

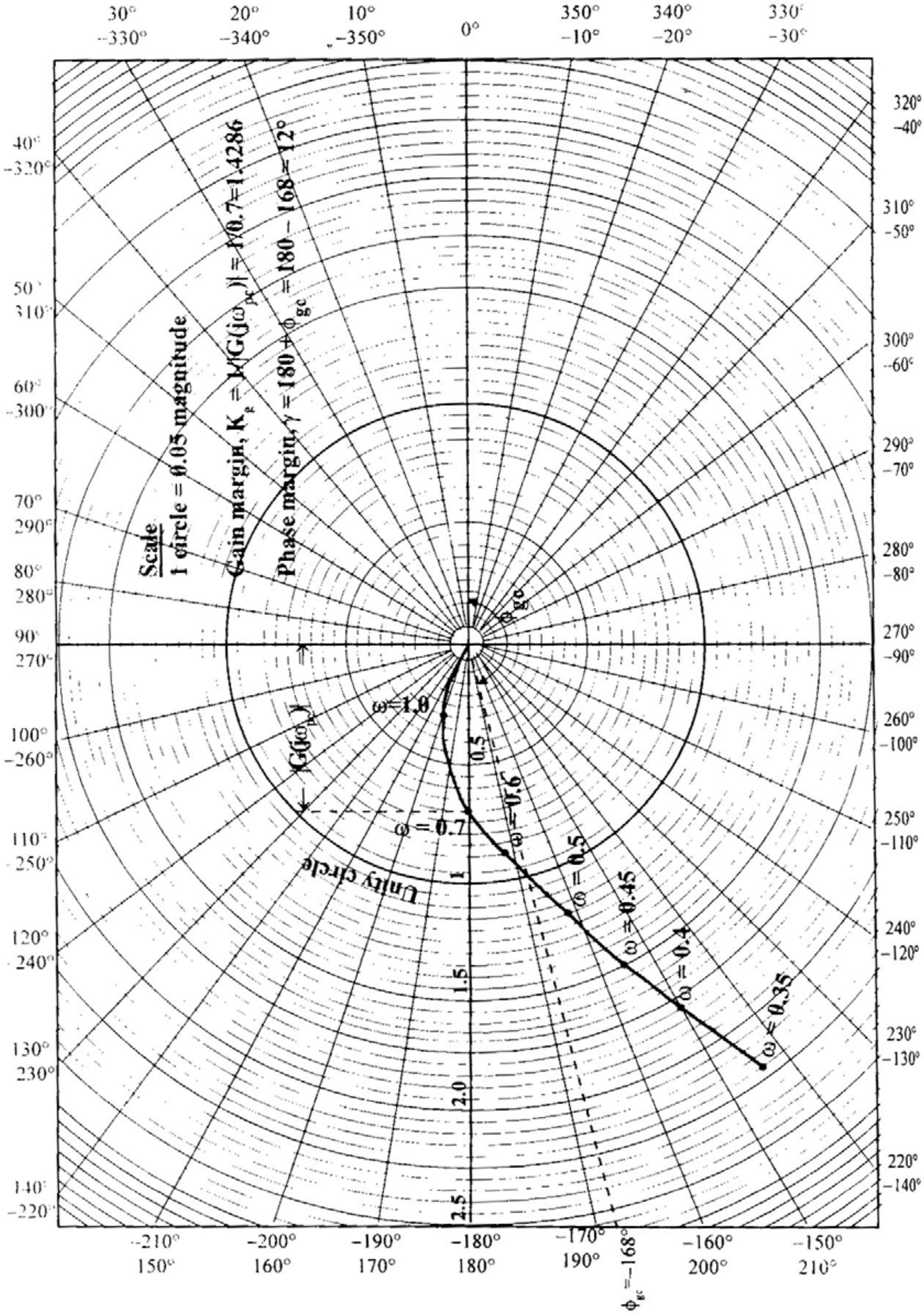


Figure1 Polar plot of $G(j\omega) = 1/j\omega(1 + j\omega)(1 + j2\omega)$ [using polar coordinates)

14] Problem

The open loop transfer function of a unity feedback system is given by G(s) = $1/s^2(1+s)(1+2s)$. Sketch the polar plot and determine the gain margin and phase margin.

Solution

Given that, G(s)= $1/s^2(1 + s)(1 + 2s)$

Put s = jω

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+j2\omega)}$$

The corner frequencies are ω_{c1} = 1 rad/sec and ω_{c2} = 2 rad/sec. The magnitude and phase angle of G(jω) are calculated for the corner frequencies and frequencies around corner frequencies and tabulated in table 1. Using the polar to rectangular conversion, the polar coordinates listed in table 1 are converted to rectangular coordinates and tabulated in table 2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in figure 1. The polar plot using rectangular coordinates is sketched on an ordinary graph sheet as shown in figure 2.

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+j2\omega)}$$

$$= \frac{1}{\omega^2\angle 180\sqrt{1+\omega^2}\angle tan^{-1}\omega\sqrt{1+4\omega^2}\angle tan^{-1}2\omega}$$

$$G(j\omega) = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}\angle(-180 - \tan^{-1}\omega - \tan^{-1}2\omega)$$

$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} = \frac{1}{\omega^2\sqrt{(1+\omega^2)(1+4\omega^2)}}$$

$$= \frac{1}{\omega^2\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(J\omega) = -180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

Table 1: Magnitude and phase plot of G(jω) at various frequencies

ω Rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
|G(jω)|	3.3	2.5	1.9	1.5	1.2	0.97≈1	0.8	0.3
∠G(jω) deg	-246	-251	-256	-261	-265	-269	-273	-288

Table 2: Real and imaginary part of g(jω) at various frequencies

ω Rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$G_R(j\omega)$	-1.34	-0.81	-0.46	-0.23	-0.1	-0.02	0.04	0.09
$G_I(j\omega)$	3.01	2.36	1.84	1.48	1.2	1.0	0.8	0.29

RESULT

Gain margin, K_g = 0

Phase margin, γ = -90°

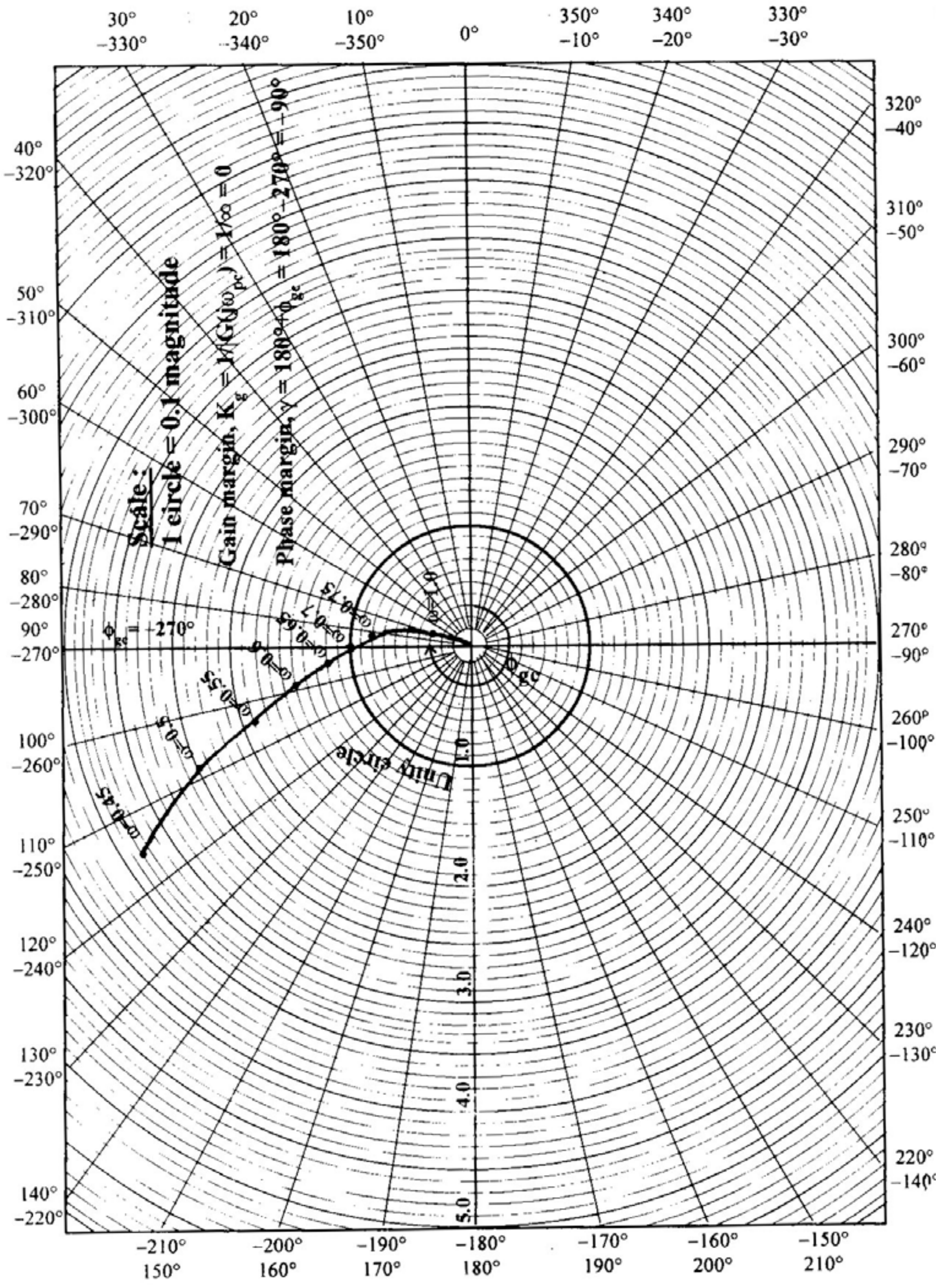


Figure 1 Polar plot of G(jω) = 1/(jω)2 (1 + jω) (1 + j2ω) (1 + j2ω), Using polar coordinates)

BODE PLOT

The step by step procedure for plotting the magnitude plot is given below.

Step 1: Convert the transfer function into Bode form or time constant form.

The Bode form of the transfer function is

$$G(s)=\frac{K(1+sT_1)}{s(1+sT_2)\left[1+\frac{s^2}{\omega_n^2}+2\zeta\frac{s}{\omega_n}\right]}$$

$$G(J\omega)\ \frac{K(1+j\omega\,T_1)}{j\omega\ (1+j\omega\,T_2)\left[1+\frac{\omega^2}{\omega_n^2}+j\,2\zeta\frac{\omega}{\omega_n}\right]}$$

Step 2: List the corner frequencies in the increasing order and prepare a table as shown below.

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dee

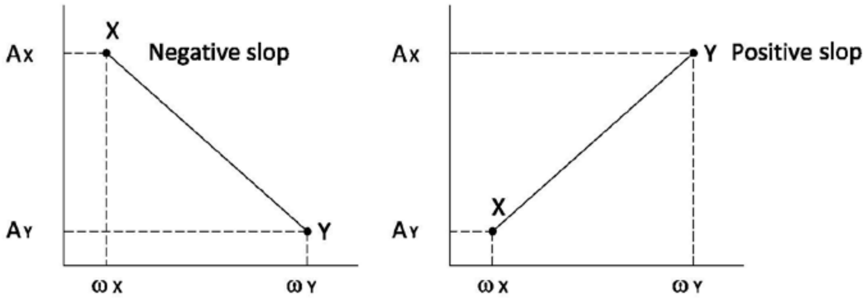
In the above table enter K or $K/(j\omega)^n$ or $K(j\omega)^n$ as the first term and the other terms in the increasing order of corner frequencies. Then enter the corner frequency, slope contributed by each term and change in slope at every corner frequency.

Step 3 : Choose an arbitrary frequency ω which is lesser than the lowest corner frequency. Calculate the db magnitude of K or $K/(j\omega)^n$ or $K(j\omega)^n$ at ω_l and at the lowest corner frequency.

Step 4 : Then calculate the gain (db magnitude) at every corner frequency one by one by using the formula,

Gain at ω_y = change in gain from $\omega_x to \omega_y + Gain at \omega_x$

$$= \left[Slope from \omega_x to \omega_y \times \log\frac{\omega_y}{\omega_x}\right] + Gain at \omega_x$$



Step 5 : Choose an arbitrary frequency ω_h which is greater than the highest corner frequency. Calculate the gain at ω_h by using the formula in step 4.

Step 6 : In a semi log graph sheet mark the required range of frequency on x-axis (ordinary scale) after choosing proper scale.

Step 7 : Mark all the points obtained in steps 3, 4, and 5 on the graph and join the points by straight lines. Mark the slope at every part of the graph.

(Note : The magnitude plot obtained above is an approximate plot. If an exact plot is needed then appropriate correction should be made at every corner frequencies)

Procedure for Phase Plot of Bode Plot

The phase plot is and no approximations are made while drawing the phase plot. Hence the exact phase angles of G(jω) are computed for various values of ω and tabulated. The choices of frequencies are preferably the frequencies chosen for magnitude plot. Usually the magnitude plot and phase plot are drawn in a single semilog-sheet on a common frequency scale.

Take another y-axis in the graph where the magnitude plot is drawn and in this y-axis mark the desired range of phase angles after choosing proper scale. From the tabulated values of $\omega$and phase angles, mark all the points on the graph. Join the points by a smooth curve.

Determination of Gain Margin and Phase Margin from Bode Plot

The gain margin in db is given by the negative of db magnitude of $G(j\omega)$at the phase cross-over frequency, ω_{pc}.theω_{pc} is the frequency at which phase of $G(j\omega)$is $-180°$. If the db magnitude of $G(j\omega) at\ \omega_{pc}$ is negative then gain margin is positive and vice versa.

Let ϕ_{gc} be the phase angle of $G(j\omega)$ at gain cross over frequency ω_g.Theω_{gc} is the frequency at which the db magnitude of $G(j\omega)$ is zero. Now the phase margin γ is given by, $\gamma = 180° + \phi_{gc}$.If$\phi_{gc}$ is less negative than $-180°$ than phase margin is positive and vice versa.

The positive and negative gain margins are illustrated in figure.

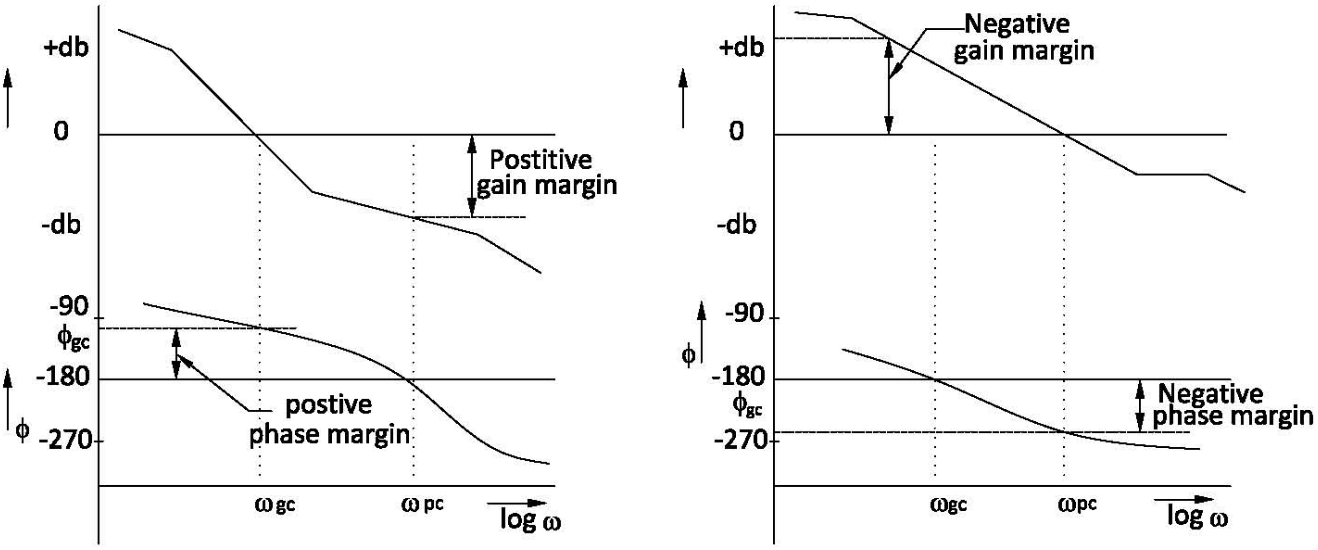


Figure2.8 *Bode plot showing phase margin (γ) and gain margin (K_g)*

15] Problem

Sketch Bode plot for the following transfer function and determine the system gain K for the gain cross over frequency to be 5 rad/sec

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

SOLUTION

The sinusoidal transfer function $G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$

Let K =1, $\therefore G(j\omega) = \frac{(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$

MAGNITUDE PLOT

The corner frequencies are

ω_{c1} = 1/0.2 = 5 rad/sec and ω_{c2} = 1/0.02 = 50 rad/sec

The various terms of G(jω) are listed in table 1 in the increasing order of their corner frequency. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

Table 1

Term	Corner frequency	Slope	Change in slope

	rad/sec	db/sec	db/dec
$(j\omega)^2$	-	+40	------
$\dfrac{1}{1+j\,0.2\omega}$	$\omega_{c1} = \frac{1}{0.2} = 5$	- 20	40 – 20 = 20
$\dfrac{1}{1+j\,0.02\omega}$	$\omega_{c2} = \frac{1}{0.02} = 50$	- 20	20 – 20 = 0

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ωh such that $\omega_h > \omega_{c2}$.

Let ω_l = 0.5 rad/sec and ω_h = 100 rad/sec

Let A = $|G(j\omega)|$ in db

Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h.

At $\omega = \omega_l$, A = 20 log$|(j\omega)^2|$ = 20 log $(\omega)^2$ = 20 log $(0.5)^2$ = - 12 db

At $\omega = \omega_{c1}$, A = 20 log$|(j\omega)^2|$ = 20 log $(\omega)^2$ = 20 log $(5)^2$ = 28 db

At $\omega = \omega_{c2}$, A = $\left[slop\ from\ \omega_{c1}\ to\ \omega_{c2} \times log\,\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(at\,\omega=\,\omega_{c1})}$

= 20 x log = $\frac{50}{5}$ + 28 = 48 db

At $\omega = \omega_h$, A = $\left[slop\ from\ \omega_{c2}\ to\ \omega_h \times log\,\frac{\omega_h}{\omega_{c2}}\right] + A_{(at\,\omega=\,\omega_{c2})}$

= 20 x log = $\frac{100}{50}$ + 48 = 48 db

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1 unit = 10 db on y – axis. Fix the point a, b, c and d on the graph. Joint the points by straight lines and mark the slope on the respective region.

PHASE PLOT

The phase angle of G(jω) as a function of ω is given by

$\phi = \angle g(J\omega) = 180° - \tan{-1}\ 0.2\omega - \tan{-1}\ 0.02\omega$

The phase angle of G(jω) are calculated for various values of ω and listed in table 2.

Table 2.

ω rad/sec	**$\tan^{-1} 0.2\omega$ Deg**	**$\tan^{-1} 0.02\omega$ deg**	**$\phi = \angle G(j\omega)$ deg**
0.5	5.7	0.6	173.7 ≈ 174
1	11.3	1.1	167.6 ≈ 168
5	45	5.7	129.3 ≈ 130
10	63.4	11.3	105.3 ≈ 106
50	84.3	45	50.7 ≈ 50
100	87.1	63.4	29.5 ≈ 30

One the same semilog sheet choose a scale of 1 unit = 20°, on the y- axis on the right side of semilog sheet. Mark the calculated phase angel on the graph sheet. Joint the points by a smooth curve.

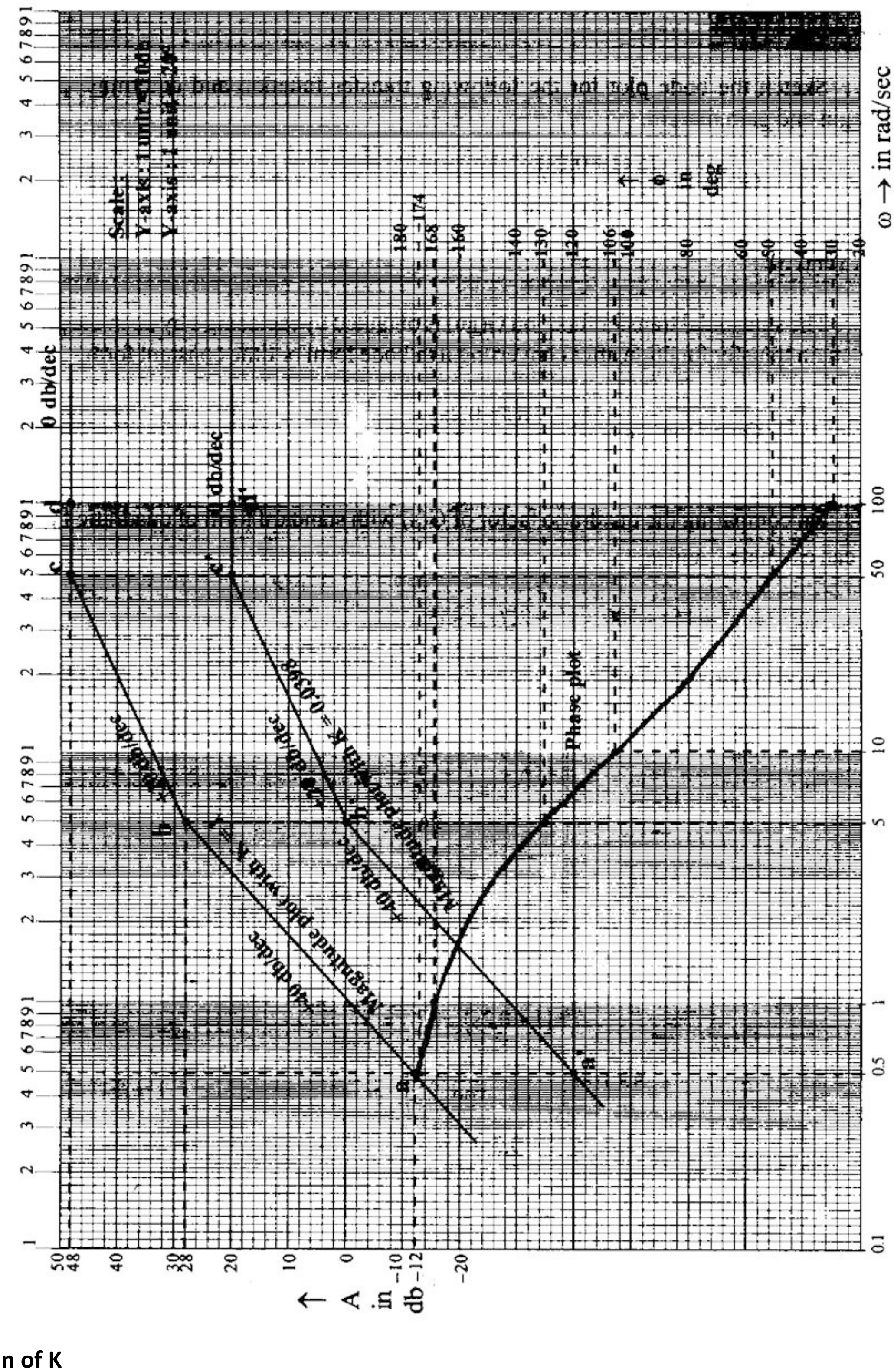


Figure 1 Bode plot of transfer function $G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$

Calculation of K

Given that the gain crossover frequency is 5 rad/sec. At ω = 5 rad/se the gain is 28 db. If gain crossover frequency is 5 rad/sec then at that frequency the db gain should be zero. Hence to every point of magnitude plot a db gain of – 28 db should be added. The addition of – 28db shifts the plot down wards. The corrected magnitude plot is obtained by shifting the plot with K = 1 by 28db downwards. The magnitude correction is independent of frequency. Hence the magnitude of -28db is contributed by the term K. The value of K is calculated by equation 20 logK to -28 db.

$\therefore$ 20 log K = - 28 db

Log K = -28/20 $\therefore$ K = $10^{-\frac{28}{20}}$ = 0.0398

The magnitude plot with K = 1 and 0.0398 and the phase plot are shown in figure

Note

The frequency ω = 5 rad/sec is a corner frequency. Hence in the expect plot the db gain at ω = 5 r5ad/sec will be 3 db less than the approximate plot. Therefore for exact plot the 20 log K will contribute a gain of – 25dp

$\therefore$ 20 log K = - 25 db

Log K = -25/20 $\therefore$ K = $10^{-\frac{25}{20}}$ = 0.562

16] Problem

Sketch the bode plot for the following transfer function and determine phase margin and gain margin.

G(s) $=\frac{75(1+0.2s)}{s(s^2+16s+100)}$

SOLUTION

The sinusoidal transfer function G(jω) is obtained by replacing s by jω in the given s-domain transfer function after converting it to bode from or time constant from.

Given that G(s) = $\frac{75(1+0.2s)}{s(s^2+16s+100)}$

On comparing the quadratic factor of G(s) with standard from of quadratic factor we can estimate ζ and ω_n

$\therefore$ s2 + 16s + 100 = s2 + 2$\zeta\omega$ns + ω_n^2

On comparing we get,

+ ω_n^2 = 100 ; $\therefore \omega_n$ = 10

$2\zeta\omega_n$ = 16

$\therefore \zeta = \frac{16}{2\omega_n}$

$= \frac{16}{2\times 10} = 0.8$

$$G(s) = \frac{75(1+0.2s)}{s\times 100\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)}$$

$$= \frac{0.75(1+0.2s)}{s\ (1+0.01s^2+0.16s)}$$

$$\therefore G(j\omega) = \frac{0.75(1+0.2j\omega)}{j\omega\ (1+0.01(j\omega)^2+0.16j\omega)} = \frac{0.75(1+j0.2\omega)}{j\omega\ (1-0.01\omega^2+j0.16\omega)}$$

Magnitude plot

The corner frequencies are $\omega_{c1} = \frac{1}{0.2}$= 5 rad/sec and $\omega_{c2} = \omega_n$ = 10 rad/sec

Note: For the quadratic factor the corner frequency is ω_n

The various terms of G(Jω) are listed in table 1 in increasing order of their corner frequencies. Also the table shows the slop contributed by each term and the change in slop at the corner frequency.

Table 1

Term	Corner frequency rad/sec	Slope dp/dec	Change in slop db/dec
$\frac{0.75}{j\omega}$	-	-20	
1+ j0.2 ω	ωc1 = $\frac{1}{0.2}$ = 5	20	-20 + 20 = 0
$\frac{1}{1-0.01\omega^2+0.16\omega}$	ωc2 = ωn =10	-40	0 – 40 = -40

Choose a low frequency ω_l such that $\omega_l < \omega_{cl}$ and choose a high frequency ωh such that $\omega_h > \omega_{c2}$

Let ωl = 0.5 rad/sec and ωh = 20 rad/sec

Let A = |G(jω)| in db; Let us calculate A at ω_l, ω_{cl}, ω_{c2} and ω_h.

At, $\omega = \omega_l$, A = 20 log $\left|\frac{0.75}{j\omega}\right|$ = 20 log $\frac{0.75}{0.5}$ = 3.5 db

At, $\omega = \omega_{cl}$, A = $\left[slop\ form\ \omega_{cl} to\ \omega_{c2} \times log\frac{\omega_{c2}}{\omega_{c1}}\right]$ + A (at$\omega = \omega_{cl}$)

= 0 x log $\frac{10}{5}$ + (-16.5) = - 16.5 db

At, $\omega = \omega_h$, A = $\left[slop\ form\ \omega_{c2} to\ \omega_h \times log\frac{\omega_h}{\omega_{c2}}\right]$ + A (at$\omega = \omega_{c2}$)

= -40 x log $\frac{20}{10}$ + (-16.5) = - 28.5 db

Let the point a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ωh respectively on the magnitude plot. In a semi log graph sheet choose a scale of 1 unit = 5 dp on y-axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on logarithmic scales in x-axis. Fix the points a, b, c and d on the graph. Join the points by straight lines and mark the slop on the respective region.

PHASE PLOT

The phase angle of G(jω) as a function of ω is given by

$\phi = \angle G(j\omega) = tab^{-1}\ 0.2\omega - 90° - tan^{-1}\frac{0.16\omega}{1-0.01\omega^2}$ For $\omega \leq \omega_n$

$\phi = \angle G(j\omega) = tab^{-1}\ 0.2\omega - 90° - tan^{-1}\frac{0.16\omega}{1-0.01\omega^2} + 180°$ For $\omega \leq \omega_n$

The phase angle of G(jω) are calculated for various values of ω and listed in table 2

Note: in quadratic factors the angle varies from 0° to 180°. But the calculator calculates tan-1 only between 0° to 90°. Hence a correction factor of 180° should be added to the phase angle after corner frequency.

On the same semilog sheet choose a scale of 1 unit = 20° on the y-axis on the right side of semi log sheet. Mark the calculated phase angle on the graph sheet. Joint the points by a smooth curve.

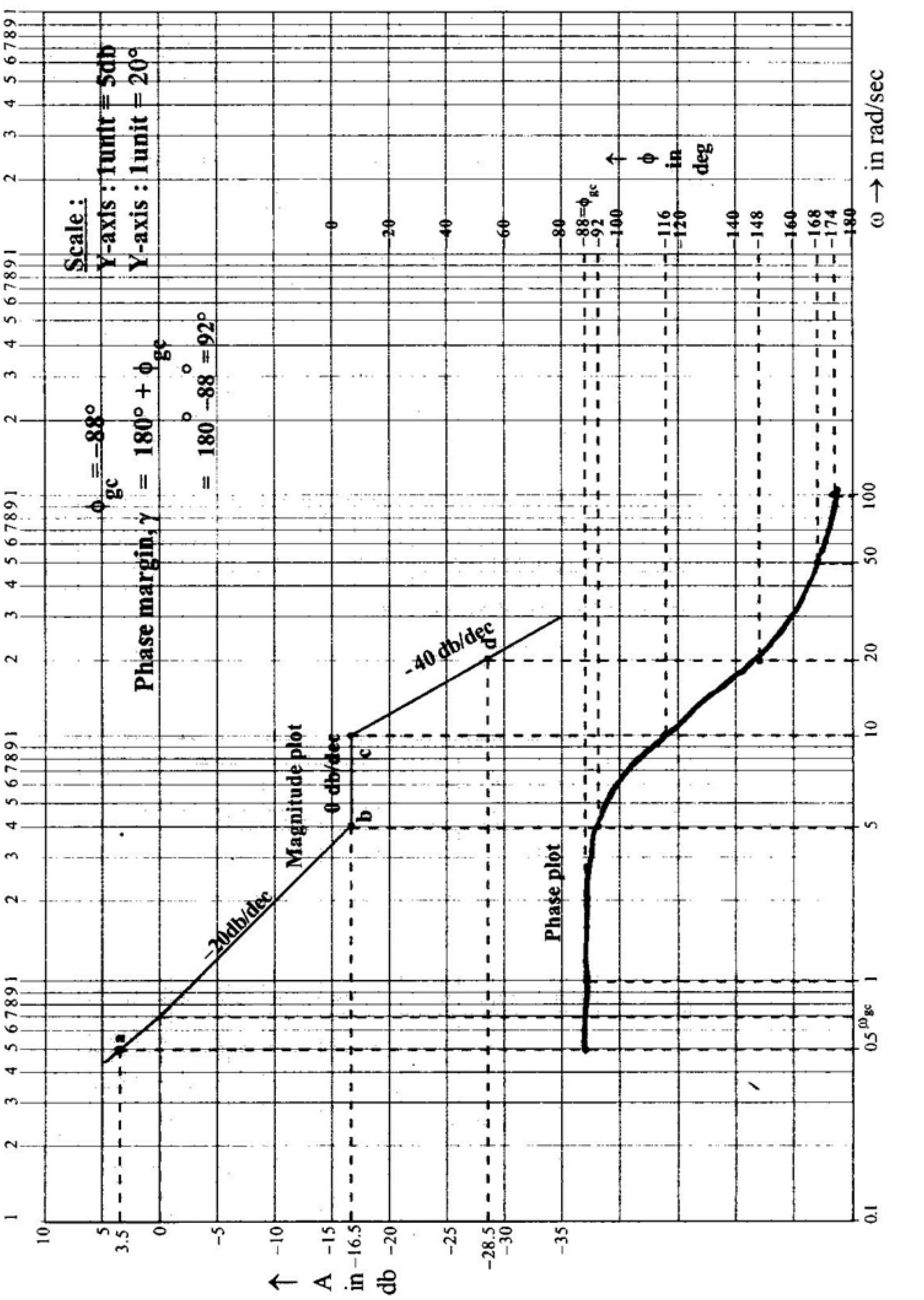


Figure 1: Bode plot of transfer function, G(jω) = $\frac{0.75(1+j0.2\omega)}{j\omega\ (1-0.01\omega^2+j0.16\omega)}$

Table 2

ω rad/sec	Tan-1 0.2 ω deg.	-tan$^{-1}\frac{0.16\omega}{1-0.01\omega^2}$	ϕ = ∠G(jω)
0.5	5.7	4.6	-88.9 ≈-88
1	11.3	9.2	-87.9 ≈ -88
5	45	46.8	-91.8 ≈ -92
10	63.4	90	-116.6 ≈-116
20	75.9	-46.8 + 180 = 133.2	-147.3 ≈ -148
50	84.3	-18.4 + 180 = 161.6	-167.3≈ -168
100	87.1	-9.2 + 180 = 170.8	-173.7 ≈ -174

The magnitude plot and the phase plot are shown in figure 1 from the figure 1 we find that the phase angle at gain crossover frequency (ω_{gc}) = ϕ_{gc} = 88°

∴ Phase Margin, γ = 180° + ϕgc = 180° - 88° = 92°

Here, Gain margin = + ∞.

The phase plot crosses - 180° only at infinity. The |G(jω)| at infinity is - ∞ db. Hence gain margin is + ∞.

NICHOLS PLOT

The Nichols plot is a frequency response plot of the open loop transfer function of a system. The Nichols plot is a graph between magnitude of G(jω) in db and the phase of G(jω) in degree, plotted on a ordinary graph sheet.

To plot the Nichols plot, first compute the magnitude of G(jω)in db and phase of G(jω) in deg for various values of ω and tabulate them. Usually the choices of frequencies are corner frequencies. Choose appropriate scales for magnitude on y-axis and phase an x-axis. Fix all the points on ordinary graph sheet and join the points by smooth curve. Write the frequency corresponding to each point of the plot.

In another method, first the Bode plot of G(jω) is sketched. From the Bode plot the magnitude and phase for various values of frequency, ω are noted and tabulated. Using these values the Nichols plot is sketched as explained earlier.

Determination of Gain Margin and Phase Margin From Nichols Plot

The gain margin in db is given by the negative of db magnitude of G(jω) at the phase crossover frequency, ω_{pc}.

The ω_{pc} is the frequency at which phase of G(jω) is –180°. If the db magnitude of G(jω) at ω_{pc} is negative then gain margin is positive and vice versa.

Let ϕ_{gc} be the phase angle of G(jω) at gain cross over frequency ω_{gc}. The ω_{gc} is the frequency at which the db magnitude of G(jω) is zero. Now the phase margin,γ is given by $\gamma = 180° + \phi_{gc}$. If ϕ_{gc} is less negative than –180° then phase margin is positive and vice versa.

The positive and negative gain margins are illustrated in figure1

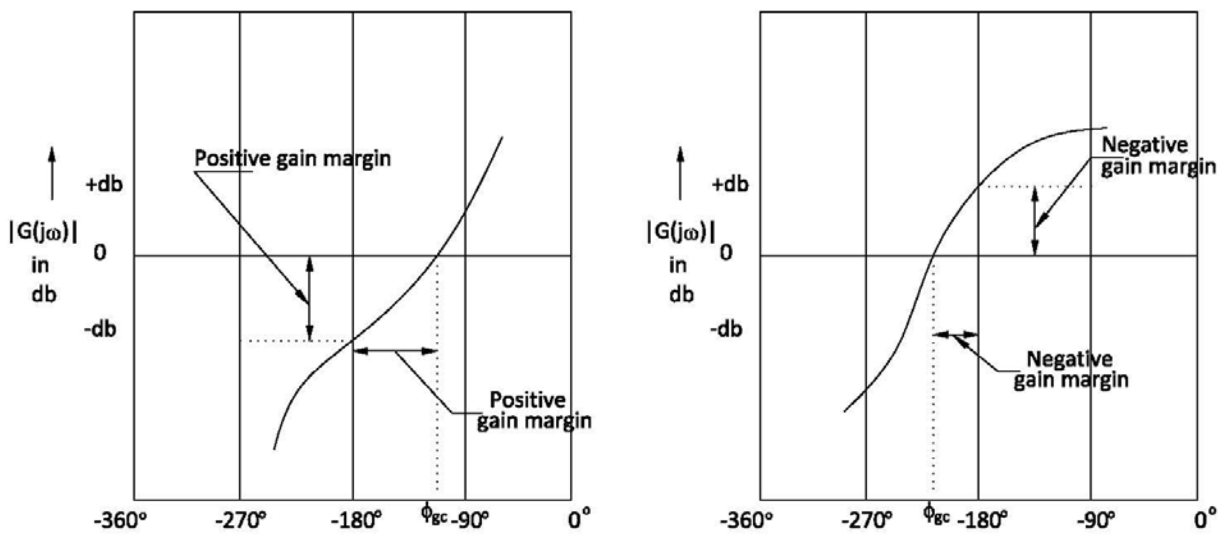


Figure 1 *Nichols plot showing phase margin and gain margin*

Gain Adjustment In Nichols Plot

In the open loop transfer function, G(jω) the constant K contributes only magnitude. Hence by changing the value of K the system gain can be adjusted to meet the desired specifications. The desired specifications are gain margin and phase margin.

In a system transfer function if the value of K required to be estimated to satisfy a desired specification then draw the Nichols plot of the system with K = 1. The constant K can add 20log to every point of the plot. Due to this addition the Nichols plot will shift vertically up or down. Hence shift the plot vertically up or down tomeet the desired specification. Equate the vertical distance by which the Nichols plot is shifted to 20 log K and solve for K.

Let x = change in db (x is positive if the plot is shifted up and vise versa)

Now, 20 log K = x

log K = $\frac{X}{20}$

$$\therefore K = 10^{\frac{x}{20}}$$

17] Problem

Consider a unit feedback system have an open loop transfer function

G(s) = $\frac{K(1+10s)}{s^2(1+s)(1+2s)}$. Sketch the Nichols plot and determine the value of K so that (i) Gain margin is 10db, (ii) Phase margin is 10°

SOLUTION

Given that G(s) $= \frac{K(1+10s)}{s^2(1+s)(1+2s)}$

The sinusoidal transfer function G(jω) is obtained by letting to s = jω. Also put K = 1

$$\therefore G(j\omega) = \frac{(1+j10\omega)}{(j\omega)^2(1+j\omega)(1+J2\omega)}$$

$$= \frac{\sqrt{1+(10\omega)^2}\angle tan^{-1}10\omega}{\omega^2\angle 180^\circ\sqrt{1+\omega^2}\ \angle tan^{-1}\omega\ \sqrt{1+(2\omega^2}\angle tan^{-1}2\omega}$$

$$G(j\omega) = \frac{\sqrt{1+100\omega^2}}{\omega^2\sqrt{1+\omega^2 J}\sqrt{1+4\omega^2}}$$

$$\therefore |G(j\omega)| = 20\log\frac{\sqrt{1+100\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

$\angle$G(jω) tan^{-1} 10ω - 180°- tan^{-1}ω - tan^{-1} 2ω

The magnitude of G(jω in db and phase of G(jω) in deg are calculated for various values of ω and listed in the following table. The Nichols plot of G(jω) with K = 1 is sketched as shown in figure 4.13.1

ω rad/sec	0.2	0.4	0.6	0.8	1.0	1.5	2.0	3.0	4.0
$\lvert G(j\omega)\rvert$ db	34.1	25.4	19.3	14.3	10	1.4	-5.3	-15.2	-22.5
$\angle$G(jω)	-150	-164	-181	-194	-204	-222	-232	-244	-250

From the Nichols plot the gain margin and phase margin of the system when K = 1 are

Gain margin = - 19.5 db

Phase margin = - 45°

Gain Adjustment for Required Gain Margin

For a gain margin of 10 db, the magnitude of G(jω) should be – 10 db, when the phase is - 180°. Hence if we add -29.5 db to every point of G(jω), then the plot shifts down wards and it will cross - 180° axis at a magnitude of -10db. The magnitude correction is independent of frequency and so this gain can be contributed by the term K. Let this value of K be K_1. The value of K_1 is calculated by equating 20 log K_1 to -29.5 db.

$\therefore$ 20 log K_1 = -29.5 db

Log K_1 = -29.5/20

$K_1 = 10^{\frac{-29.5}{20}} = 0.0335$

Gain adjustment for required phase margin

Let ϕ_{gc2} = phase of G(jω) at gain crossover frequency for a phase margin of 10°

$\therefore$ Phase margin, $\gamma_2 = 180 + \phi_{gc2}$

$\therefore \phi_{gc2} = \gamma_2 - 180° = 10 - 180 = -170°$

When K = 1, the magnitude of G(jω) is +23 db corresponding to phase of - 170°. But for a phase margin of 10°, this gain should be made zero. Hence if we add -23 db to every point of G(jω) locus then the plot shifts downwards and it will cross - 170° axis at magnitude of 0 db. The magnitude correction is independent of frequency and so this gain can be contributed by the term K. Let this value of K be K_2. The value of K_2 is calculated by equation 20 log K_2 to – 23 db.

$\therefore$ 20 log K_2 = –23

Log K_2 = –23/20

$K_2 = 10^{\frac{-23}{20}} = 0.07$

RESULT

When K = 1,

Gain margin	= –19.5db
Phase margin	= –45°
For gain margin of 10db, K = K_1	= 0.0335
For a phase margin of 10°, K = K_2	= 0.07

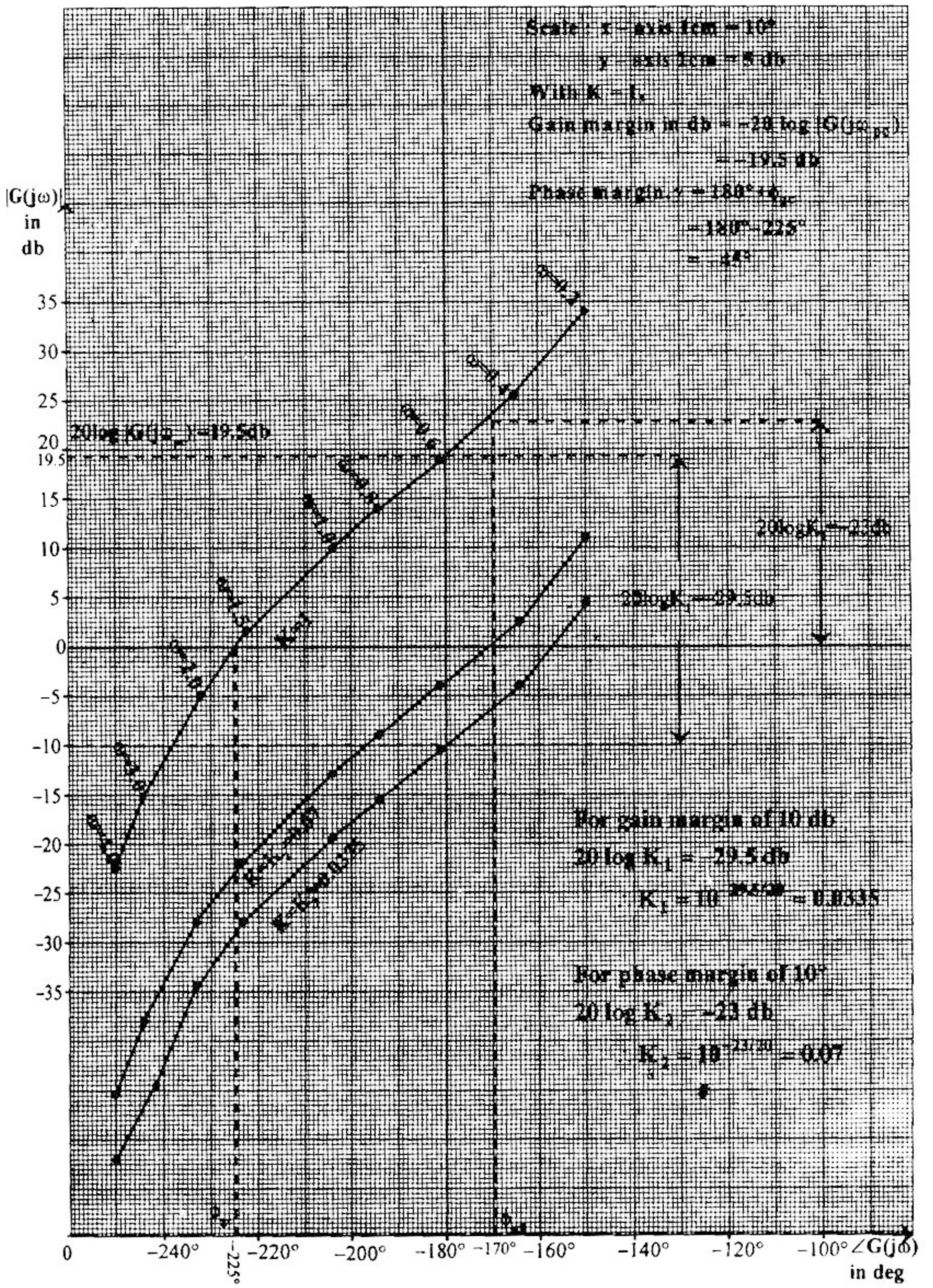


Figure1 Nichols plot of $G(j\omega) = \frac{(1+j10\omega)}{(j\omega)^2(1+j\omega)(1+j2\omega)}$

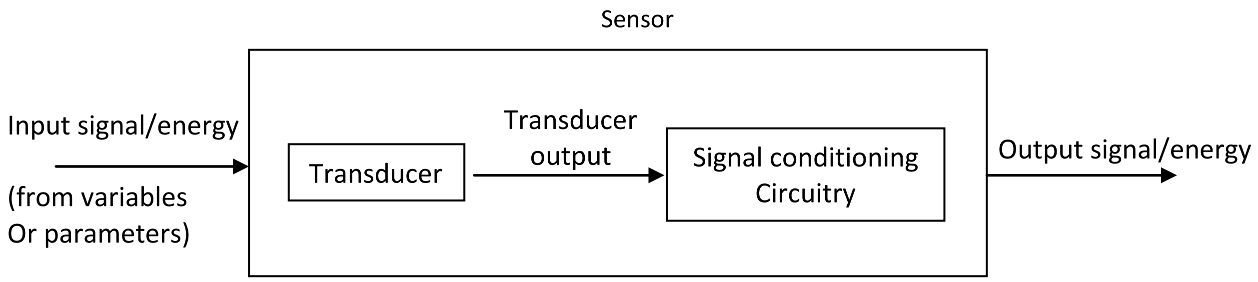
Unit III

Sensors and Transducers

Sensors are devices which produce a proportional output signal (mechanical, electrical, magnetic, etc.) when exposed to a physical phenomenon (pressure, temperature, displacement, force, etc.). Many devices require sensors for accurate measurement of pressure, position, speed, acceleration or volume. Transducers are devices which converts an input of one form of energy into an output of another form of energy. The term transducer is often used synonymously with sensors. However, ideally, the word 'transducer' is used for the sensing element itself whereas the term 'sensor' is used for the sensing element plus any associated signal conditioning circuitry. Typically, a transducer may include a diaphragm which moves or vibrates in response to some form of energy, such as sound.

Some common examples of transducers with diaphragms are microphones, loudspeakers, thermometers, position and pressure sensors. Sensors are transducers when they sense one form of energy input and output in a different form of energy. For example, a thermocouple responds to a temperature change (thermal energy) and outputs a proportional change in electromotive force (electrical energy). Therefore, a thermocouple can be called a sensor and or transducer.

Figure illustrates a sensor with sensing process in terms of energy conversion. The form of the output signal will often be a voltage analogous to the input signal, though sometimes it may be a wave form whose frequency is proportional to the input or a pulse train containing the information in some other form.



Selection of Sensors

In selecting a sensor for a particular application there are a number of factors that need to be considered:

1. The nature of the measurement required, e.g., the variable to be measured, its nominal value, the range of values, the accuracy required, the required speed of measurement, the reliability required, the environmental conditions under which the measurement is to be made.
2. The nature of the output required from the sensor, this determining the signal conditioning requirements in order to give suitable output signals from the measurement.
3. The possible sensors can be identified taking into account such factors as their range, accuracy, linearity, speed of response, reliability, maintainability, life, power supply requirements, ruggedness, availability and cost.

The selection of sensors cannot be taken in isolation from a consideration of the form of output that is required from the system after signal conditioning and thus there has to be a suitable marriage between sensor and signal conditioner.

Classification and characteristics of Sensors

Sensors are generally classified into two types based on its power requirement: *passive* and *active.* In active sensors, the power required to produce the output is provided by the sensed physical phenomenon itself (Examples: thermocouples, photovoltaic cells, piezoelectric transducers, thermometer etc.) whereas the passive sensors require external power source (Examples: resistance thermometers, potentiometric devices, differential transformers, strain gage etc.). The active sensors are also called as *self-generating* transducers. Passive sensors work based on one of the following principles: resistance, inductance and capacitance.

Sensors can also be classified as analog or digital based on the type of output signal. Analog sensors produce continuous signals that are proportional to the sensed parameter. These sensors generally require analog-to-digital conversion before sending output signal to the digital controller (Examples: potentiometers, LVDTs (linear variable differential transformers), load cells, and thermistors, bourdon tube pressure sensor, spring type force sensors, bellows pressure gauge etc.). Digital sensors on the other hand produce digital outputs that can be directly interfaced with the digital controller (Examples: incremental, encoder, photovoltaic cells, piezoelectric transducers, phototransistors, photodiodes etc.). Often, the digital outputs are produced by adding an analog-to-digital converter to the sensing unit. If many sensors are required, it is more economical to choose simple analog sensors and interface them to the digital controller equipped with a multi-channel analog-to-digital converter.

Another way of classifying sensor refers to as *primary* or *secondary* sensors. Primary sensors produce the output which is the direct measure of the input phenomenon. Secondary sensors on the other hand produce output which is not the direct representation of the physical phenomenon. Mostly active sensors are referred as primary sensors where as the passive sensors are referred as secondary sensors.

1. Static Characteristics

Static character sties of an instrument are the parameters which are more or less constant or varying very slowly with time. The following characteristics are static characteristics.

Range

Every sensor is designed to work over a specified range i.e. certain maximum and minimum values. The design ranges are usually fixed, and if exceeded, result in permanent, damage to or destruction of it sensor. For example, a thermocouple may have a range of-100 to 1260 °C.

Span

It represents the highest possible input value which can be applied to the sensor without causing unacceptably large inaccuracy. Therefore, it is the difference between maximum and minimum values of the quantity to be measured.

Span = Maximum value of the input - Minimum value of the input

Errors

Error is the difference between a measured value and the true input value.

Error = Measured value - True input value

Accuracy

A very important characteristic of a sensor is accuracy which really means inaccuracy. Inaccuracy is measured as a ratio of the highest deviation of a value represented by the sensor to the ideal value. The accuracy of a sensor is inversely proportional to error, i.e., a highly accurate sensor produces low errors.

Sensitivity

Sensor sensitivity is defined as the change in output per change in input. The factor may be constant over the range of the sensor (linear), or it may vary (nonlinear).

$$\text{Sensitivity} = \frac{\text{Change in output}}{\text{Change in output}} = \frac{\Delta\theta_o}{\Delta\theta_i}$$

When an instrument consists of different elements connected in series and have static sensitivities of S_1, S_2, S_3, ... etc, then the overall sensitivity is expressed as follows.

$$S_1 = \frac{\theta_1}{\theta_i},\ S_2 = \frac{\theta_2}{\theta_i},\ S_3 = \frac{\theta_3}{\theta_i},\ \ldots.$$

Overall sensitivity, $S = \frac{\theta_0}{\theta_i} = \frac{\theta_1}{\theta_i} \times \frac{\theta_2}{\theta_i} \times \frac{\theta_3}{\theta_i} \times \ldots. = S_1 \times S_2 \times S_3 \ldots\ldots$

Hysteresis

Hysteresis is defined as the maximum differences in output for a given input when this value is approached from the opposite direction. It is a phenomenon which shows different outputs when loading and unloading. Simply, hysteresis means that both the loading and unloading curves do not coincide. Figure 1 shows that the deviation of unloading from loading condition due to hysteresis effect.

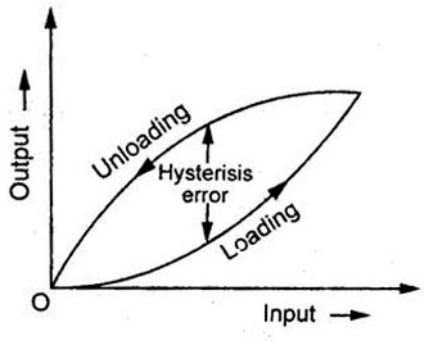


Figure 1 *Hysteresis effects*

Linearity

Linearity of a sensor refers to the output that is directly proportional to input over its entire range, so that the slope of a graph of output versus input describes a straight line. If the response of the system to input A is output A, and the response to input B is output B, then the response to input C (= input A + input B) will be output C (= output A + output B).

Non-linearity

Non-linearity of a sensor refers to the output that is not proportional to input over its entire range, so that the slope of a graph of output versus input describes a curve. Non-linearity error is the deviation of output curve from a specified straight line as shown in Figure 2.

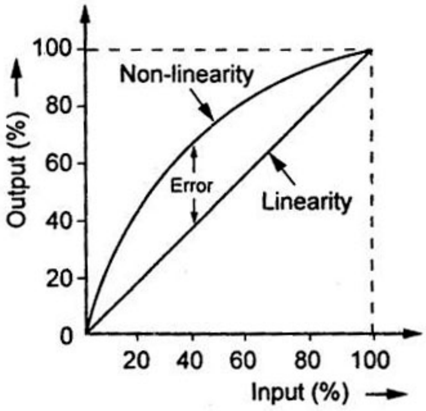


Figure 2 *Non-linearity errors*

Repeatability and reproducibility

Repeatability may be defined as the ability of the sensor to give same output reading when the same input value is applied repeatedly under the same operating conditions.

Reproducibility may be defined as the degree of closeness among the repeated measurements of the output for the same value of input under the same operating conditions at different times.

Stability

Stability means the ability of the sensor to indicate the same output over a period of time for a constant input.

Dead band/time

Dead band of a sensor is the range of input values for which the instrument does not respond. The dead band is typically a region of input close to zero at which the output remains zero.

Dead time is the time taken by the sensor from the application of input to begin its response and change.

Resolution

Resolution is defined as the smallest change that can be detected by a sensor. It can also be defined as the minimum value of the input required to cause an appreciable change or an increment in the output.

Zero Drift

Drift is the variation of change in output for a given input over a period of time. When making a measurement it is necessary to start at a known datum, and it is often convenient to adjust the output of the instrument to zero at the datum. The signal level may vary from its set zero value when the sensor works. This introduces an error into the measurement equal to the amount of variation or drift. Zero drift may result from changes of temperature, electronics stabilizing, or aging of the transducer or electronic components.

Output impedance

Impedance is the ratio of voltage and current flow for a sensor. Two types of impedance are important in sensor applications: input impedance and output impedance. Input impedance is a measure of how much current must be drawn to power a sensor. Output impedance is a measure of a sensor's ability to provide current for the next stage of the system.

2. Dynamic characteristics

Sensors and actuators respond to inputs that change with time. Any system that changes with time is considered a dynamic system. Dynamic characteristics of aninstrument are the parameters which are varying with time. The following characteristics are dynamic characteristics.

Response time

The time taken by a sensor to approach its true output when subjected to a step input is sometimes referred to as its response time. It is more usual, however, to quote a sensor as having a flat response between specified limits of frequency. This is known as the frequency response, and it indicates that if the sensor is subjected to sinusoidally oscillating input of constant amplitude, the output will faithfully reproduce a signal proportional to the input.

Time constant

It is the time taken by the system to reach 63.2% of its final output signal amplitude i.e. 62.3% of response time. A system having smaller time constant reaches its final output faster than the one with larger time constant. Therefore possesses higher speed of response.

Rise time

It is the time taken by the system to reach 63.2% of its final output signal.

Setting time

It is the time taken by a sensor to be within a close range of its steady state value.

TYPES OF SENSORS

1. Inductive Displacement Sensors

The most widely used variable-inductance displacement transducer in industry is LVDT (Linear Variable Differential Transformer). It is a passive type sensor. It is an electro-mechanical device designed to produce an AC voltage output proportional to the relative displacement of the transformer and the ferromagnetic core.

The physical construction of a typical LVDT consists of a movable core of magnetic material and three coils comprising the static transformer as shown in Figure 1.39. One of the three coils is the primary coil or excitation coil and the other two are secondary coils or pick-up coils. An AC current (typically 1 kHz) is passed through the primary coil, and an AC voltage is induced in the secondary coils. The magnetic core inside the coil winding assembly provides the magnetic flux path liking the Primary and secondary Coils.

When the magnetic core is at the centre position or null position the output voltages are equal and opposite in polarity and, therefore, the output voltage is zero. The Null Position of an LVDT is extremely stable and repeatable. When the magnetic core is displaced from the Null Position, a certain number of coil windings are affected by the proximity of the sliding core and thus an electromagnetic imbalance occurs. This imbalance generates a differential AC output voltage across the secondary coil which is linearly proportional to the direction and magnitude of the displacement. The output voltage to displacement plot is a straight line within a specified range. Beyond the nominal range, the output deviates from a straight line in a gentle curve as shown in Figure 1.40.

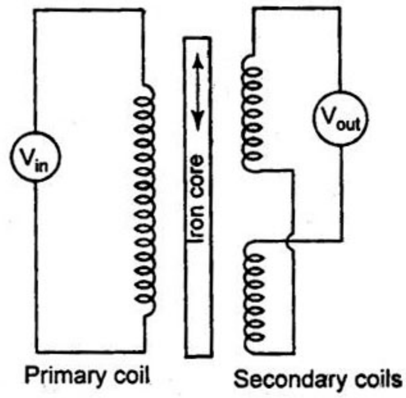


Figure 3 LVDE (Linear variable differential transformer)

The *Rotational Variable Differential Transformer (RVDT)* is used to measure rotational angles and operates under the same principles as the LVDT sensor. Whereas the LVDT uses a cylindrical iron core, the RVDT uses a rotary ferromagnetic core. A schematic of RVDT is shown in Figure 1.41.

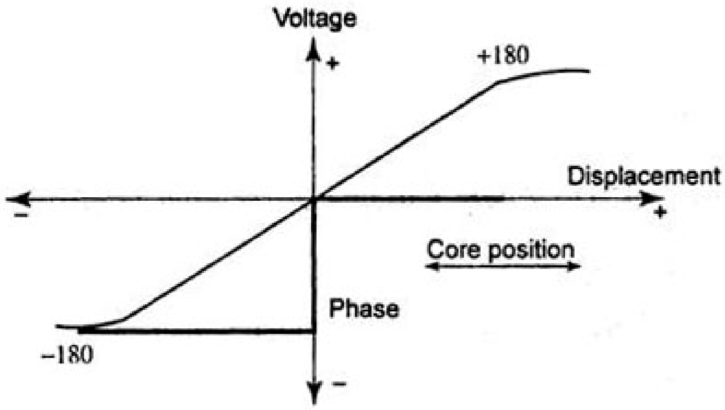


Figure 4 LVDT output

Calculation of output voltage

Motion of a magnetic core changes the mutual inductance of two secondary coils relative to a primary coil

Primary coil voltage: $V_{in} = \sin(\omega t)$

Secondary coils induced emf:

$V_1 = k_1 \sin(\omega t)$ and

$V_2 = k_2 \sin(\omega t)$

The value of k_1 and k_2 depend on the amount of coupling between the primary and the secondary coils, which is proportional to the position of the coil.

When the coil is in the central position, $k_1=k_2$

$$V_{out} = V_1 - V_2 = 0$$

When the coil is displaced x units, $k_1 \neq k_2$

$$V_{out} = (k_1 - k_2) \sin(\omega t)$$

Positive or negative displacements are determined from the phase of V_{out}.

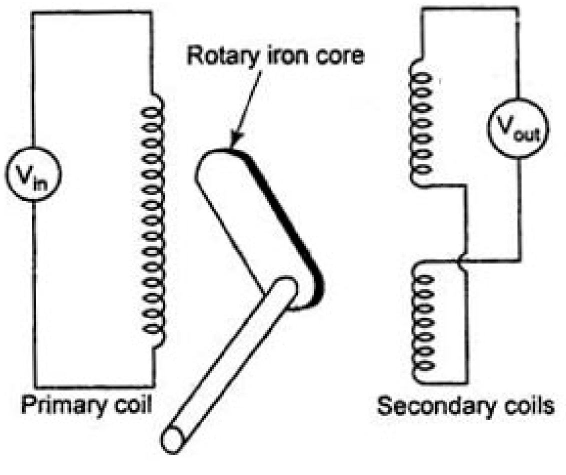


Figure 5 RVDT (Rotary variable differential transformer)

Applications

LVDT can be used to measure the displacement, deflection, position and profile of a workpiece.

Advantages

- Relative low cost due to its popularity.
- Solid and robust, capable of working in a wide variety of environments.

- No friction resistance, since the iron core does not contact the transformer coils, resulting in an infinite (very long) service life.
- High signal to noise ratio and low output impedance.
- Negligible hysteresis.
- Short response time, only limited by the inertia of the iron core and the rise time of the amplifiers.
- No permanent damage to the LVDT if measurements exceed the designed range.
- It can operate over a temperature range of - 265 °C to 600°C.
- High sensitivity up to 40 V/mm.
- Less power consumption (less than 1W)

Disadvantages:

- The performance of these sensors is likely affected by vibration etc.
- Relatively large displacements are required for appreciable output.
- Not suitable for fast dynamic measurements because of mass of the core.
- Inherently low in power output.
- Sensitive to stray magnetic fields but shielding is not possible.

2. Inductive proximity sensor

Inductive proximity sensors are today the most commonly employed industrial sensors for detection of ferrous metal objects over short distances. Inductive proximity sensors operate under the electrical principle of inductance. Inductance is the phenomenon where a fluctuating current, which by definition has a magnetic component, induces an electromotive force (emf) in a target object.

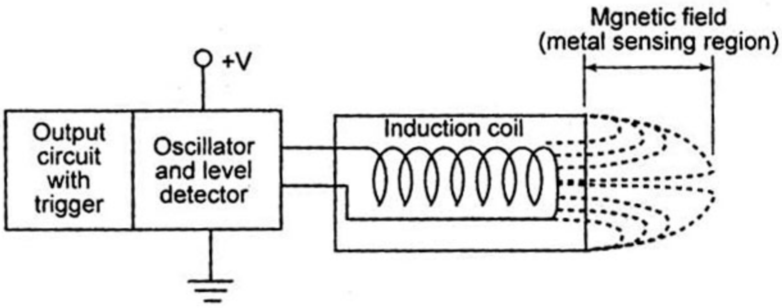


Figure 6 Inductive proximity sensor

An inductive proximity sensor has four components; the induction coil, oscillator, detection circuit and output circuit as shown in Figure 6. The oscillator generates a fluctuating magnetic field the shape of a doughnut around the winding of the coil that locates in the device's sensing face. When a metal object moves into the magnetic field of detection, eddy circuits build up in the metallic object. These eddy currents produce a secondary magnetic field that interacts with field of the probe, thereby loading the probe oscillator. The effective impedance of the probe coil changes, resulting in an oscillator frequency shift (or amplitude change). The sensor's detection circuit monitors the oscillator's strength and triggers an output signal from the output circuitry proportional to the sensed gap between probe and target.

3. Pyro-electric Sensors

The pyroelectric sensor is made of a crystalline material that generates a surface electric charge when exposed to heat source. Example of crystalline material is lithium tantalite. When this type of material is heated below a temperature known as Curie point, a large spontaneous electrical polarization is exhibited from the material in response to a temperature change. The change in polarization is observed as an electrical voltage signal if electrodes are placed on opposite faces of a thin slice of the material. Figure 7 (a) shows that the charges in the pyroelectric material are balanced if there is no infrared radiation from the heat source falls on the materials surface. When the material is exposed to the infrared radiation from the heat source the charges in the pyroelectric material are not balance and hence there is some excess charge in the material as shown in Figure 7 (b).

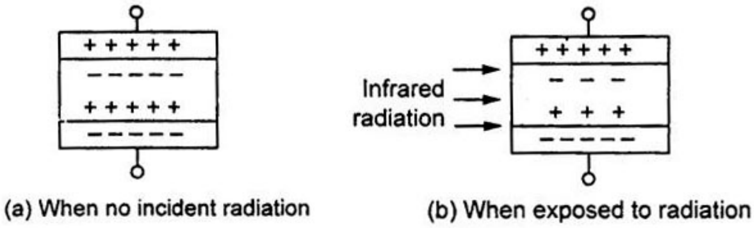


Figure 7 *Pyroelectric effect*

The design can be thought of as a typical form of a capacitor circuit. Figure 8 shows the equivalent circuit of a pyroelectric sensor. It essentially consists of a capacitor charged by the excess charge with a resistance R to represent the internal leakage combined with the input resistance of an external circuit.

For detection of a human motion or intrusion in the country borders, the pyroelectric are sensors used. In such, applications, the sensing element has to differentiate between general background heat radiation and a moving heat source. Therefore, a single pyroelectric sensor is not capable to use and *dual pyroetectric* sensors are used as shown in Figure 9. In this dual pyroelectric sensor, the sensing element has the one front electrode and two back electrodes. When two sensors are connected, both sensors receive the same heat signal and their outputs are cancelled. When a heat source moves from its position the heat radiation moves from one of the sensing elements to the other. Then the current alternates in one direction first and then reversed to the other direction. When the amount of infrared radiation from heat source striking the crystal, the electric charge also changes and can then be measured with a sensitive FET device built into the sensor.

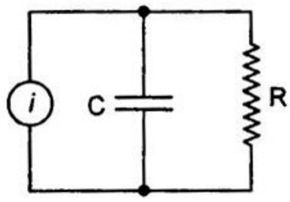


Figure 8 *Equivalent circuit of a pyroelectric sensor*

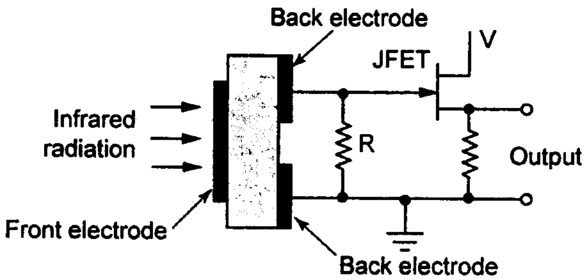


Figure 9 *Dual pyroelectric sensor*

4. Force Sensors

Force sensors are used in many mechanical equipments and aggregates for an accurate determination of forces applied in the system. The force sensor outputs an electrical signal corresponding to the force applied. Force sensors are commonly used in many applications such as automotive brakes, suspension, transmission, speed control, lifts, aircrafts, digital weighing systems etc. Most of the force sensor uses displacement as the measure of the force. The simplest form of force sensor is the spring balance in which a force is applied to the one end of the spring causes displacement of the spring. This displacement is the measure of the force applied. A common force sensor is a strain gauge load cell which is explained under.

5. Strain gauge load cell

A load cell is an electromechanical transducer that converts load acting on it into an analog electrical signal. Load cells provide accurate measurement of compressive and tensile loads. Load cells commonly function by utilizing an internal strain gauge that measure deflection. Because the modulus of elasticity of a load cell is constant the amount of strain can be calibrated to determine the force upon the load cell. Typically the force creates the train in the load cell which is measured by strain gauge transducer.

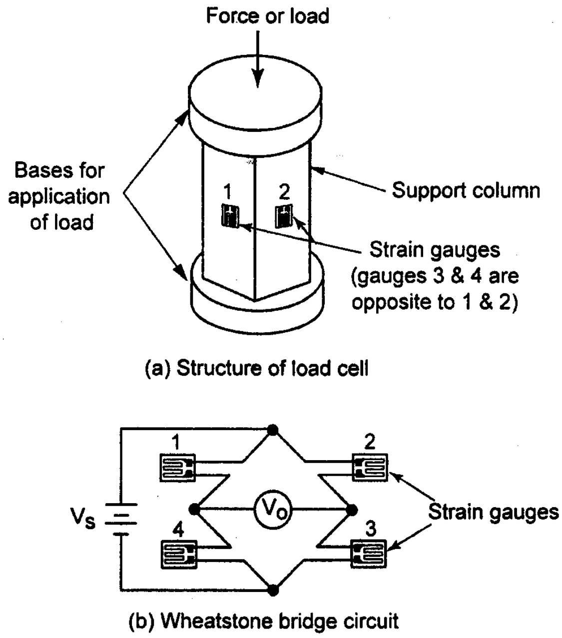


Figure 10 Strain gauge load cell

Strain gauge is attached to the object or the strained element where the force is being applied. As the object is stressed due to the applied force, the resulting strain deforms the strain gauge attached with it. This causes an increase in resistivity of the gauge which produces electrical signal proportional to the deformation. The measurement of resistivity is the measure of strain which in turn gives the measurement of force or load applied on the object. The change of resistance is generally very small and is usually measured using a Wheatstone bridge circuit where the strain gauges are connected into the circuit. The strain gauges are serving as resistors in the circuit. The Wheatstone bridge circuit produces analog electrical output signal. In a typical strain gauge load cell for measuring force, four strain gauges are attached to the surface of the counterforce and are electrically connected in a full Wheatstone bridge circuit as shown in Figure 10. Load cells have different shapes (cylindrical tubes, rectangular or square beams, and shaft) for different applications and load requirements to ensure that the desired component of force is measures, thus strain gauges having different shapes are positioned in various orientations upon the load cell body. The different configurations of strain gauges are already discussed under *strain gauges displacement sensors.*

6. Fluid Pressure Sensors

Pressure is an expression of the force required to stop a fluid from expanding, and is usually expressed in terms of *force per unit area. A* pressure sensor measures pressure of gases or liquids. These sensors generate a signal as a function of the pressure applied by the fluid. Pressure sensors are used in many applications such as automotive vehicles, hydraulic systems, engine testing etc. Pressure sensors may required to measure different types of pressures: 1. Absolute pressure where the pressure is measured relative to the perfect vacuum or zero-pressure, 2. Gauge pressure where the pressure is measured relative to the atmospheric pressure, and 3. Differential pressure where a pressure difference is measured. The devices which are used to measure fluid pressure in industrial processes are:

1. Diaphragm pressure sensor
2. Capsule pressure sensor
3. Bellows pressure sensor
4. Bourdon tube pressure sensor
5. Piezoelectric sensor
6. Tactile Sensor

The construction and working principle of these sensors are explained here.

7. *Diaphragm pressure sensor*

The diaphragm pressure sensor uses the elastic deformation of a diaphragm (i.e. membrane) to measure the difference between an unknown pressure and a reference pressure. Diaphragm is a thin circular elastic membrane made of generally silicon as show in Figure 11. As pressure changed, the diaphragm moves, and this motion is the measure of differential pressure. Diaphragms are popular because they require less space and the motion they produce is sufficient for operating electronic transducers. They also are available in a wide range of materials for corrosive service applications.

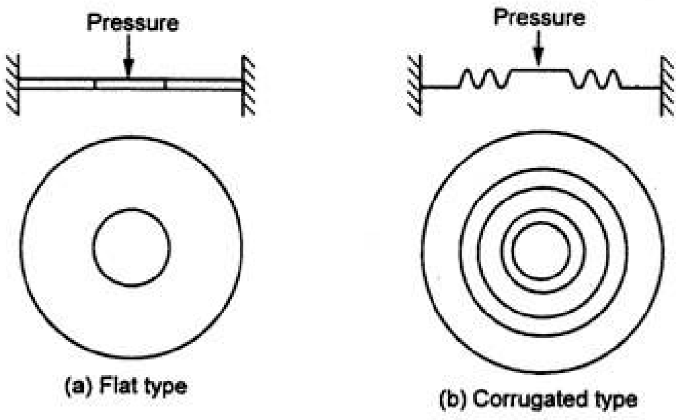


Figure 11. Diaphragm

A typical diaphragm pressure gauge contains a chamber divided by a diaphragm, as shown in the Figure 12. One side of the diaphragm is open to the external targeted pressure. P_{Ext}, and the other side is connected to a known pressure, P_{Refr}. The pressure difference, P_{Ext}, P_{Ref}, mechanically deflects the diaphragm.

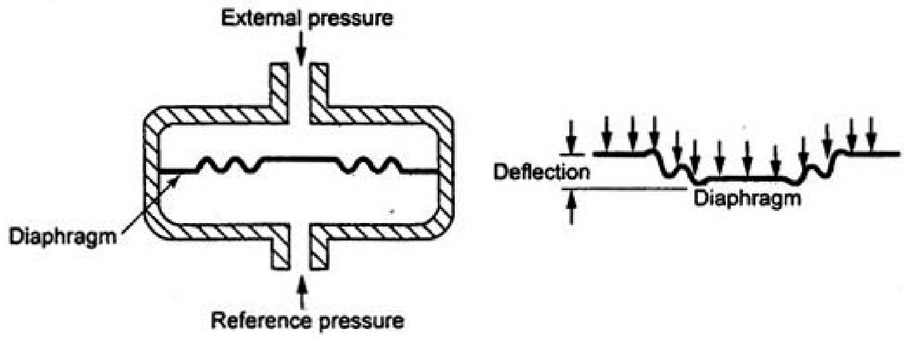


Figure 12 *Typical diaphragm pressure gauge*

The diaphragm deflection can be measured in any number of ways. For example, **i** can be detected via a mechanically-coupled indicating needle, an attached strain gauge [refer Figure 13 (a)], a linear variable differential transformer (LVDT) [refer Figure 13 (b)], or with many other displacement/velocity sensors. Once known, the deflection can be converted to a pressure loading using plate theory.

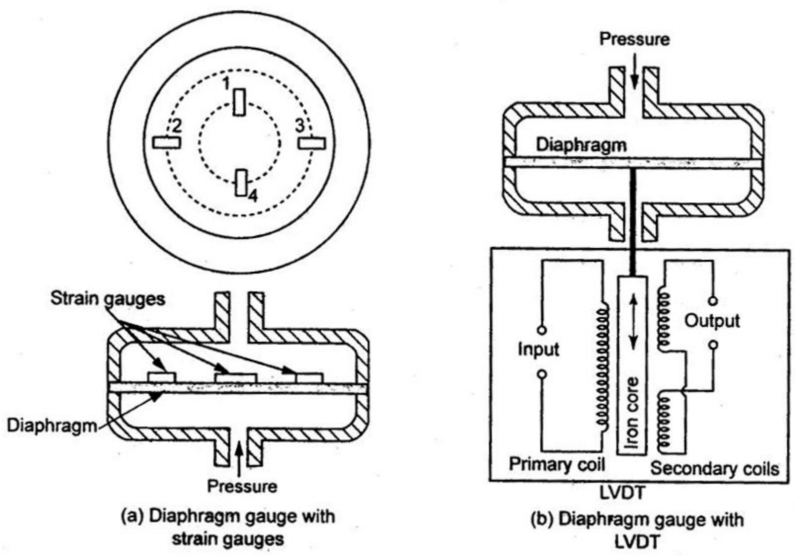


Figure 13

Strain gauge arrangement consists of four strain gauges with, two measuring the strain in a circumferential direction while the remaining two measure strains in a radial direction. The four strain gauges are connected to form the arms of a Wheatstone bridge. The sensitivity of pressure gauges using LVDTs is good and, therefore, stiff primary sensors with very little movement can be used to reduce environmental effects. Frequency response is also good.

Advantages:

* Much faster frequency response than U tubes.
* Accuracy up to ±0.5% of full scale.
* Good linearity when the deflection is no larger than the order of the diaphragm thickness.

Disadvantages:

* More expensive than other pressure sensors.

8. Capsule pressure sensor

In order to improve the sensitivity, two corrugated diaphragms are combined by arranging these in back-to-back and sealed together at the periphery to obtain shell like shape as shown in Figure 1.65. These are called as *capsules.* One of the diaphragms is provided with a central reinforced port to allow the pressure to be measured, and the other is linked to a mechanical element. The difference in pressure between inner and outer surface of the capsule produces displacement. These capsules can also be attached with the LVDT as described in the diaphragm pressure gauge.

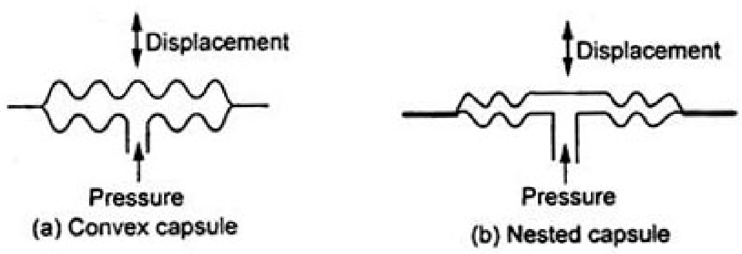


Figure 14 Capsule pressure sensor

9. Bellows pressure sensor

The bellows is a one-piece, collapsible, seamless metallic unit that has deep folds formed from very thin-walled tubing. It looks like a stake of capsules. It is more sensitive than the diaphragm and capsule pressure sensors. The diameter of the bellows ranges from 1.2 to 30 cm and may have as many as 24 folds. System pressure is applied to the internal volume of the bellows. As the inlet pressure varies, the bellows will expand or contract. The moving end of the bellows is connected to a mechanical linkage assembly. The deflection can be measured in any number of ways. For example, it can be detected via a mechanically-coupled indicating needle [refer Figure 15 (a)], a linear variable differential transformer (LVDT) as described in the diaphragm pressure gauge [refer Figure 13 (b)], a potentiometer [refer Figure 14 (b)], or with many other displacement sensors. As the bellows and linkage assembly moves, either an electrical signal is generated or a direct pressure indication is provided. Figure 14 shows a bellows pressure sensing element along with the potentiometer.

The potentiometric bellows pressure sensor provides a simple method for obtaining an electronic output from a mechanical pressure gauge. The device consists of a precision potentiometer, whose wiper arm is mechanically linked bellows or Bourdon-element. The movement of the wiper arm across the potentiometer converts the mechanically detected sensor deflection into a resistance measurement, using a Wheatstone bridge circuit.

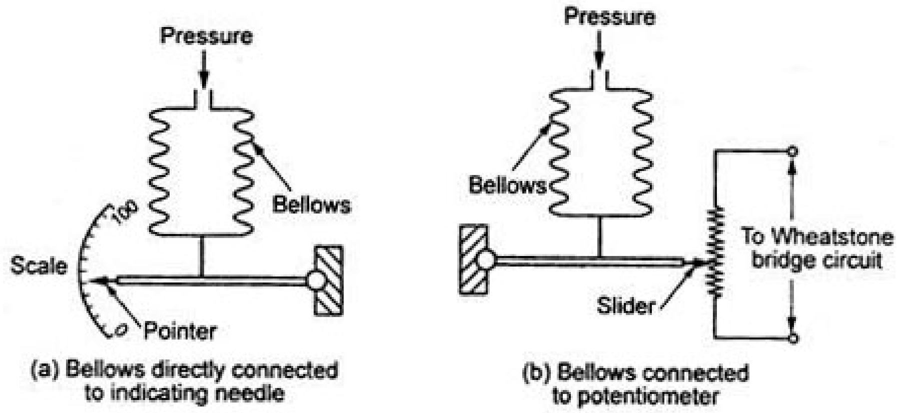


Figure 15 Bellower pressure sensor

The flexibility of a metallic bellows is similar in character to that of a helical, coiled compression spring. Up to the elastic limit of the bellows, the relation between increments of load and deflection is linear. In practice, the bellows must always be opposed by a spring, and the deflection characteristics will be the resulting force of the spring and bellows.

10. B*ourdon tube pressure sensor*

The bourdon tube pressure instrument is one of the oldest pressure sensing instruments in use today. It is widely used in applications where inexpensive static pressure measurements are needed, the bourdon tube consists of a thin-walled C-shaped tube that is flattened diametrically on opposite sides to produce a cross-sectional area elliptical in shape, having two long flat sides and two short round sides. The tube is bent lengthwise into an arc of a circle of 270 to 300 degrees. Bourdon tube is open to external pressure input on one end and is coupled mechanically to an indicating needle on the other end as shown schematically in Figure 16. Pressure applied to the inside of the tube causes distention of the flat sections and tends to restore its original round cross-section. This change in cross-section causes the tube to straighten slightly. Since the tube is permanently fastened at one end, the tip of the tube traces a curve that is the result of the change in angular position with respect to the center. Within limits, the movement of the tip of the tube can then be used to position a pointer or to develop an equivalent electrical signal to indicate the value of the applied internal pressure.

The deflection of the Bourdon tube can be measured in any number of ways. For example, it can be detected via a mechanically-coupled indicating needle [refer Figure 16], a linear variable differential transformer (LVDT) as described in the diaphragm pressure gauge [refer Figure 12 (b)], a potentiometer [refer Figure15(b)], or with many other displacement sensors.

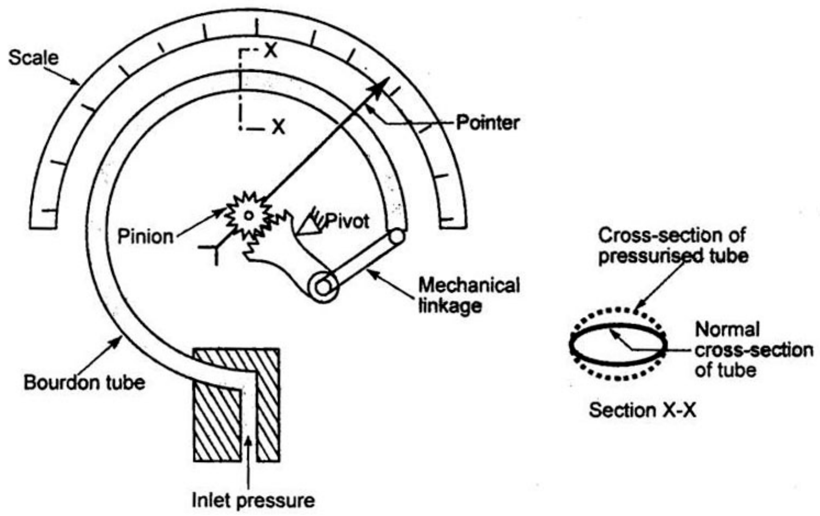


***Figure 16** C-shaped Bourdon tube pressure sensor*

To increase their sensitivity, Bourdon tube elements can be extended into spirals or helical coils [Figures 17 (a) and (b)]. This increases their effective angular length and therefore increases the movement at their tip, which in turn increases the resolution of the transducer.

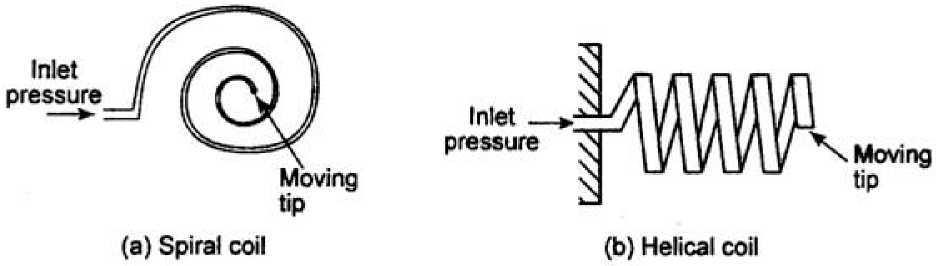


Figure 17 Spiral and helical coil Bourdon tubes

Advantages:

- Portable
- Convenient to use
- No leveling required

Disadvantages:

- Limited to static or quasi-static measurements.
- Accuracy may be insufficient for many applications. A mercury barometer can be used to calibrate and check Bourdon Tubes.

11. *Piezoelectric sensors*

A piezoelectric sensor is a device that uses the piezoelectric effect to measure pressure, acceleration, strain or force. When pressure, force or acceleration is applied to piezoelectric materials such as quartz crystal, PZT ceramic, tourmaline, gallium phosphate, and lithium sulfate, an electrical charge is developed across the crystal that is proportional to the force applied (Figure 18 (a)). When pressure is applied to a crystal, it is elastically deformed. This deformation results in a flow of electric charge (which lasts for a period of a few seconds). The" resulting electric signal can be measured as an indication of the pressure which was applied to the crystal.

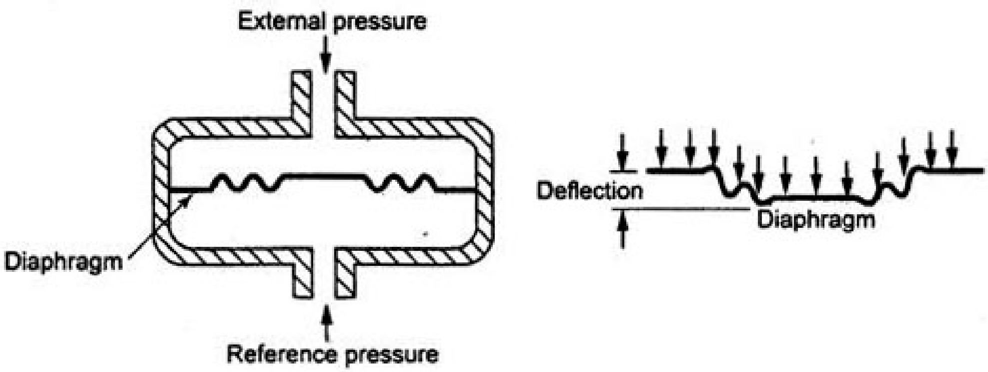


Figure 18 *Piezoelectric sensors*

The net electrical charge *(q)* produced in the crystal is proportional to the deformation of the crystal *(x)* due to the applied pressure and the stiffness of the material *(k).* Since the deformation is proportional to the applied pressure or force *(P),* the net electric charge is given by the equation:

$$q = k \times x = S \times P$$

where S is the charge sensitivity.

The piezoelectric sensors are attached with the diaphragm pressure sensing element to measure the pressure as shown in Figure 18 (b).

The output electrical signal of the piezoelectric sensor is related to the t mechanical force or pressure as if it had passed through the *equivalent circuit* as shown in Figure 19. The model of the equivalent circuit includes the following components:

C represents the capacitance of the sensor surface itself;

R is the insulation leakage resistance of the transducer; and

q is the charge generator

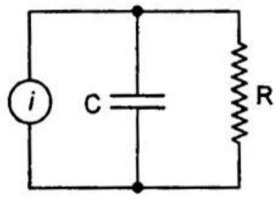


Figure 19 *Equivalent circuit of Piezoelectric sensor*

If the sensor is connected to a load resistance, this also acts in parallel with the insulation resistance.

The fundamental difference between these piezoelectric sensors and static-force devices such as strain gauges is that the electric signal generated by the piezoelectric sensors decays rapidly. This characteristic makes these sensors unsuitable for the measurement of static forces or pressures but useful for dynamic measurements.

Piezoelectric pressure sensors do not require an external excitation source and are very rugged. These sensors, however, do require charge amplification circuitry and very susceptible to shock and vibration.

The desirable features of piezoelectric sensors include their rugged construction, small size, high speed, and self-generated signal. On the other hand, they are sensitive to temperature variations and require special cabling and amplification.

12. Tactile sensors

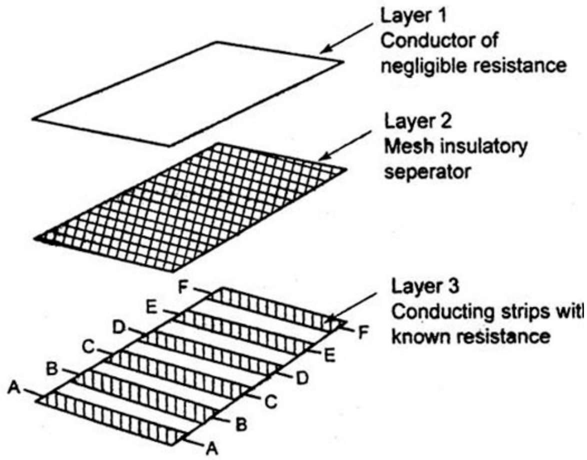


Figure 20 Tactile pressure sensor

Tactile pressure sensors are used to detect the pressure distribution between a sensor and a target. They are often used on the robot grippers or flat tactile arrays to identify whether the finger is in touch with the target object or not. These sensors are also used in touch screen display of laptops, ATM machines, mobiles etc. Most tactile pressure sensors use resistive-based technologies where the sensor acts as a variable resistor in an electrical circuit. A small deflection of the diaphragm causes implanted resistors to exhibit a change in resistance value. The sensor converts this change in resistance into a voltage that is interpreted as a continuous and linear pressure reading. When tactile pressure sensors are unloaded, their resistance is very high. When force is applied, their resistance decreases. Pressure sensitive film is used to create a direct, visual image of the pressure distribution. Active pressure sensor arrays consist of multiple sensing elements packaged in a single sensor.

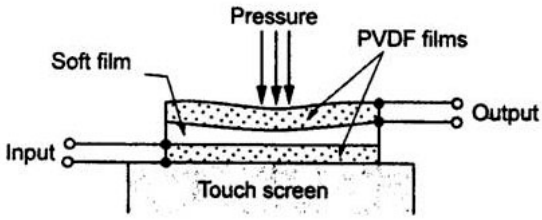


Figure 21 PVDF Tactile sensor

There are many different forms of tactile sensors. One form of tactile pressure sensor includes upper and lower conductive layers separated by an intermediate insulating layer which is formed as a separating mesh (Refer to Figure 20). The upper conductive layer is of negligible resistance. The lower conductive layer is formed of a plurality of conductive strips (A-F) separated by insulating strips. Each conductive strip (A-F) has a known resistance. An electrical signal is applied to the conductive strips (A-F) in turn and the electrical path between the upper and lower conductive layers then determined. The electrical resistance of the conductive path establishes the location of the pressure point at which bridging occurs and from this it is possible to establish the location and size of the pressure area.

Figure 22 shows another form of tactile sensor. It uses piezoelectric material of polyvinylidene fluoride (PVDF) film. Two layers of PVDF films are used and they are separated by a soft film which transmits vibrations. When the alternating voltage is supplied in the lower PVDF film it results in mechanical oscillations of the film. The intermediate film transmits these vibrations to the upper PVDF film. Due to the piezoelectric effect the vibrations formed cause an-alternating voltage to be produced across the upper film. So, pressure is applied to the upper PVDF film and its vibrations affect the output voltage.

SWITCHES

Proximity Switches

There are a number of forms of switch which can be activated by the presence of an object in order to give a proximity sensor with an output which is cither ON or OFF.

The micro switch is a small electrical switch which requires physical contact and a small operating force to close the contact and a small operating force to close the contacts. For example, in the case of determining the presence of an item on a conveyor belt, this might be actuated by the weight of the item on the belt depressing the belt and hence a spring-loaded platform then closing the switch. Figure 3.1 shows examples of ways such switches can be actuated.

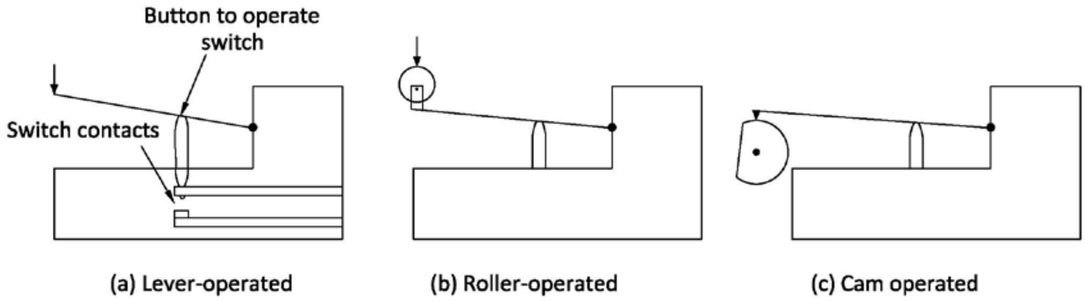


Figure 22. *Various types of switches*

Figure 23 shows the basic form of a reed switch. It consists of two magnetic switch contacts scaled in a gas tube. When a magnet is brought close to the switch, the magnetic reeds are attracted to each other and close the switch contacts. It is a non-contact proximity switch. Such a switch is very widely used for checking the closure of doors. It is also used with such devices as tachometers which involve the rotation of a toothed wheel past the reed switch. If one of the teeth has magnet attached to it, then every time it passes the switch it will momentarily close the contacts and hence produce a current/voltage pulse in the associated electrical circuit.

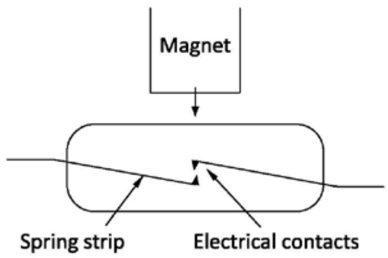


Figure 23 *Reed switch*

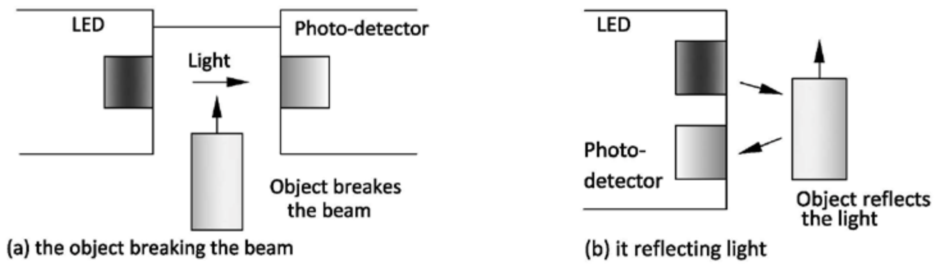


Figure 24. *Using photoelectric sensor to detect objects*

Photosensitive devices can be used detect the presence of an opaque object by it breaking a beam of light, or infrared radiation, falling on such a device or by detecting the light reflected back by the object (figure 24)

Inputting Data by Switches

Mechanical switches consist of one or more pairs of contacts which can be mechanically closed or opened and in doing so make or break electrical circuits. Thus 0 or 1 signals can be transmitted by the act of opening or closing a switch.

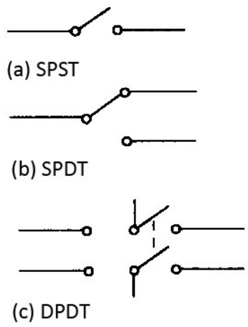


Figure 25. *Switches*

Mechanical switches are specified in terms of their number of poles and throws. Poles are the number f separate circuits that can be completed by the same switching action and throws are the number of individual contacts for each pole. Figure 25(a) shows a single pole-single throw (SPST) switch. Figure 25(b) a single pole-double throw (DPDT) switch and figure 25(c) double pole-double throw switch.

De-bouncing

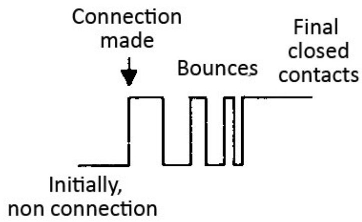


***Figure** 26 Switch bounce on closing a switch*

A problem that occurs with mechanical switches is switch bounce. When a mechanical switch is switched to close the contacts, we have one contact being moved towards the other. It hits the other and, because the contacting elements are elastic, bounces. It may bounce a number of times (figure 26) before finally settling to its closed state after, typically, some 20 m. Each of the contacts during this bouncing time can register as a separate contact. Thus, to a microprocessor, it might appear that perhaps two or more separate switch actions have occurred. Similarly, when a mechanical switch is opened, bouncing can occur. To overcome this problem either hardware or software can be used.

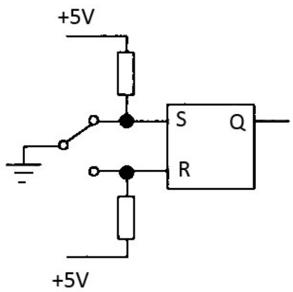


***Figure 27** Debouncing a SPDT switch*

With software, the microprocessor is programmed to detect if the switch is closed and then wait, say 20 m. After checking that bouncing has ceased and the switch is in the same closed position, the next part of the program can take place.

The hardware solution to the bounce problem is based on the use of a flip-flop. Figure 27 shows a circuit for debouncing a SPDT switch which is based on the use of a SR flip-flop. As shown we have S at 0 and R at 1 with an output of 0. When the switch is moved to its lower position, initially S becomes 1 and R becomes 0. This gives an output of 1. Bouncing in changing S from 1 to 0 to 1 to 0, etc. gives no change in the output. Such a flip-flop can be derived from two NOR or two NAND gates. A SPDT switch can be de-bounced by the use of a D flip-flop figure 28 shows the circuit. The output from such a flip-flap only changes when the clock signal changes. Thus by choosing a clock period which is greater than the time for which the bounces last, say 20m, then the bounce signals will be ignored.

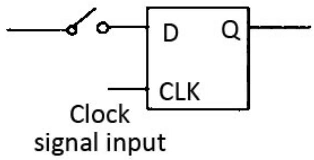


Figure 28. *Debouncing a SPDT switch*

Keypads

A keypad is an array of switches, perhaps the keyboard of a computer or the touch input membrane pad for some device such as a microwave oven. A contact type key of the form generally used with a keyboard is shown in figure 29(a) is built up from two wafer-thin plastic films on which conductive layers have been printed. These layers are separated by a spacer layer. When the switch area of the membrane is pressed, the top contact layer closes with the bottom one to make the connect in and then opens when the pressure released.

While each switch in such arrays could be connected to individually give signals when closed, a more economical method is to connect them in an array in that an individual output is not needed for each key but each key gives a unique row-column combination. Figure 29 shows the connections for a 16 way keypad.

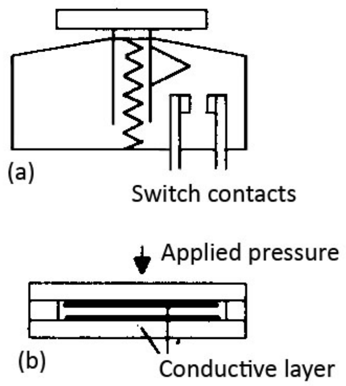


Figure 29 *(a) contact key, (b) membrane key*

SIGNAL CONDITIONING

The output signal from the sensor of a measurement system has generally to be processed in some way to make it suitable for the next stage of the operation. The signal may b, for example, too small and have to be amplified, contain interference which has to be removed, be non-linear and require linearization, be analogue and have to be made digital, be digital and have to be made analogue, be a resistance change and have t be made into a current change, be a voltage change and have to be made into a suitable size current change, etc. All these change can be referred to as signal conditioning. For example, the output from a thermocouple is a small voltage, a few mill volts. A signal conditioning module might then be used to convert this into a suitable size current signal, provide noise rejection, linearization and cold junction compensation (i.e., compensating for the cold junction not being at 0°C)

Signal-conditioning processes

The following are some of the processes that can occur in conditioning a signal:

1. Protection to prevent damage to the next element e.g., a microprocessor, as a result of high current or voltage. Thus there can be series current-limiting resistors, fuses to break if the current is too high, polarity protection and voltage limitation circuits.
2. Getting the signal into the right type of signal. This can mean making the signal into a dc voltage or current. Thus, for example, the resistance change of a strain gage has to be converted into a voltage change. This can be done by the use of a Wheatstone bridge and using the out-of-balance voltage. It can mean making the signal digital or analogue.
3. Getting the level of the signal right. The signal from athermocouple might be just a few millivolts. If the signal is to be fed into an analogue-to-digital converter for inputting to a microprocessor then it needs to be made much larger, volts rather than millivolts. Operational amplifiers are widely used for amplification.
4. Eliminating or reducing noise. For example, filters might be used to eliminate mains noise from a signal.
5. Signal manipulation, e.g., making it a linear function of some variable. The signals from some sensors, e.g., a flow meter, are non-linear and thus a signal conditioner might be used so that the signal fed on to the next element is linear.

The Operational Amplifier

The basis of many signal conditioning modules is the operational amplifier. The operational amplifier is a high gain dc amplifier, the gain typically being of the order of 100,000 or more, than is supplied as an integrated circuit on a silicon chip. It has two inputs, known as the inverting input (-) and the non-inverting input (+). The output depends on the connections made to these inputs. There are other inputs to the operational amplifier, namely a negative voltage supply, a positive voltage supply and two inputs termed offset null, these being to enable

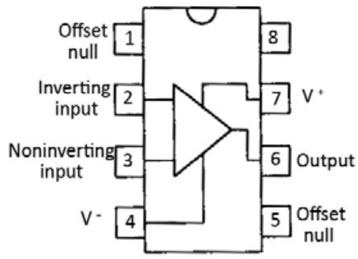


Figure 30 *Pin connections for a 741 operational amplifier*

corrections to be made for the non-idle behavior of the amplifier. Figure 30 shows the pin connections for a 741 type operational amplifier

The following indicates the types of circuits that might be used with operational amplifiers when used as signal conditioners. For more details the reader is referred to more specialist texts.

Inverting amplifier

Figure 31 shows the connections made to the amplifier when used as an inverting amplifier. The input is taken to the inverting input through a resistor $R_1$with the non-inverting input being connected to ground. A feedback path is provided from the output via the resistor R_1 to the inverting input. The operational amplifier has a voltage gain of about 1,00,000 and the change in output voltage is typically limited to about ± 10 V. The input voltage must then be between +0.0001 V and – 0.0001 V. This is virtually zero and so point x is at virtually earth potential. For this reason it is called a virtual earth. The potential difference across R_1 is ($V_{in} – V_x$). Hence, for an ideal operational amplifier with an infinite gain, and hence V_x = 0, the input potential V_{in} can be considered to be across R_1. Thus

$$V_{in} = I_1 R_1$$

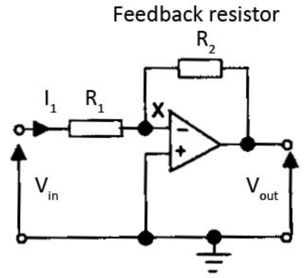


Figure 31 *Inverting amplifier*

The operational amplifier has a very high impedance between its input terminals; for a 741 about 2 MΩ. Thus virtually no current flows through X into it. For an ideal operational amplifier the input impedance is taken to be infinite and so there is no current flow through X. Hence the current I_1 through R_1 must be the current through R_2. The potential difference across R_2 is (V_x – V_{out}) and thus, since V_x is zerofor the ideal amplifier, the potential difference across R_2 is – V_{out}. Thus

$$-V_{out} = I_1 R_2$$

Dividing these two equations:

$$\text{Voltage gain of circuit} = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

Thus the voltage gain of the circuit is determined solely by the relative values of R_2 and R_1. The negative sign indicates that the output is inverted, i.e., 180° out of phase, with respect to the input. To illustrate the above, consider an inverting operational amplifier circuit which has a resistance of 1 MΩ. What is voltage gain of the circuit?

$$\text{Voltage gain of circuit} = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} = -\frac{10}{1} = -10$$

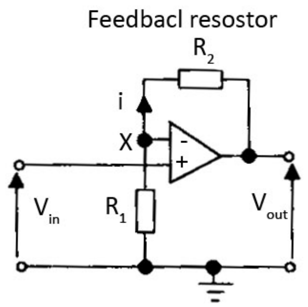


Figure 32 *Non inverting*

Non-inverting amplifier

Figure 32 shows the operational amplifier connected as a non inverting amplifier. The output can be considered to be taken from across a potential divider circuit consisting of R_1 in series with R_2. The voltage $V_x$is then the fraction R_1l(R_1+ R_2) of the output voltage.

$$V_x = \frac{R_1}{R_1 + R_2} V_{out}$$

Since there is virtually no current through the operational amplifier between the two inputs three can be virtually no potential different between them. Thus, with the ideal operational amplifier, we must have V_x = V_{in}. Hence

$$\text{Voltage gain of circuit} = \frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

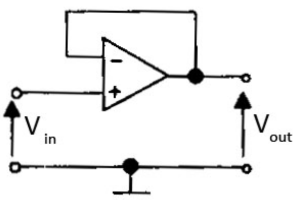


Figure 33 *Voltage follower*

A particular form of this amplifier is when the feedback loop is a short circuit, i.e., $R_2 = 0$. Then the voltage gain is 1. The input to the circuit is into a large resistance, the input resistance typically being 2 MΩ. The output resistance, the resistance between the output terminal and the ground line is, however, much smaller, e.g., 75Ω. Thus the resistance in the circuit that follows is a amplifier is referred to as a voltage follower, figure 33 showing the basic circuit.

Summing amplifier

Figure 34 shows the circuit of a summing amplifier. As with the inverting amplifier, X is a virtual earth. Thus the sum of the currents entering X must equal that leaving it. Hence

$$I = I_A + I_B + I_C$$

But $I_A = V_A/R_A$, $I_B = V_B/R_B$, $I_B = V_B/R_B$. Also we must have the same current I passing through the feedback resistor. The potential difference across R_2 is $(V_x – V_{out})$. Hence, since V_x can be assumed to be zero, it is $-V_{out}$ and so $I – V_{out}/R_2$. Thus

$$\frac{V_{out}}{R_2} = \frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}$$

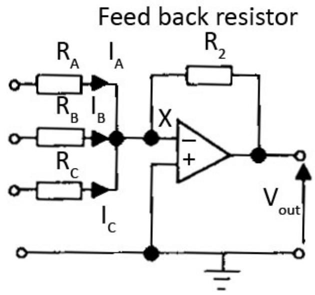


Figure 34 *Summing amplifier*

The output is thus the scaled sum of the inputs, i.e.,

$$V_{out} = -\left(\frac{R_2}{R_A}V_A + \frac{R_2}{R_B}V_B + \frac{R_2}{R_C}V_C\right)$$

If $R_A = R_B = R_C = R_1$ then

$$V_{out} = -\frac{R_1}{R_2}(V_A + V_B + V_C)$$

To illustrate the above, consider the design of a circuit that can be used t produce an output voltage which is the average of the input voltages from three sensors. Assuming that an inverted output is acceptable, a circuit of the form shown in figure 34 can be used. Each of the three inputs must be scaled to 1/3 to give an output of the average. Thus a voltage gain of the circuit of 1/3 for each of the input signals is required. Hence if the feedback resistance is 4 kΩ the resistors in each input arm will be 12 kΩ.

Integrating amplifier

Consider an inverting operational amplifier circuit with the feedback being via a capacitor, as illustrated in figure 35. Current is the rate of movement of change q and since for a capacitor the charge q = Cv, where v is the voltage across it, then the current through the capacitor I = dq/dt = C dv/dt. The potential difference across C is $(V_x – V_{out})$ and since Vx is effectively zero, being the virtual earth, it is $– V_{out}$. Thus the current through the capacitor is $– CdV_{ou}t/dt$. But this is also the current through the input resistance R. Hence

$$\frac{V_{in}}{R} = -C\frac{dV_{out}}{dt}$$

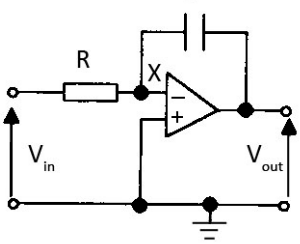


Figure 35 *Integrating amplifier*

Rearranging this gives

$$dV_{out} = -\left(\frac{1}{RC}\right)V_{in}dt$$

Integrating both sides give

$$V_{out}(t_2) = V_{out}(t_1) = -\frac{1}{RC}\int_{t_1}^{t_2} V_{in}\,dt$$

$V_{out}(t_2)$ is the output voltage at time t_2 and $V_{out}(t_1)$ is the output voltage at time t_1. The output is proportional to the integral of the input voltage, i.e., the area under a graph of input voltage with time.

A differentiation circuit can be produced if the capacitor and resistor are interchanged in the circuit for the integrating amplifier.

Differential amplifier

A differential amplifier is one that amplifies the difference between two input voltages. Figure 36 shows the circuit. Since there is virtually no current through the high resistance in the operational amplifier between the two input terminals, there is no potential drop and thus both the inputs X will be at the same potential. The voltage V_2 is across resistors R_1 and R_2 in series. Thus the potential V_x at X is

$$\frac{V_X}{V_2} = \frac{R_2}{R_1 + R_2}$$

The current through the feedback resistance must be equal to that from V_1 through R_1. Hence

$$\frac{V_1 - V_A}{R_1} = \frac{V_X - V_{out}}{R_2}$$

This can be rearranged to give

$$\frac{V_{out}}{R_2} = V_x\left(\frac{1}{R_2} + \frac{1}{R_1}\right) - \frac{V_1}{R_2}$$

Hence substituting for Vx using the earlier equation

$$V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$$

The output is thus a measure of the difference between the two input voltages.

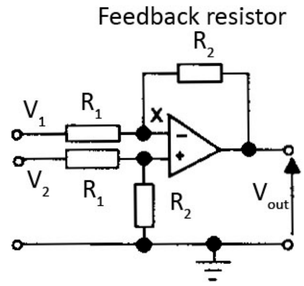


Figure 36 *Differential amplifier*

As an illustration of the use of such a circuit with a sensor, the difference in voltage between the e.m.f.s of the two junctions of the thermocouple is being amplified. The values of R_1 and R_2 can, for example, be chosen to give a circuit with an output of 10 mV for a temperature difference between the thermocouple junctions of 10°C if such a temperature difference produces an e.m.f. difference between the junctions of 530 µV. For circuit we have

$$V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$$

Logarithmic amplifier

Some sensors have outputs which are non-linear. For example the output from a thermocouple is not a perfectly linear function of the temperature difference between its junctions. A signal conditioner might then be used to linearise the output from such a sensor. This can be done using an operational amplifier circuit which is designed to have non-linear relationship between its input and output so that when its input is non-linear the output is linear. This is achieved by a suitable choice of component for the feedback loop.

The logarithmic amplifier shown in figure 3.37 is an example of such a signal conditioner. The feedback loop contains a diode (or a transistor with a grounded base). The diode has non-linear characteristic. It might be represented by V = C in I, where C is a constant. Then, since the current through the feedback loop is the same as the current through the input resistance and the potential difference across the diode is $-V_{out}$, we have

$$V_{out} = -C \text{ in } (V_{in}/R) = K \text{ In } V_{in}$$

Were K is some constant. However, if the input V_{in} is provided by a sensor with an input t, where $V_{in} = e^{at}$, with A and a being constants, then

$$V_{out} = K \text{ In } V_{in} = K \text{ In } (A\, e^{at}) = K \text{ In } A + Kat$$

The result is linear relationship between V_{out} and t.

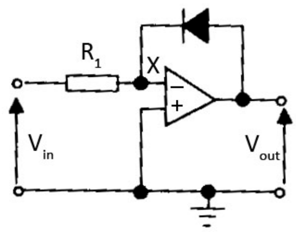


Figure 37 *Logarithmic amplifier*

Digital Signals

The output from most sensors tends to be in analogue from. Where a microprocessor is used as part of the measurement or control system the analogue output from the sensor has to be converted into a digital from before it can be used as an input to the microprocessor. Likewise, most actuators operate with analogue inputs and so the digital output from a microprocessor has to be converted into an analogue form before it can be used as an input by the actuator.

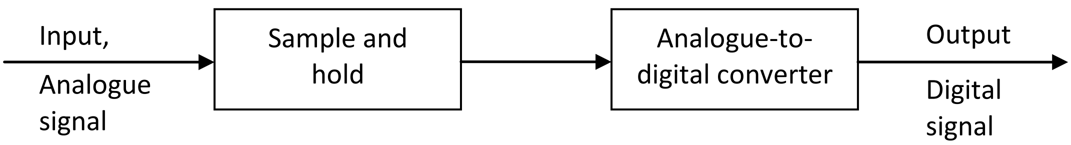
The binary system is based on just the two symbols or states 0 and 1. These are termed binary digits or bits. When number is represented by this system, the digit position in the number indicates the weight attached to each digit, the weight increasing by a factor of 2 as we proceed from right to left:

----	2^3	2^2	2^1	2^0
	bit 3	bit 2	bit 1	bit 0

for example, the decimal number 15 is $2^0 + 2^1 + 2^2 + 2^3$ = 1111 in the binary system. In binary number the bit 0 is termed the least significant bit (LSB) and the highest bit the most significant bit (MSB). The combination of bits to represent a number is termed a word. Thus 1111 is a four – bit word. The term byte is used for a group of 8 bits.

Analogue-to-digital conversion

Analogue-to-digital conversion involves converting analogue signals into binary words. The basic elements of analogue-to-digital conversion.



The procedure used is that a clock supplies regular time signal pulses to the analogue-to-digital converter (ADC) and every time it receives a pulse it sample the analogue signal. Figure 38 illustrates this analogue-to-digital conversion by showing the types of signals involved at the various stages. Figure 38a shows the analogue signal and figure 38b the clock signal which supplies the time signals at which the sampling occurs. The result of the sampling is a series of narrow pulses (figure 3.38d). A simple and hold unit is necessary becomes the analogue-to-digital converter requires a finite amount of time, termed the conversation time, to convert the analogue signal into a digital one.

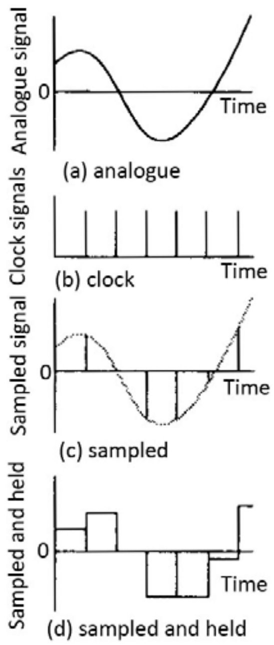


Figure 38 *signals*

The relationship between the sampled and held input and the output for an analogue-to-digital converter is illustrated by the graph shown in figure 39 for a digital output which is restricted to three bits. With three bits there are 2^3 = 8 possible output levels. Thus, since the output of the ADC to represent the analogue input can be only one of these eight possible levels is termed the quantization interval. Thus for the ADC given in figure 39, the quantization interval is 1 V. Because of the step-like nature of the relationship, the digital output is not always proportional to the analogue input and thus there will be error, this being termed the quantization error. When the input is center over the interval the quantization error is zero, the maximum error being equal to one-half of the interval or 4 ½ bit.

The word length possible determines the resolution of the element, i.e., the smallest change in input which will result in a change in the digital output. The smallest change in digital output is one bit in the least significant bit position in the word, i.e., the far right bit. Thus with a word length of n bits the full-scale analogue input V_{FS} is divided into 2^8 pieces and so the minimum change in input that can be detected, i.e., the resolution, is $V_{FS}/2^8$.

Digital-to-analogue conversion

The input to a digital-to-analogue converter (DAC) is a binary word; the output is an analogue signal that represents the weighted sum of the non-zero bits represented by the word. Thus, for example, an input of 0010 must give an analogue output which is twice that given by an input of 0001.

Consider the situation where a microprocessor gives an output of an 8-bit word. This is fed through an 8 bit digital-to-analogue converter to a control valve. The control valve requires 6.0 V to be fully open. If the fully open state is indicated by 11111111 what will be the output to the valve for a change of 1 bit?

The full-scale output voltage of 6.0 V will be divided into 2^8 intervals. A change of 1 bit is thus a change in the output voltage of $6.0/2^8 = 0.023$ V

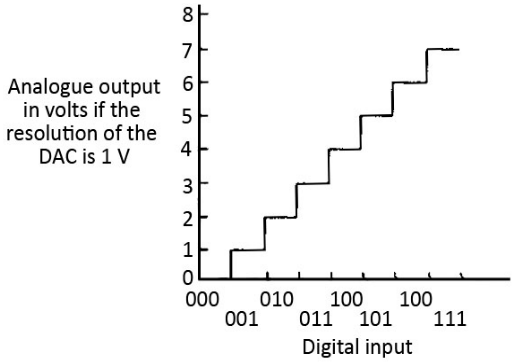


Figure 39 *Input-output for a DAC*

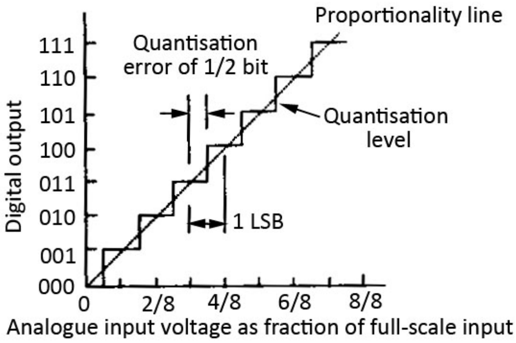


Figure 40 *Input-output for an ADC*

Figure 41 illustrates this for an input to a DAC with a resolution of 1 V for unsigned binary words. Each additional bit increase the output voltage by 1 V.

Digital-to analogue converters

A simple form of a digital-to-analogue converter uses a summing amplifier to from the weighted sum of the all the non-zero bits in the input word (figure 41). The reference voltage is connected to the resistors by means of electronic switches which respond to binary 1. The values of the input resistances depend on which bit in the word a switch is responding to, the valve of the resistor for successive bit from the LSB being halved. Hence the sum of the voltages is a weighted sum of the digits in the word. Such a system is referred to as a weighted-resistor network.

A problem with the weighted-resistor network is that accurate resistances have to be used for each of the resistors and it is difficult to obtain such resistors over the wide range needed. As a result this form of DAC tends be limited to 4 bit conversions.

Another, more commonly used, version uses a R-2R ladder network (figure 42). This overcomes the problem of obtaining accurate resistances over a wide range of valves, only two valves being required. The output voltage is generated by switching sections of the ladder to either the reference voltage or 0 V according to whether there is a 1 or 0 in the digital input.

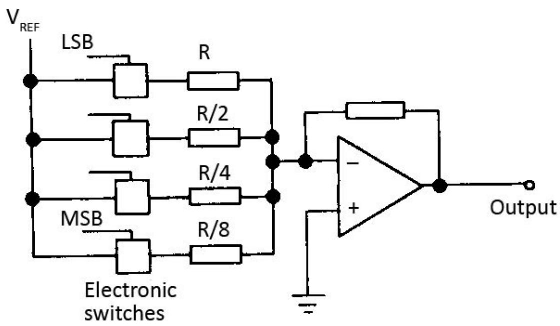


Figure 41 *Weighted-resistor DAC*

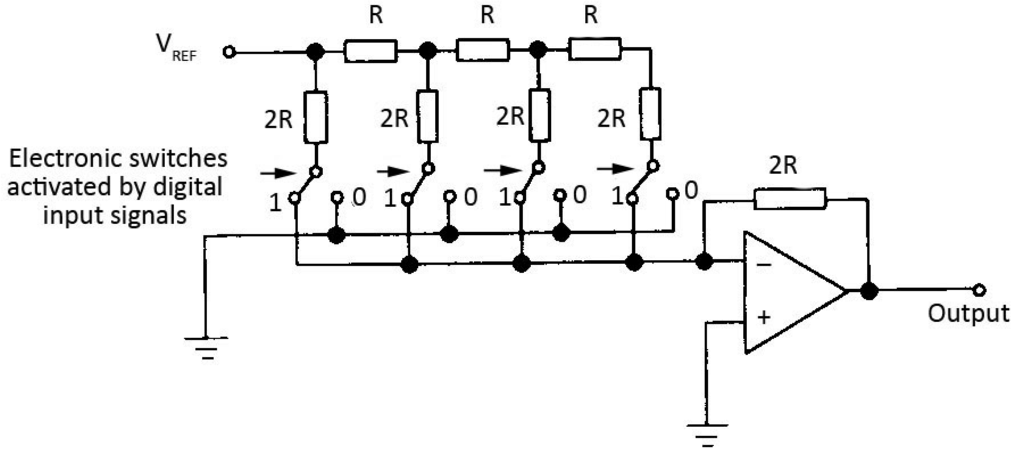


Figure 42 *R-2R Ladder DAC*

Figure 43 shows details of the GEC Plessey ZN55D 8 bit latched input digital-to-analogue converter using a R-2R ladder network. After the conversion is complete, the 8 bit result is placed in an internal latch until the next conversion is complete. Date is held is the latch when ENABLE is high, the latch being side to be transparent when ENABLE is low. A latch is just a device to retain the output until a new one replaces it. When a DAC without a latch would be connected via a peripheral interface adapter (PIA), shows how the ZN558D might be used with a microprocessor when the output is required to be a voltage which varies between zero and the reference voltage, this being termed unipolar operation. With B_{refin} = 2.5 V, the output range is +5 V when R1 = 8 Ω and R2 = 8 kΩ and the range is +10V when R1 = 16 kΩ and R2 = 5.33 kΩ.

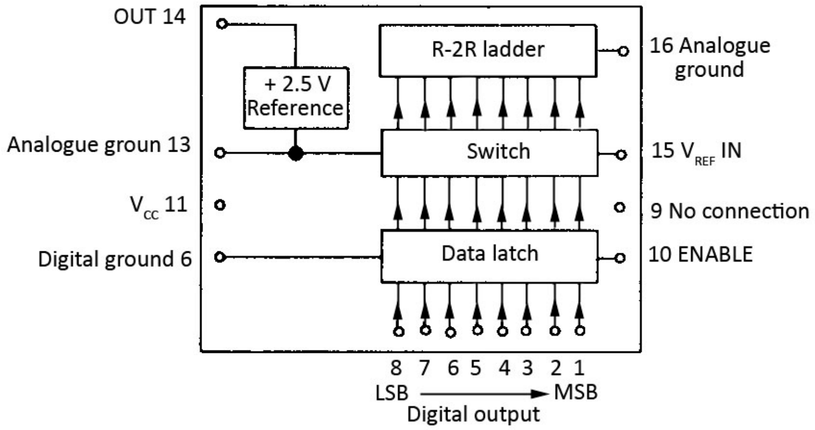


Figure 43 *ZN558D ADC*

Analogue-To-Digital Converters

The input to an analogue-to-digital converter is an analogue signal and the output is binary word that represents the level of the input signal. There are a number of analogue-to-digital converter, the most common being successive approximations, ramp, dual ramp and flash.

Successive approximations ADC

Successive approximations are probably the most commonly used method figure 44 illustrates the subsystems involved. A voltage is generated by a clock emitting a regular sequence of pluses which are counted, in a binary manner, and the resulting binary word converted into an analogue voltage by a digital-to-analogue converter. This voltage rises in steps and is compared with the analogue input voltage from the sensor. When the clock generated voltage passes the input analogue voltage the pulses from the clock are stopped from being counted by a gate being closed.

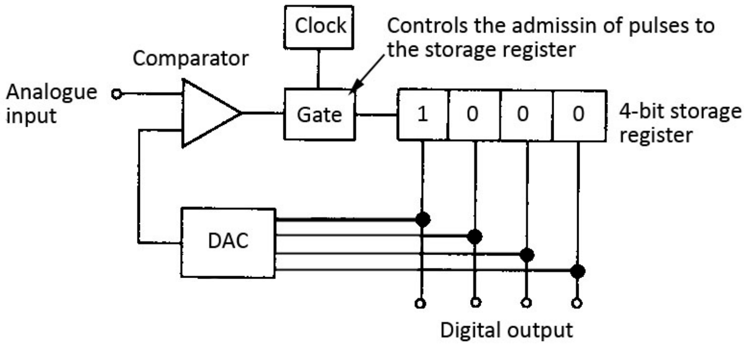


***Figure 44** Successive approximations ADC*

The output from the counter at that time is then a digital representation of the voltage. While the comparison could be accomplished by starting the count at 1, the less significant bit that is less than the analogue value, these adding successive lesser bits for which the total does not exceed the analogue value. For example, we might start the comparison with 1000. If this is too large we try 0100. If this is too small we then try 0110. If this is too large we try 0101. With an n-bit word it only takes n steps to make the comparison. Thus if the clock has a frequency f, the time between pulses is 1/f. Hence the time take to generate the word, i.e., the conversion time, is n/f.

Ramp ADC

The ramp form of analogue-to-digital converter involves as analogue voltage which is increased at a constant rate, a so-called ramp voltage and applied to a comparator where it is compared with the analogue voltage from the sensor. The time taken for the ramp voltage to increase to the value of the sensor voltage will depend on the size of the sampled analogue voltage. When the ramp voltage starts, a gate is opened which starts a binary counter counting the regular pulses from a clock. When the two voltages are equal, the gate closes and the word indicated by the counter is the digital representation of the sampled analogue voltage. Figure 45 indicates the subsystems involved in the ramp from of analogue-to-digital converter.

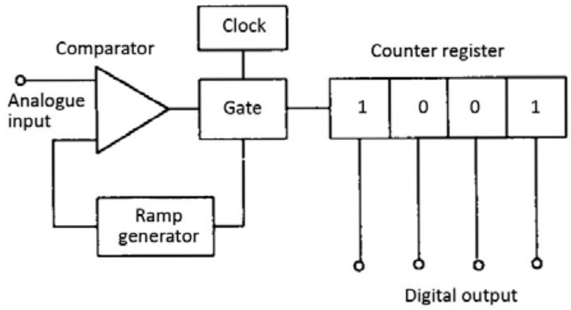


***Figure 45** Ramp ADC*

Dual ramp ADC

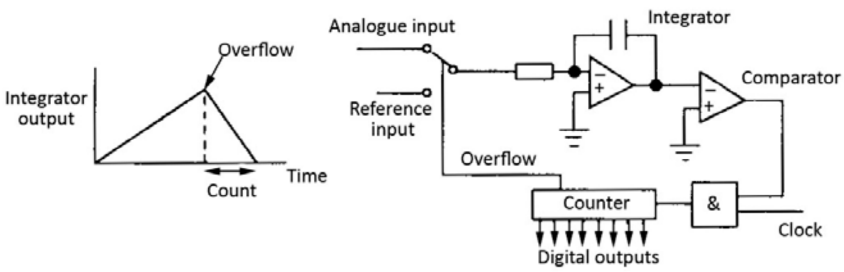


***Figure 46** Dual ramp ADC*

The dual ramp converter is more common than the single ramp. Figure 46 shows the basic circuit. The analogue voltage is applied to an integrator which drives a comparator. The output from the comparator goes high as soon as the integrator output is more than a few millivolts. When the comparator output is high, an AND gate passes pulses to a binary counter. The counter counts pulses until it overflows. The counter then resets to zero, sends a signal to a switch which disconnects the unknown voltage and connects a reference voltage, and starts counting again. The polarity of the reference voltage is opposite to that of the input voltage. The integrator voltage then decrease at a rate proportional to the reference voltage. When the integrator output low and so switching the clock off. The count is then a measure of the analogue input voltage. Duel ramp analogue-to-digital action averages out random negative and positive contributions over the sampling period. They are however, very slow.

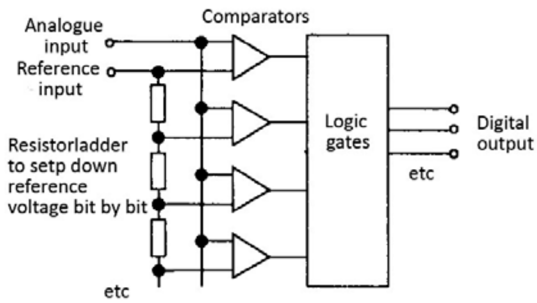


Figure 47 *Flash ADC*

Flash ADC

The flash analogue-to-digital converter is very fast. For an n-bit converter, 28 – 1 separate voltage comparators are used in parallel, with each having the analogue input voltages as one input. A reference voltage is applied to a ladder of resistors so that the voltage applied as the other input to each comparator is one bit larger in size then the voltage applied to the previous comparator in the ladder. Thus when the analogue voltage is applied to the ADC, all those comparators for which the analogue voltage is greater than the reference voltage of a comparator will give a high output and those for which it is less will be low. The resulting outputs are fed in parallel to a logic gate system which translates them into a digital word.

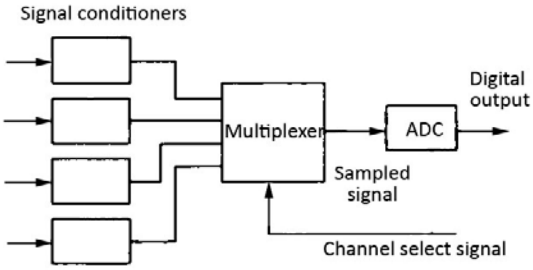


Figure 48 *Multiplexer*

Multiplexers

A multiplexer is a circuit that is able to have inputs of data from a number of sources and then, by selecting an input channel, give an output from just one of them. In applications where there is a need for measurements to be made at a number of different locations, rather than use a separate ADC and microprocessor for each measurement, a multiplexer can be used to select each input in turn and switch it through a signal ADC and microprocessor. The multiplexer is essentially an electronic switching device which enables each of the inputs to be sampled in turn.

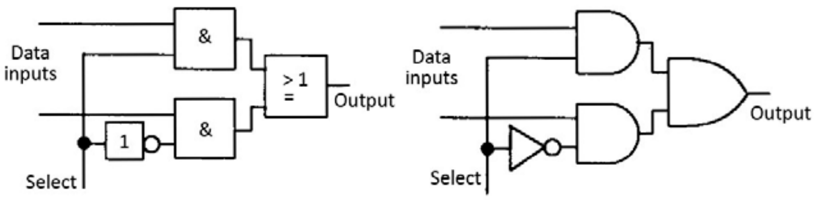


Figure 49 *Two channel multiplexer*

As an illustration of the types of analogue multiplexers available, the DG508ACI has eight input channels with each channel having a 3-bit binary address for selection purposes. The transition time between taking samples is 0.6 µs.

Figure 44 shows the basic principle of a multiplexer which can be used to select digital data inputs; for simplicity only a two input channel system is shown. The logic level applied to the select-input determines which AND gate is enabled so that its data input passes through the OR gate to the output. A number of forms of multiplexers are available in integrated packages. The 151 types enable on line from eight to be selected, the 153 type one line from four inputs which are supplied as data on two lines each, the 157 types one line from two inputs which are supplied as data on four lines.

Unit IV

ELECTRICAL ACTUATION SYSTEM

Basic Principles

Figure 1 shows the basic principle of the dc motor, a loop of wire which is free to rotate in the field of a permanent magnet. When a current is passed through the coil, the resulting forces acting on its sides at right angle to the field cause forces to act on those sides to give rotation. However, for the rotation to continue, when the coil passes through the vertical position the current direction through the coil has to be reversed.

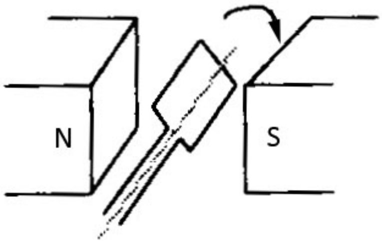


Figure 1 *DC motor basics*

In the conventional dc motor coils of wire are mounted in steps on a cylinder of magnate material called to armature. The armature is mounted on bearings and is free to rotate. It is mounted in the magnetic field produced by field poles. These may be for small motors, Permanent magnets or electromagnets with their magnetism produced by a current through the field cols. Figure 2 show the basic principle of a four-pole dc motor with the magnetic field produced by current carrying coils. The ends of each armature coil are connected to adjacent segments of a segmented ring called the commutator with electrical contacts made to the segments through carbon contacts called brushes. As the armature rotates, the commutator reverses the current in each coil as it move between the field poles. This is necessary if the force acting on the coil is to remain acting in the same direction and so the rotations continue. The direction of rotation of the DC motor can be reversed by reversing either the armature current or the field current.

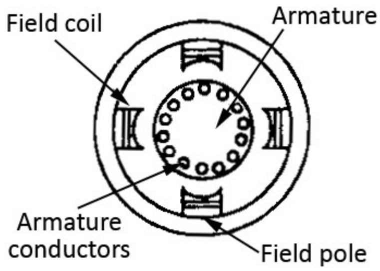


Figure 4.2 *DC motor*

Permanent Magnet Dc Motor

Consider a permanent magnet dc motor the permanent magnet giving a constant value of flux density. For an armature conductor of length L and carrying a current i the force resulting from a magnetic flux density B at right angles to the conductor is BiL. (Figure 3) With N such conductors the force is NBiL. The forces result in torque T and the coil axis of Fb, with b being the breadth of the coil. Thus:

Torque T = NbbLi = k_tj

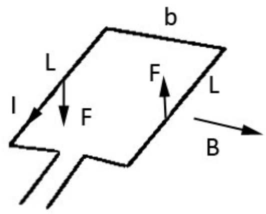


Figure 3 *Armature*

Where,k_t is the torque constant. Since an armature coil is rotating in a magnetic field, electromagnetic induction will occur and a back e.m.f will be induced. The back e.m.f. v_b is proportional to the rate at which the flux linked by the coil changes and hence, for a constant magnetic field, is proportional to the angular velocity ω of the rotation. Thus:

Back e.m.f. $v_b = k_v\omega$

Where kv is the back e.m.f. constant

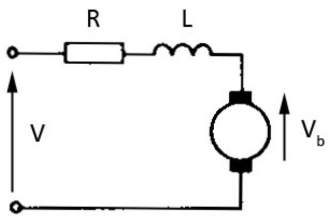


Figure 4 Equivalent circuit

We can consider a dc motor to have the equivalent circuit shown in figure 4. i.e. the armature coil being represented by a resistor R is series with an inductance L in series with a source of back e.m.g. If we neglect the inductance of the armature coil then the voltage providing the current i through the resistance is the applied voltage V minus the back e.m.f. i.e., V – v_b. Hence:

$$i = \frac{V - v_b}{R} = \frac{V - k_v(t)}{R}$$

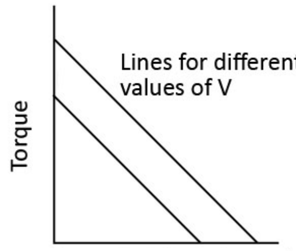


Figure 5 *Torque-speed characteristic*

The torque T is thus:

$$T = kgi = \frac{k_t}{R}\,[V - k_v\omega]$$

Graphs of the torque against the rotational speed ω are a series of straight lines for different voltage values (figure 5). The starting torque, i.e. the torque when ω = 0, is thus proportional to the applied voltage, the non-load speed is proportional to the applied voltage and the torque decreases with increasing speed.

As an example, a small permanent magnet motor S6M41 by PMI motors has k_t = 3.01 N cm/A. k_v = 3.15 V/krpm, a terminal resistance of 1.207Ω and an armature resistance of 0.940 Ω.

DC motors with field coils

DC motors with field coils are classified as series, shunt compound and separately excited according to how the filed windings and armature windings are connected (figure 7.26)

1. **Series wound motor**

With the series wound motor the armature and fields coils are in series. Such a motor exerts the highest starting torque and has the greater no-load speed. With light loads there is a danger that a series would motor might run at too high a speed. Reversing the polanty of the supply to the coils has no effect on the direction of rotation of the motor, it will continue rotating in the same direction since both the field and armature currents have been reversed.

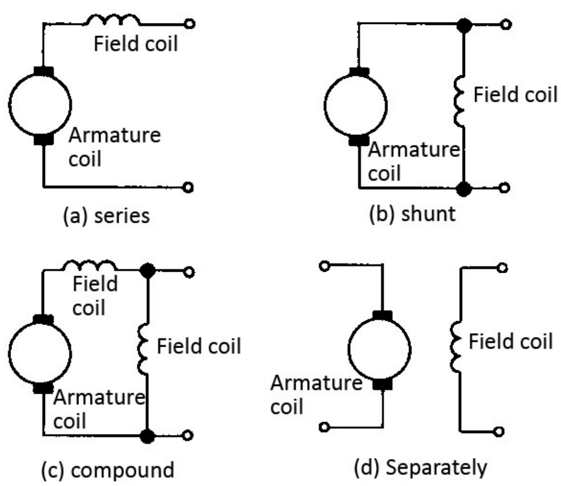


Figure 6 *DC motors wound*

2. **Shunt wound motor**

With the shunt would motor the armature and field coils are in parallel. It provides the lowest starting torque a much lower no-load speed and has good speed regulation. Because of this almost constant speed regardless of load, shunt would motors are very widely used. To reverse the direction of rotation, either the armature or field supplied must be reversed. For this reason, the separately excited windings are preferable for such a situation.

3. **Compound motor**

The compound motor has two field windings, one in series with the armature and one in parallel. Compound wound motors aim to get the best features of the series and shunt would motors, namely a high starting torque and good speed regulation.

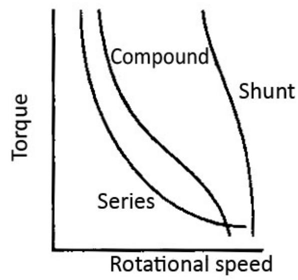


Figure 7 *Torque-speed characteristics*

4. **Separately excited motor**

The separately excited motor has separate control of the armature and field currents and can be considered to be a special case of the shunt wound motor.

Figure 7 indicates the torque-speed characteristics of the above motors. The speed of such dc motor can be changed by either changing the armature current or the field current. Generally it is the armature current that is varied. The choice of motor will depend on its application. For example, with a robot manipulator, the robot wrist might use a series would motor because the speed decreases as the load increases. A shunt would motor would be used where a constant speed was required, regardless of the load.

Brushless permanent magnet dc motors

A problem with dc motors is the they require a commutator and brushes in order to periodically reverse the current through each armature coil. The brushes make sliding contacts with the commutator and as a consequence sparks jump between to two and they suffer wear. Brushes thus have to be periodically changed and the commutator resurfaced. To avoid such problems brushless motors have been designed.

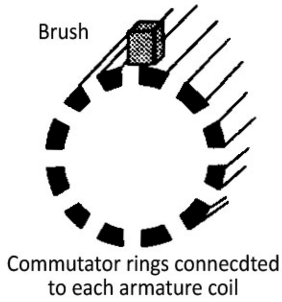


Figure 8 *Commutator*

Essentially they consist of a sequence of stator coils and permanent magnet rotor. A current carrying conductor in a magnetic field experiences a force likewise, as a consequence of Newton's third law of motion, the magnet will also experience an opposite and equal force. With the conventional dc motor the magnet is fixed and the current-carrying conductors mad to move. With the brushless permanent magnet dc motor the reverse is the case, the current carrying conductors are fixed and the magnet moves. The rotor is a ferrite or ceramic permanent magnet figure 9 shows the basic form of such a motor. The current to the stator oils is electronically switched by transistors in sequence round the coils, the switching being controlled by the position of the rotor so that there are always forces acting on the magnet causing it to rotate in the same direction. Hall sensor are generally used to sense to position of the rotor and initiate the switching by the transistors, the sensors being positioned around the stator.

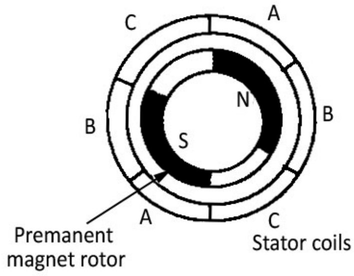


Figure 9 *Brushless permanent magnet dc motor*

Figure 8 shows the transistor switching circuits that might be used with the motor shown in figure 7. To switch the coils in sequence we need to supply signals to switch the transistors on in the right sequence. This is provided by the outputs from the three sensors operating though a decoder circuit to give the appropriate base currents. Thus when the rotor is in the vertical position, i.e., 0°, there is an output from sensor c but non from 'a' and 'b'. This is used to switch on transistors A+ and B- for the rotor in the 60° position there are signals from the sensors b and c and transistors A+ and C- are switched on. The entire circuit for controlling such a motor is available on the single integrated circuit.

AC motors

Alternating current motors can be classified in the groups single phase and polyphase with each group being further subdivided into induction and synchronous motors. Single-phase motor lend to be used for low power requirements while polyphase motors are used for higher powers. Induction motors tend to be cheaper than synchronous motors and are thus very widely used.

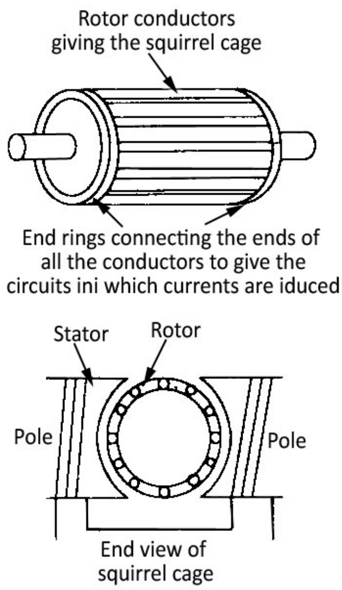


Figure 10 *Single-phase induction motor*

The single-phase squirrel-cage condition motor consists of a squirrel-cage motor, this being copper or aluminium bars that fit into slots in end rings to form complete electrical circuits (figure 10). There are no external electrical connections to the rotor. The basic motor consists of this rotor with a stator having a set of windings. When an alternating current passes through the stator windings and alternating magnetic field is proceed. As a result of electromagnetic inductione.m.f.s are induced in the conductors of the rotor and currents flow in the rotor. Initially when the rotor is rotor is stationary, the forces on the current carrying conduction of the rotor in the magnetic field of the stator are such as to result is in not torque. The motor is not self-starting. A number of methods are used to make the motor self-starting and give this initial impetus to start it, one is to use an auxiliary staring winding to give the rotor an initial push. The rotor rotates at a speed determined by the frequency of the alternating current applied to the stator. For a constant frequency supply to a two-pole single-phase motor the magnetic field will alternate at this frequency. This speed of rotation of the magnetic field is termed the synchronous sped. The rotor will never quit match this frequency of rotation, typically differing from it by about 1 to 3%. This difference is termed slip. Thus for a 50 Hz supply the speed of rotation of the rotor will be almost 50 revolutions per second.

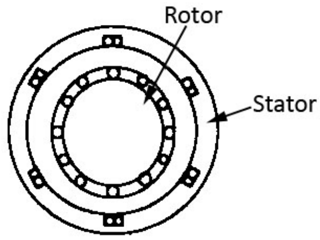


Figure 11 *Three-phase induction motor*

The three-phase induction motor (figure 12) is similar to the single-phase induction motor but has a stator with three windings located 120° apart, each winding being connected to one of the three lines of the supply. Because the three phases reach their maximum currents at different times, the magnetic field can be considered to rotate round the stator poles, completing one rotation in one fuel cycle of the current. The rotation of the field is much smoother than with the signal-phase motor. The three-phase motor has a great advantage over the single-phase motor of being self-starting. The direction of rotation is reversed by interchanging any two of the line connections, this changing the direction of rotation of the magnetic field.

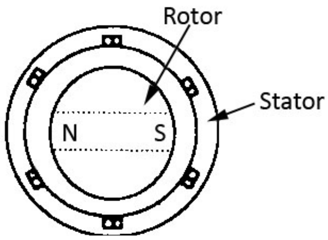


Figure 12 *Three-phase synchronous motor*

Synchronous motors have stators similar to those described above for induction motors to a rotor which is a permanent magnet (figure 12). The magnetic field produced by the stator rotates and so the magnet rotors with it. With one pair of poles per phase of the supply, the magnetic field rotates through 360° in one cycle of the supply and so the frequency of rotation with this arrangement is the same as the frequency of the supply. Synchronous motors are used when a precise speed is required. They are not self-starting and some system has to be employed to start them.

AC motors have the great advantage over dc motors of being cheaper, more rugged, reliable and maintenance free. However sped control is generally more compels than with dc motors and as a consequence a speed-controlled dc drive generally works out cheaper than a speed controlled ad drive through the price difference is steadily dropping as a result of technological developments and the reduction in price of solid-state devices. Speed control of ac motors is based around the precision of a variable frequency supply, since the speed of such motors is determined by the frequency is constant. Thus to maintain a constant torque at the different speeds when the frequency is varied the voltage applied to the stator less also to be varied, with one method, the ac is first rectified to dc by a converter and then inverted back to ac again but at a frequency that can be selected (figure 13) another method that is often used for operating slow-speed motors is the cycloconverter. The converts ac at one frequency directly to ac at another frequency with the intermediate dc conversion.

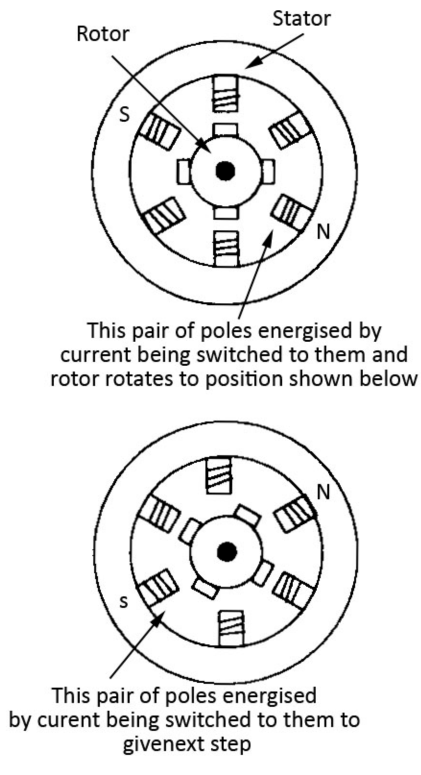


Figure *13 Variable reluctance stepper motor*

Stepper motors

The stepper motor is a device that produces rotation through equal angles, the so-called steps for each digital pulse supplied to its input. Thus, for example, if with such a motor 1 pulse produces a rotation of 6° then 60 pulses will produce a rotation through 360°. There are a number of forms of stepper motor.

Variable reluctance stepper

Figure 13 shows the basic form of the variable reluctance stepper motor. With this form the rotor is made of soft steel and is cylindrical with four poles, i.e. fewer poles than on the stator. When an opposite pair of windings has current switched to them a magnetic field is produced with line of force which pass from the stator poles though the nearest set of poles on the rotor. Since lines of force can be considered to be rather like elastic thread and always trying to shorten themselves, the rotor will move until the rotor and stator poles line up. This is termed the position of minimum reluctance. This form of stepper generally gives step angles of 7.5° or 15°.

Permanent magnet stepper

Figure 14 shows the basic form of the permanent magnet motor. The motor shown has a stator with four poles. Each pole is wound with a field winding, the coils on opposite pairs of poles being in series. Current is supplied from a dc source to the windings through switches. The rotor is a permanent magnet and thus when a pair of stator poles has a current switched to it, the rotor will move to line up with it. Thus for the currents giving the siltation shown in the figure the rotor moves to the 45° position. If the current is then switched so that the polarities are reversed, the rotor will move a further 45° in order to line up again. Thus by switching the currents through the coils the rotor rotates in 45° steps. With this of motor, step angles are commonly 1.8°, 7.5°, 15°, 30°, 34° or 90°.

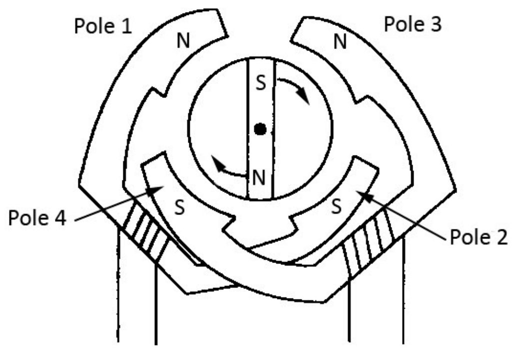


Figure 14 *Permanent magnate stepper motor*

Hybrid stepper

Hybrid stepper motors combine the features of both the variable reluctance and permanent magnet motors, having a permanent magnet encased in iron caps which are cut to have teeth (figure 15). The rotor sets itself in the minimum reluctance position in response to a pair of stator coils being energized. Typical step angles are 0.9° and 1.8°. Such stepper motors are extensively used in high accuracy positioning applications, e.g in computer hard disc drive.

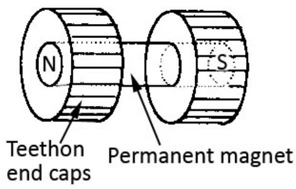


Figure 15 *Hybrid motor rotor*

STEPPER MOTOR SPECIFICATIONS

The following are some of the terms commonly used in specifying stepper motors

1. Phase

This term refers to the number of independent windings on the stator, e.g. a four-phase motor. The current required per phase and its resistance and inductance will be specified so that the controller switching output is specified Two-phase motors, e.g., figure 16 tend to be used in light-duty applications, three-phase motors tend to be variable reluctance steppers e.g., figure 7.40, and four-phase motors tend to be used for higher power applications

2. Step angle

This is the angle through which the rotor rotates for one switching change for the stator coils.

3. Holding torque

This is the maximum torque that can be applied to powered motor without moving it from its rest position and causing spindle rotation.

4. Pull-in-torque

This the maximum torque against which a motor will start for a given pulse rate and reach synchronism without losing a step.

5. Pull-out torque

This the maximum torque that can be applied to a motor, running at a given stepping rate, without losing synchronism

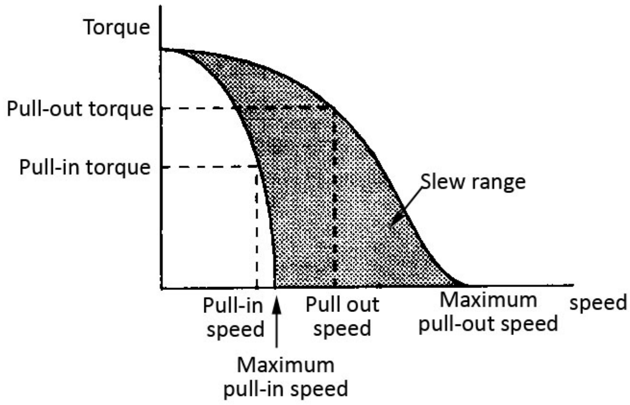


Figure 16 *Stepper motor characteristics*

6. Pull-in-rate

This is the maximum switching rate at which a loaded motor can start without losing a step.

7. Pull-out rate

This is the switching rate at when a loaded motor will remain in synchronism as the switching rate is reduced.

8. Slew range

This is the range of switching rates between pull-in and pull-out within which the motor runs in synchronism but cannot start up or reverse.

Pneumatic and Hydraulic Systems

Most of mechatronics systems work based oil motion or action by means of sort. This motion or actuation is caused either by a torque or force from which displacement and acceleration can be obtained. To obtain this force or acceleration, abators are mainly used. Actuator is a device which provides enough force needed start the mechatronics systems. At the same, power should be supplied to the actuator to activate it. The power supplied to actuators might be anyone of the flowing forms such as compressed air, pressurized fluid, electric power and mechanical power. If compressed air is supplied to the system, it is called as pneumatic system. But if pressurized fluid is supplied to flow the system, it is called is hydraulic system. Electrical actuator system is with electrical power and mechanical system is with mechanical power. Among these, hydraulic and pneumatic systems are quiet effective and efficient way of getting motion and action which can be used in mechatronics systems.

1. Introduction to Pneumatic Systems

Fluid power technology over the years has continuous development involved the applications of pneumatic and hydraulic systems in several areas, like

(i) Manufacturing,

(ii) Process industries,

(iii) Transportation systems, and

(iv) Utilities.

The fluid power systems are also used:

(i) to carry out mechanical works such as linear, swivel and rotary motion for plant equipment and machinery, clamping, shifting and positioning, packaging, feeding sorting, stamping, drilling, turning, milling and sawing etc.

(ii) to obtain control application such as controlling of plant, process and equipment to take necessary corrective action,

(iii) to measure process parameter to act on necessary output.

Advantages of fluid systems:

(i) Air is available every where in enormous quantities.

(ii) Transporting air and hydraulic fluid will be easy through pipe line over large distances,

(iii) Storing of compressed air will be easy in a reservoir and removed as required. Hydraulic oil can be stored in accumulators,

(iv) Compressed air is too sensitive with temperature fluctuations but hydraulic fluids are insensitive,

(v) Compressed air offers minimal risk of explosion or fire,

(vi) The construction of components in fluid system is simple in construction and cheap.

Disadvantages:

(i) Good preparation of compressed air and hydraulic fluid required to remove the dirt and condensate,

(ii) Speed fluctuation will always be with pneumatic systems,

(iii) The working pressure of compressed air is limited to 6-7 bar.

(iv) The exhaust air will release with very high noise thereby leading noise pollution,

(v) Producing compressed air and hydraulic fluid are expensive.

2. Pneumatic systems

In pneumatic systems, force is produced by gas. It is mainly by air pressure acting on the surface of a piston or valve.

Compressed air is produced in a compressor and stored in a receiver. From compressor, it is send to valves which control the direction of fluid flow. Also, flow control valves control the amount of power produced by the cylinders. The force acting on the piston is given by the equation:

$$\text{Force} = \text{Pressure} \times \text{Area} = p \times A$$

3. Basic Elements of Pneumatic System

The basic components of a fluid power system are essentially the same, regardless of whether the system uses a hydraulic or a pneumatic medium. A pneumatic system essentially has the following components as shown in Figure 17

(i) Compressor and Motor

(ii) Pressure relief valve and Check valve

(iii) Cooler, filter and water trap

(iv) Air receiver

(v) Directional control valves

(vi) Actuator or pneumatic cylinder

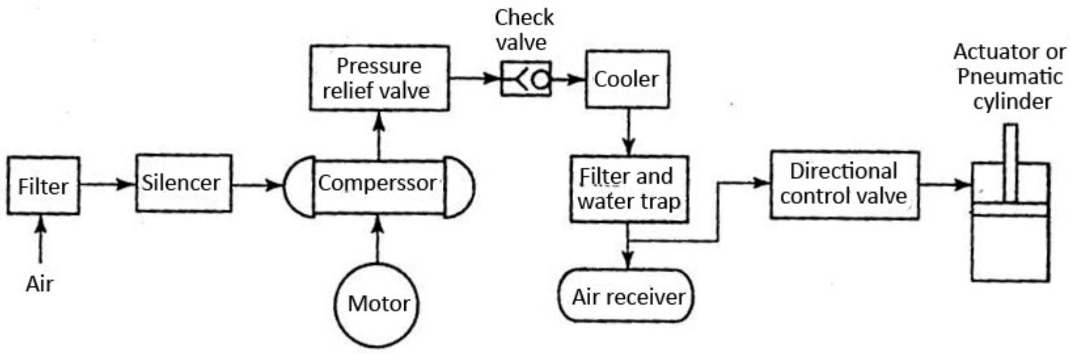


Figure 17 *Basic elements of a pneumatic system*

The fresh atmosphere air is not sent directly to the compressor to use in pneumatic systems. First, it is filtered by filters. Then, filtered atmospheric air is supplied to the compressor through silencer to reduce noise level. Then it is compressed.

Pressure relief valve is used to avoid the damage of compressor due to excess pressure raise in the system. Check valve is a one-way valve that allows pressurized air to enter the pneumatic system, but prevents backflow and loss of pressure into the compressor when it is stopped.

The cooler is used to cool the compressed air which is usually very hot. The filter is used to remove contamination in the compressed air and water trap is used to remove water particles.

The pressurized air is stored in a device called an air receiver, preventing surges in pressure and relieving the duty cycle of the compressor.

Directional control valves are used to control flow of pressurized air from the source to the selected port. These valves can be actuated either manually or electrically.

Actuator or pneumatic cylinder converts energy stored in the compressed air into mechanical motion.

4. Hydraulic Systems

A hydraulic system uses force which is applied at one point and transmitted to-another point using an incompressible fluid, plant equipment and machinery. In this type of machine, high-pressure liquid called *hydraulic fluid* is transmitted throughout the machine to various hydraulic motors and hydraulic cylinders. The fluid is controlled directly or automatically by control valves and distributed through hoses and tubes. The popularity of hydraulic machinery is due to large amount of power that can be transferred through small tubes and flexible hoses, and the high power density and wide array of actuators that can make use of this power.

5. Basic Elements of Hydraulic System

The necessary components of any hydraulic systems are

(i) Hydraulic pump unit

(ii) Control valves

(iii) Reciprocating or rotary unit

➢ ***Hydraulic pump unit***

A pump is a device in which mechanical energy is converted into fluid energy. The pump is connected with the reservoir called *fluid tank.*

➢ ***Control valves***

The flow of pressurized fluid by a pump is controlled by the following valves such as:

(i) Pressure relief valves control the fluid pressure.

(ii) Non return valve controls the back flow of fluid.

(iii) Directional control valves control the direction of fluid.

- ***Hydraulic actuator or cylinder***

The actuator is the element which converts hydraulic power into mechanical power. The pressurized fluid by the pump is supplied to either rotary pump or hydraulic cylinder based on the type of motion needed. Rotary pumps are used to get rotary motion and hydraulic cylinder is used to obtain linear motion.

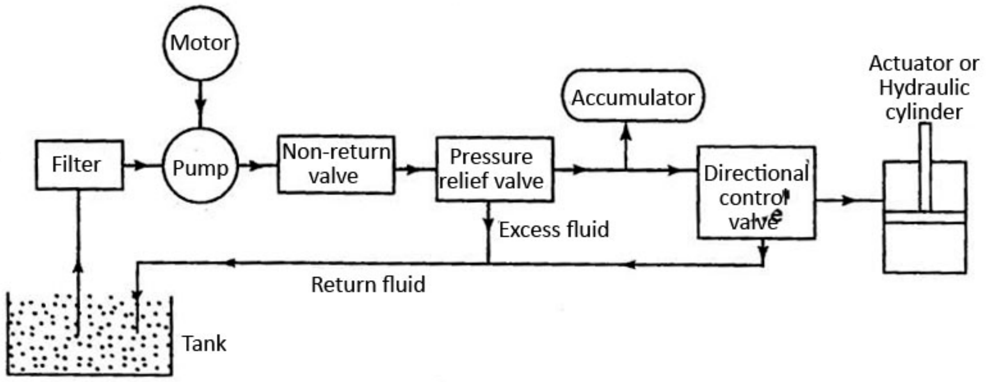


Figure 18 Basic elements of a hydraulic system

Working of hydraulic system (Figure 18)

The fluid stored in the tank is send to the filter to remove dust and foreign particles. After the fluid is filtered, it is sucked by the pump which is driven by a motor. During pumping, the p. assure of the fluid will increase and it is released with high pressure to the accumulator through non return valve. One pressure relief valve is connected at the exit to control the delivery pressure of fluid. The fluid with high pressure is supplied to the hydraulic cylinder through directional control valve.

6. Hydraulic Accumulator

A hydraulic accumulator is an energy storage device. It is a pressure storage reservoir in which a fluid is held under pressure by compressed gas or a spring or a raised weight. The main reasons that an accumulator is used in a hydraulic system are:

(i) the pump does not need to be so large to cope with extremes of demand,

(ii) the supply circuit can respond more quickly to any temporary demand and to smooth pulsations.

Compressed gas accumulators are the most common type which is shown in Figure 19. A compressed gas accumulator consists of a cylinder with two chambers that are separated by a totally enclosed bladder. One chamber contains hydraulic fluid and is connected to the hydraulic line. The other chamber contains an inert gas (mostly nitrogen) under pressure that provides the compressive force on the hydraulic fluid. As the volume of the compressed gas changes the pressure of the gas, and the pressure on the fluid, changes inversely.

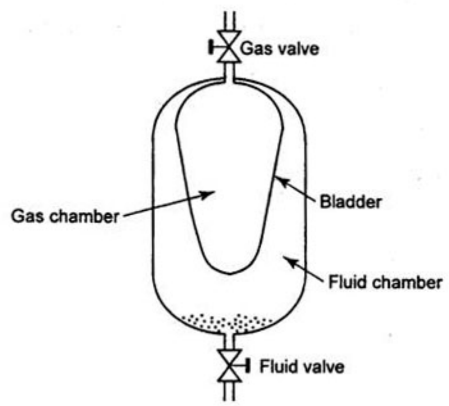


***Figure 19** Hydraulic accumulator*

7. Hydraulic Pumps

In general, a *pump* is a device which converts the mechanical energy supplied into hydraulic energy by lifting water to higher levels. Here, hydraulic energy refers ID potential and kinetic energy of a liquid. Hydraulic pumps are the energy-absorbing machines. Since, it requires mechanical power to drive. Lifting of water to higher levels is carried out by the various actions of pumps such as centrifugal action, reciprocating action etc., The symbol of a pump is shown in Figure 20.

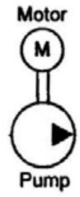


Figure 20 Symbol of a pump

Power required by a pump

The motor power required to derive a pump is given by

$$\text{Power, P} = \frac{Work}{Time} = \frac{Force \times Distance}{Time}$$

$$\text{Power} = \frac{Pressure \times Area \times l}{Time} \quad [\ \therefore \text{force = Pressure} \times \text{area}]$$

$$Q = A \times V = \text{Area} \times \frac{l}{Time}$$

$$\text{Power, P} = p \times Q$$

8. Advantages and Disadvantages of Hydraulic Systems

Advantages of hydraulic systems

1. It is easy to produce and transmit hydraulic power.
2. Hydraulic systems are uniform and smooth.
3. Balancing hydraulic forces is easier.
4. Weight-to-power ratio is less.
5. It is easy to maintain.
6. Systems are cheaper.
7. Hydraulic systems are safe and compact.
8. Frictional resistance is less.

Disadvantages of hydraulic systems

1. Manufacturing cost of the system is quiet high.
2. Hydraulic elements should be kept free from dirt, corrosion, rust etc.,
3. Petroleum based hydraulic systems more prone to fire hazards.
4. Hydraulic power is not readily available as pneumatic

Microcomputer Structure

Computers have three sections: a control processing unit (CPU) to recognize and carry out program instructions, input and output circuitry interfaces to handle communications between the computer and the outside world, and memory to hold the program instructions and data. Digital signal move from one section to another along paths called buses. A bus, in the physical sense, is just a number of conductors along which electrical signals can be carried. It might be tracks on a printed circuit board or wires in a ribbon cable. The data associated with the processing function of the CPU is carried by the data bus, the information for the address of a specify memory location for the accessing of stored data is carried by the address bus and the signal relating to control actions are carried by the control bus. Figure 21 illustrates the general arrangement. In some cause a microprocessor chip constitutes just the CPU while in other cause it might have all the components necessary for the complete computer on one chip. Microprocessor which have memory and various input/output arrangements all on the same chip are called microcontrollers. They are effectively a microcomputer on a single chip.

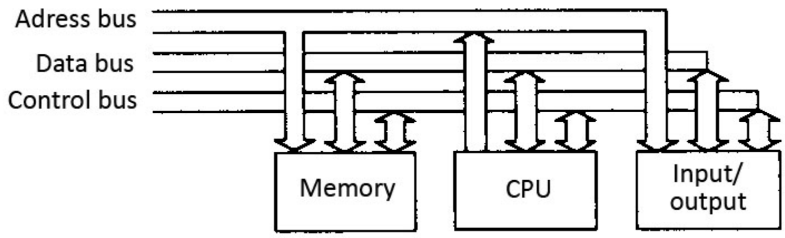


Figure 21 *General form of a computer*

Buses

The data bus is used to transport a word to or from the CPU and the memory or the input/output interfaces. Word length used may be 4, 8, 16, 32 or 64. Each wire in the bus carries a binary signal, i.e., a 0 or a 1. Thus with a for-wire bus we might have the word 1010 being carried, each bit being carried by a separate wire in the bus as

Word	Bus wire
0 (least significant bit)	Firstdata bus wire
1	Second data bus wire
0	Third data bus wire
1(most significant bit)	Fourth data bus wire

The more wires the data bus has the longer the word length that can be used. The range of values which a single item of data can have is restricted to that which can be represented by the word length. Thus with a word of length 4 bits the number of values is 24 = 16. Thus if the data is to represent, say, a temperature, then the range of possible temperatures must be divided into 16 segments if we are to represent that range by a 4-bit word. The earliest microprocessors are still widely used in such devices as toys, washing machines and domestic central heating controllers. They were followed by 8-bit microprocessor, e.g, the Motors 6800, the Intel 8085A and the Zilog Z80. Now, 16-bit, 32-bit and 64-bit microprocessor are available, however, 8-bit microprocessor are still widely used for controllers.

The address bus carriers signals which indicate where data is to be found and so the selectionof certain memory location or input or output ports. When a particular address is selectedby its address being placed on the address bus, only that location is open to the communicationfor the CPU. The CPU is thus able to communicate with just one location at a time. A computer with an 8-bit data bus has typically a 1-bit wide address bus, i.e 16 wirers. This size of address bus enable 2^{16} location to be addressed 2^{16} is 65 536 location sand is usually written as 64K, where K is equal to 1024. The more memory that can be addressed the greater the volume of data that can be stored and the larger and more sophisticated the programs that can be used.

The control bus is the means by which signals are sent to synchronies the separate elements. The system clock signals are carried by the control bus. These signals generate time intervals during which system operations can take place. The CUP sends some control signals to other elements to indicate the type of operation being performed e.g. whether it needs to READ (receive) a signal or WRITE (send) a signal.

CPU

The CPU is the section of the processor which processes the data, fetching instruments from memory, decoding them and executing them. It can be considered to consist of a control unit, arithmetic and logic unit (ALU) and registers (figure 15.2). it is the bit which is the microprocessor.

The control unit determines the timing and sequence of operations. It generates the time signals used to fetch a program instrument form memory and execute it. The Motorola 6800 used a clock with a maximum frequency of 1 MHz, i.e., a clock period of 1 μs, and instructions require between two and twelve clock cycles. Operations involving the microprocessor are reckoned in terms of the number of cycles they take. The arithmetic and logic unit is responsible for performing the actual data manipulation. Internal data that the CPU is currently using in temporarily held in a group of registers while instructions are being execute.

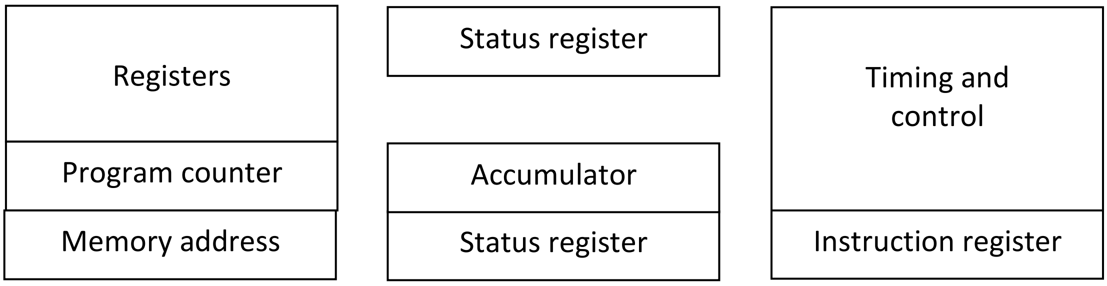


Figure 22 *Factures of a CPU*

There are number of types of register, the number, size and type of registers varying from one microprocessor to another. Te following are common types of registers.

1. Accumulator

The accumulator register (AO is where data for an input to the arithmetic and logic unit is temporarily stored. In order for the CPU to be able to access, i.e. read, instructions or data in the memory it has to supply the address of the required memory word using the address bus. When this has been done, the required instructions or data can be read into the CPU using the data bus. Since only one memory location can be addressed at once, temporary storage has to be used when, for example, numbers are combined. For example, in the addition of two numbers, one of the numbers is fetched from one address and placed in the accumulator register while the CPU fetches the other number form the other memory address. Then the two numbers can be processed by the arithmetic and logic section of the CPU. The result is then transferred back into the accumulator register. The accumulator register is thus a temporary holding register for data to be operated on by the arithmetic and logic unit and also, after the operation, the register for holding the results. It is thus involved in all data transfers associated with the execution of arithmetic and logic operations.

2. Status register, or condition code register or flag register

The contains information concerning the result of the latest process carried out in the arithmetic and logic unit. It contains individual bits with each bit having special significance. The bits are called flags. The status of the latest operation is indicated by each flag with each flag being set or reset to indicate a specific status. For example, they can be used to indicate whether the last operation resulted in a negative result, a zero result, a carry output occurs an overflow occurs or the program is to be allowed to be interrupted to allow as external event to occur.

3. Program counter register (PC) or instruction pointer (IP)

This is the register used to allow the CPU to keep track of its position in a program. This register contains the address of the memory location that contains the next program instruction. As each instruction is executed the program counter register is updated so that it contains the address of the memory location where the next instruction to be executed is stored. The program counter is incremented each time so that the CPU executes instructions sequentially unless an instruction, such as JUMP or BRANCH, changes the program counter out of the sequence.

4. Memory address register (MAR)

The contains the address of data. Thus, for example, in the summing of two number the memory address register is loaded with the address of the first number. The data at the address is then moved to the accumulator. The memory address of the second number is then loaded into the memory address register.

The data at this address is then added to the data in the accumulator. The result is then stored in a memory location addressed by the memory address register.

5. Instruction register (IR)

This stores an instruction after fetching an instruction from the memory, the CPU stores it in the instruction register. It can then be decoded and used to execute an operation.

6. General-purpose registers

These may serve as temporary storage for data or addresses and be used in operations involving transfers between various other registers.

7. Stack pointer register (SP)

The contents of this register depends on the microprocessor concerned. For example, the Motorola 6800 microprocessor (figure 21) has two accumulator registers, a status register, and index register a stack pointer register and a program counter register. The status register has flag bits to show negative, zero, carry, overflow, half-carry and interrupt. The Motorola 6802 is similar but includes a small amount of RAM and a built-in clock generator.

The Intel 8085A microprocessor (figure 24) has sic general-purpose registers, a stack pointer, a program counter and two temporary registers.

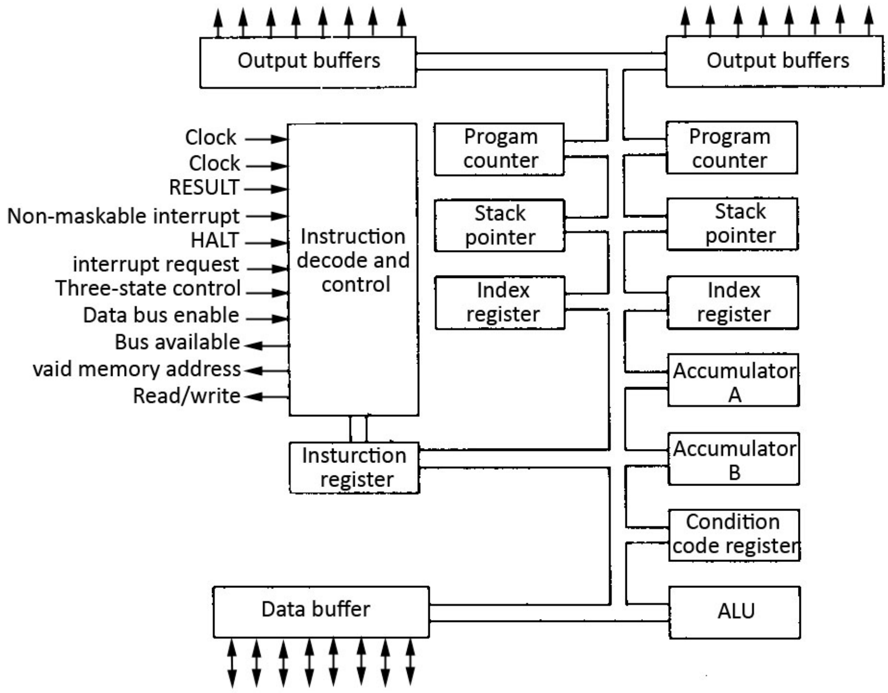
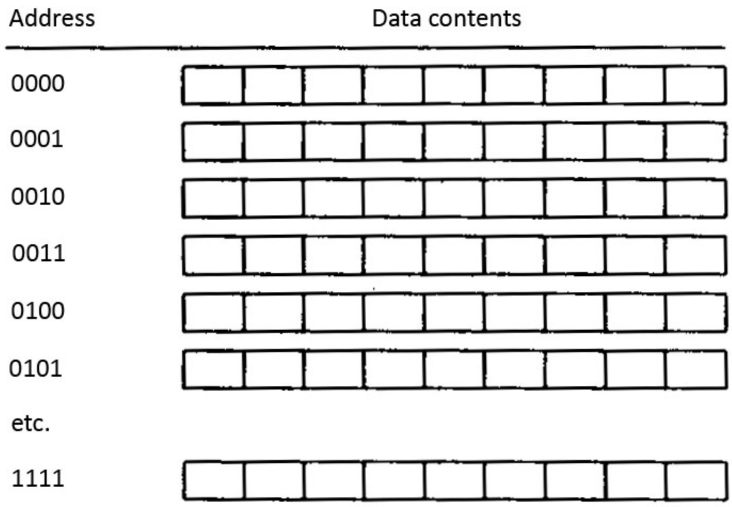


Figure 23 *Motorola 6800 architecture*

MEMORY

The memory unit stores binary data and takes the form of one or more integrated circuits. The data may be program instruction codes or number being operated on. The size of the memory is determined by the number of wires in the address bus. The memory elements is a unit consist essentially of large number of storage cells with each cell capable of storing either a 0 or a 1 bit. The storage cells are grouped in locations with each location capable of storing one word. In order to access the stored word, each location is identified by a unique address. Thus with a 4-bit address bus we can have 16 different address with each perhaps, capable of storing one byte, i.e., a group of eight bits.



The size of a memory unit is specified in terms of the number of storage locations available; 1 k is 2^{10} = 1024 locations and thus a 4 K memory has 4096 locations. There are a number of forms of memory unit:

1. ROM
2. PROM
3. EPROM
4. EEPROM
5. RAM

ROM

For data that is stored permanently a memory device called a read-only memory (ROM) is used. ROMs are programmed with the required contents during the manufacture of the integrated circuit. Not data can that be written into this memory while the memory chip is in the computer. The data can only be read and is used for fixed programs such as computer operating systems and programs for dedicated microprocessor applications. They do not lose their memory when power is removed. Figure 15.5 shows the pin connections of a typical ROM chip which is capable of storing 1 K x 8 bits.

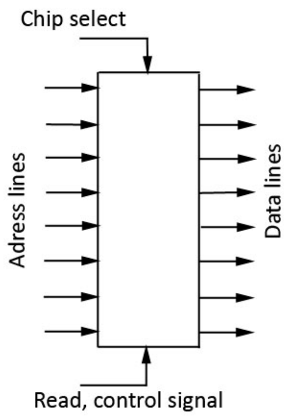


Figure 25 *ROM chip*

PROM

The term programmable ROM (PROM) is used for ROM clips that can be programmed by the user. Initially every memory cell has a fusible link which keeps its memory at 0. The 0 is permanently changed to 1 by sending a current through the fuse to permanently open it. One the fusible like has been opened the data is permanently stored in the memory and cannot be further changed.

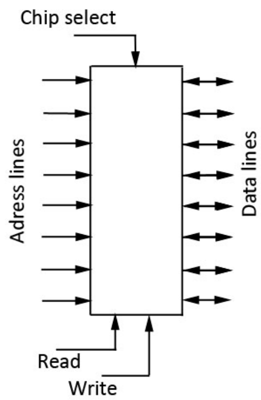


Figure 26 *RAM chip*

EPROM

The term erasable and programmable ROM (EPROM) is used for ROMs that can be programmed and their contents altered. A typical EPROM chip contains a series of small electronic circuits, cell, which can store change. The program is stored by applying voltage to the integrated circuit connection pins and producing a pattern of charged and uncharged cells. The pattern remains permanently in the chip until erased by shining ultraviolet light through a quartz window on the top of the device. This cases all the cells to become discharged. The chip can then by reprogrammed. The Intel 2716 EPROM has 11 address pins and a single chip enable pin which is active when taken low.

EEPROM

Electrically enable PROM (EEPROM) is similar to EPROM. Erasure is by applying a relatively high voltage rather than using ultraviolet light.

RAM

Temporary data, i.e. data currently being operated on, is stored in a read/write memory referred to as a random-access memory (RAM). Such a memory can be read or written to figure 15.6 shows the typical pin connections for a 1 k x 8-bit RAM chip. The Motorola 6810 RAM chip has seven address pins and six chip select pins of the which four are active when low and two active when high and all must be made simultaneously activate enable the RAM.

Microcontroller

For a microprocessor to give a working microcomputer system which can be used for control, additional chips are necessary, e.g., memory device for program and data storage and input/output ports to a flow it to communicate with the external world and receive signals from it. The microcontroller is the integration of a microprocessor with memory and input/output interfaces, and other peripherals such as timers, on a signal chip. Figure 28 shows the general block diagram of a microcontroller.

The general microcontroller has pins for external connections of inputs and outputs, power clock and control signals. The pins for the inputs and outputs are grouped into units called input/output ports. Usually such ports have eight line in order to be able to transfer an 8-bit word of data. Two ports may be used for a 16-bit word, one to transmit the lower 8 bits and the other the upper & bits. The ports can be input only, output only or programmable to be either input or output.

The Motorola 68HC11 and the Intel 8051 are example of 8-bit microcontrollers in that the data path is 8 bits wide. The Motorola 68HC16 is an example of a 16-bit microcontroller and the Motorola 68300 a 32-bit microcontroller.

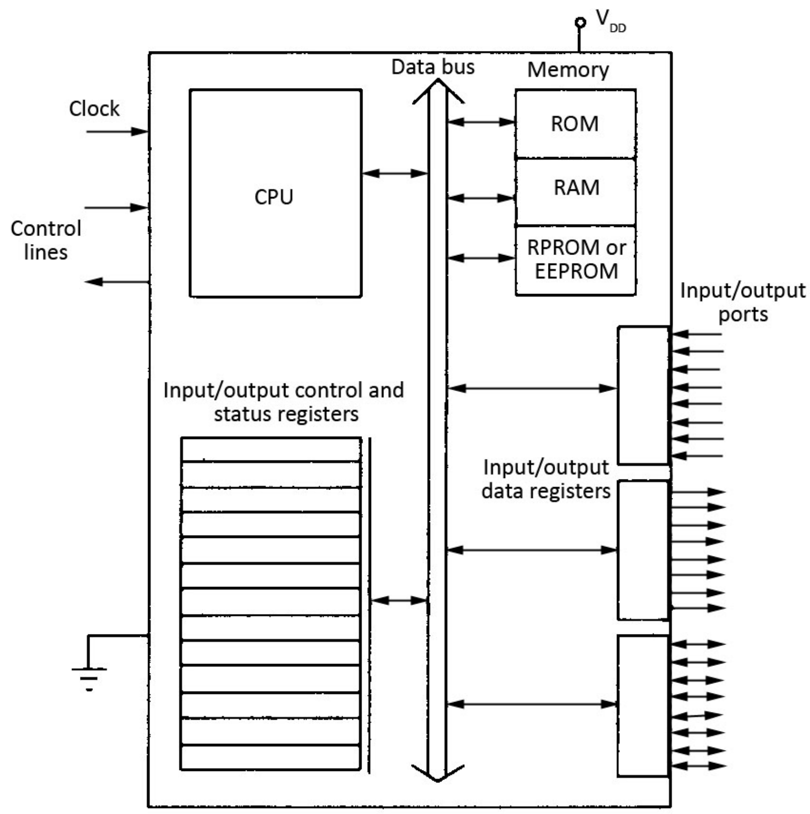


Figure 28 *Block diagram of a microcontroller*

Selecting a microcontroller

In selecting a microcontroller the following factors need to be considered:

1. Name of input/output pins
   How many input/output pins are going to be needed for the tank concerned?
2. Interfaces required

   What interfaces are going to be required? For example, is pulse width modulation required? Many microcontrollers have PWM outputs, e.g., the PICI7C42 has two.

3. Memory requirements
   What size memory is required for the task?
4. The number of interrupts required
   How many events will need interrupt?
5. Processing speed required

   The microprocessor takes time to execute instructions this time being determined by the processor clock.

Unit V

Temperature Measurement System

As a brief indication of how a microcontroller might be used, figure 1 shows the main elements of a temperature measuring system using a MC568HC11. The temperature sensor gives a voltage proportional to the temperature. The output from the temperature sensor is connected to an ADC input line of the microcontroller. The microcontroller is programmed to convert the temperature into a BDC output which can be used to switch on the elements of a two-digital seven-element display. However, because the temperature may be fluctuating it is necessary to use a storage register which can hold data long enough for the display to be read. The storage register, 74HCT273, is an octal D-type flip-flop which is reset on the next positive-going edge of the clock input from the microcontroller.

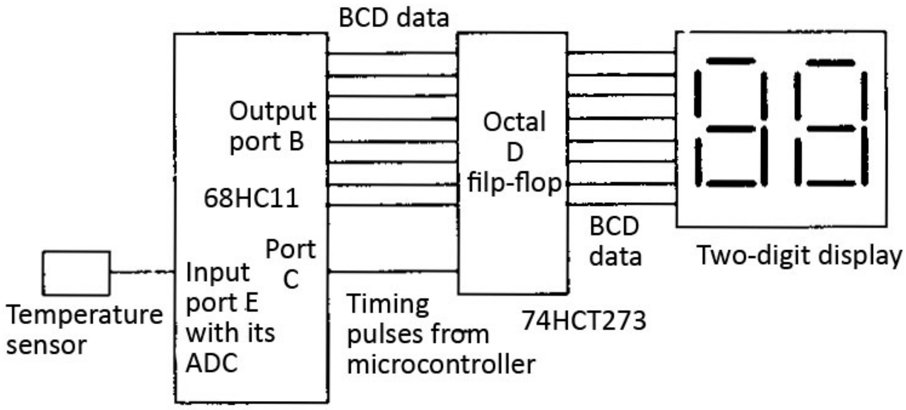


Figure 1. *Temperature measurement system*

Domestic Washing Machine

Figure 2 shows how a microcontroller might be used as the controller for a domestic washing machine. The microcontroller often used is the Motorola M68HCIO5B6; this is simpler and cheaper than the Motorola M68HC11microcontroller discussed earlier in this chapter and is widely used for low cost applications. The inputs from the sensors for water temperature and motor speed are via the analogue-digital input port. Port A provides the output for the various actuators used to control the machine and also the input for the water level switch. Port B gives outputs the display. Port C gives outputs to the display and also receives inputs from the keyboard used to input to the machine the various program selections. The PWM section of the timer provides a pulse width modulated signal to control the motor speed. The entire machine program is interrupted and stopped if the door of the washing machine is opened.

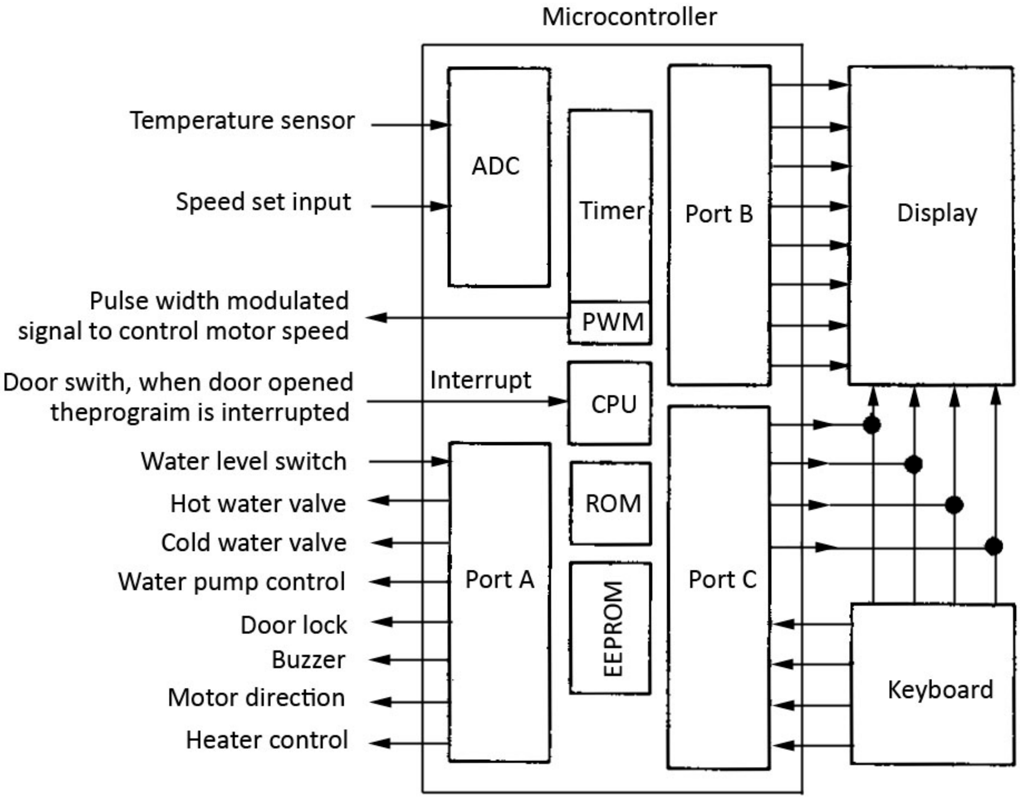


Figure 2. *Washing machine*

Programming

A commonly used method for the development of program follows the steps:

1. Define the problem, stating quite clearly what function the program is to perform, the inputs and outputs required, any constraints regarding speed of operation, accuracy., memory size, etc.
2. Define the algorithm to be used. An algorithm is the sequence of steps which define a method of solving the problem.
3. For systems with fewer then thousand of instructions a useful aid is to represent the algorithm by means of a flow chart. Figure 3 shows the standard symbols used in the preparation of flow charts. Each step of an algorithm is represented by one or more of these symbols and linked together by lines to represent the program flow. Another useful design tool is pseudo code. Pseudo code is way of describing the steps in an algorithm in an informal way which can later the translated in to a program.

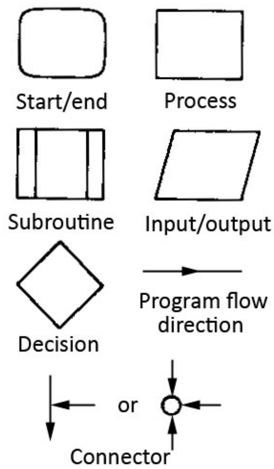


Figure 3. *Flow chart symbols*

Programmable Logic Controllers

A programmable logic controller (PLG) can be defined as a digital electronics device that uses a programmable memory to store instructions and to implement functions such as logic, sequencing, timing, counting and arithmetic in order to control machines and process. The term logic is used because the programming is primarily concerned wi5th implementing logic and switching operations. Inputs devices, e.g. switches, and output devices, e.g. motors, being controlled are connected to the PLC and these the controller monitors the inputs and outputs according to this program stored in the PLC by the operator and so controls the machine or process. Originally they were designed as a replacement for hard-wired relay and timer logic control systems. PLCs have the great advantage that it is possible to modify a control system without having to rewire the connections to the input and output devices, the only requirement being that an operator has to key in a different set of instructions. The result is a flexible system which can be used to control system which very quite widely in their nature and complexity.

PLCs are simpler to computers but have certain features w3hich are specific to their use as controllers. These are:

1. They are rugged and designed to withstand vibrations, temperature, humidity and noise.
2. They are easily programmed and have an easily understood programming language. Programming is primarily concerned with logic and switching operations.

Basic Structure

Figure 4 shows the basic internal structure of a PLC. It consists essentially of a central processing unit (CPU), memory, and input/output circuitry. The CPU controls and processes all the operations within the PLC. It is supplied with a clock with a frequency of typically between 1 and 8 MHz. This frequency determines the operating speed of the PLC and provides the timing and synchronization for all elements in the system. A bus system carries information and data to and from the CPU, memory and input/output units. There are several memory elements a system ROM to give permanent storage for the operating system and fixed data, RAM for the user's program, and temporary buffer stores for the input/output channels.

Architecture of a PLC

The programs in RAM can be changed by the user. However, to prevent the loss of the programs when the power supply is switched off, a battery is likely to be used in the PLC to maintain the RAM contents for a period of time. After a program has been developed in RAM it may be loaded in to an EPROM memory chip and so made permanent. Specification for small PLCs often specify the program memory size in the terms of the number of program steps that can be stored. A program step is an instruction for some event to occur. A program task might consist of a number of steps and could be, for example: examine the state of switch A, examine the switch B, if A and B are closed then energies solenoid P which then might result in the operation of some actuator. When this happens another task might then be started. Typically the number steps that can be handled by a small PLC is of the order of 300 to 1000, which is generally more than adequate for most control situations.

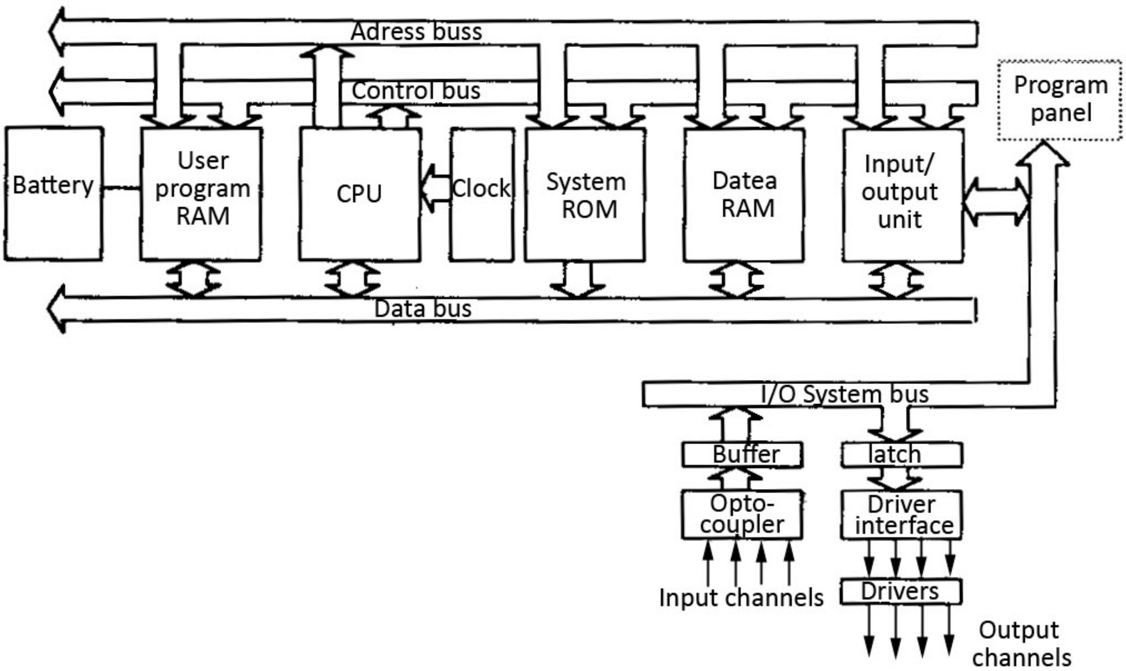


Figure 4. *Architecture of a PLC*

The input/output unit provides the interface between the system and the outside world. Programs are centered into the input/output unit from the panel which can vary from small keyboards with liquid crystal displays to those using a visual display unit (VDU) with keyboard and screen display. Alternatively the programs can be entered in to the system by means of a link to a personal computer (PC) which is loaded with an appropriate software package.

The input/output channels provide signal conditioning and isolation functions so that sensors and actuators can be generally directly connected to them without the need for other circuitry. Figure 19.3 shows the basic form of an input channel. Common input voltages are 5 V and 24 V.

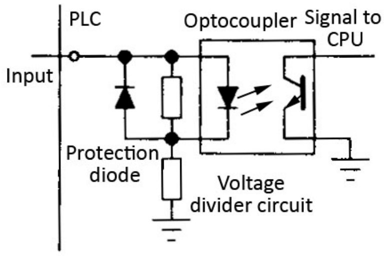


Figure 5. *Input channel*

Programming

PLC programming based on the use of ladder diagrams involves writing a program in a similar manner to drawing a switching circuit. The ladder diagram consists of two vertical lines representing the power rails. Circuits are connected as horizontal lines, i.e. the rungs of the ladder, between these two verticals Figure 19.6 shows the basic standard symbols that are used and an example of rungs in ladder diagram.

In drawing the circuit line for a rung, inputs most always precede outputs and there must be at least one output on each line. Each rung must start with an input or a series of inputs and end with an output.

The inputs and outputs are numbered, the notation used depending on the PLC manufacturer, e.g. the Mitsubishi F series of PLCs pressed input elements by an X and output elements by a Y and uses the following numbers:

Inputs	X400 – 407, 410 – 413
	X500 – 507, 510 – 513
	(24 possible inputs)
Outputs	Y430 – 437
	Y350 – 537
	(16 possible outputs)

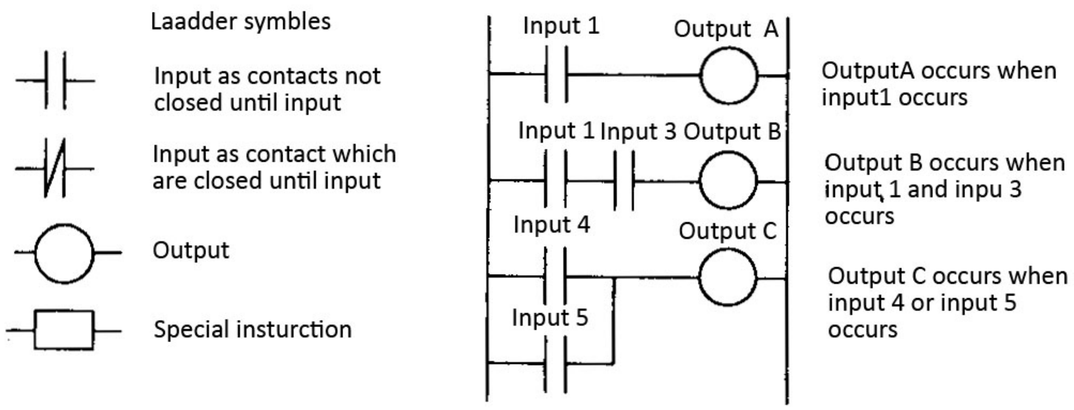


Figure 6. *Ladder diagram*

To illustrate the drawing of a ladder diagram, consider a situation where the output from the PLC is to energies a solenoid when a normally open start switch connected to the input is activated by being closed (Figure 7a). The program required is shown in Figure 7b. Starting with the input, we have the normally open symbol ll. This might have an input address X400. The line terminates with the output, the solenoid, with the symbol O. This might have the output address Y430. To indicate the end of the program the end rung is marked. When the switch is closed the solenoid is activated. This might, for example, be a solenoid valve which opens to allow water to enter a vessel.

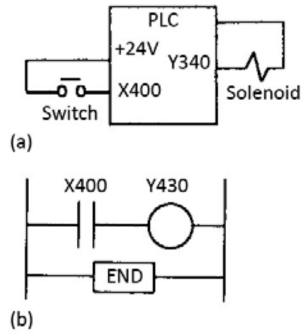


Figure 7. *Switch controlling a solenoid*

Another example might be an on-off temperature control in which the input goes from low to high when the temperature sensor reaches the set temperature. The output is then to go from on to off. The temperature sensor shown in the figure is a thermistor connected in a bridge arrangement with out put to an amplifier connected as a comparator. The program shows the input as a normally closed pair of contacts, so giving the on signal and hence an output. When the contacts are opened to give the off signal then the output is switched off.

Such ladder programs can be entered from special keypads or selected from a monitor screen by using a mouse. They can also be specified by using a mnemonic language. However they are centered, the programs are then translated by the PLC into machine language for the benefit of the microprocessor and its associated elements.

Designing

This chapter is a brief review of the design process and brings together many of the topics discussed in this book in the consideration of both traditional and mechatronics solutions to design problems and case studies. The design processes can be considered as a number of stages:

1. The need

The design process begins with a need from, perhaps, a customer or client. This may be identified by market research being used to establish the needs of potential costumers.

2. Analysis of the problem

The first stage in developing a design is to find out the true nature of the problem, i.e. analyzing it. This is an important stage in that not defining the problem accurately can lead to wasted time on designs that will not fulfill the need.

3. Preparation of a specification

Following the analysis a specification of the requirements can be prepared. This will state the problem, any constraints placed on the solution, and the criteria which may be used to judge the quality of the design. In stating the problem, all the functions required of the design, together with any desirable features, should be specified. Thus there might be a statement of mass, dimensions, types and range of motion required, accuracy, input and output requirement of elements, interfaces, power requirements, operating environment, relevant standards and codes of practice, etc.

4. Generation of possible solutions

This is often termed the conceptual stage. Outline solutions are prepared which are worked out in sufficient detail to indicate the means of obtaining each of the required functions, e.g. approximate sizes, shapes, materials and costs. It also means finding out what has been done before for similar problems, there is no sense in reinventing the wheel.

5. Selections of a suitable solution

The various solutions are evaluated and the most suitable one selected.

6. Production of a detailed design

The detail of the selected design has now to be worked out. This might require the production of prototypes or mock-ups in order to determine the optimum details of a design.

7. Production of working drawings

The selected design is then translated into working drawings, circuit diagrams, etc. so that the item can be made. In should not be considered that each stage of the design process just flows on stage by stage. There will often be the need to return to an earlier stage and give it further consideration. Thus when at the stage of generating possible solutions there might be a need to go back and reconsider the analysis of the problem.

Designing Mechatronics Systems

The design process consists of the following stages (refer Figure 8):

Stage 1: *Need for design*

The design process begins with a need. Needs are usually arise from dissatisfaction with an existing situation. Needs may come from inputs of operating or service personal or from a customer through sales or marketing representatives. They may be to reduce cost, increase reliability or performance or just change because of public has become bored with the product.

Stage 2: *Analysis of problem*

Probably the most critical step in a design process is the analysis of the problem i.e., to find out the true nature of the problem. The true problem is not always what it seems to be at the first glance. Its importance is often overlooked because this stage requires such a small part of the total time to create the final design. It is advantageous to define the problem as broadly as possible. If the problem is not accurately defined, it will lead to a waste of time on designs and will not fulfill the need.

Stage 3: *Preparation of specification*

The design must meet the required performance specifications. Therefore, specification of the requirements needs to be prepared first. This will state the problem definition of special technical terms, any constraints placed on the solution and the criteria that will be used to evaluate the design. Problem statement includes all the functions required of the design, together with any desirable features. The following are some of the statements about the problem:

- Mass and dimensions of design.
- Type and range of motion required.
- Accuracy of the element.
- Input and output requirements of elements.
- Interfaces.
- Power requirements.
- Operating environment.
- Relevant standards and code of practice, etc.

Stage 4: *Generation of possible solution*

This stage is often known as *conceptualisation stage.* The conceptulisation step is to determine the elements, mechanisms, materials, process of configuration that in some combination or other result in a design that satisfies the need. This is the key step for employing inventiveness and creativity.

A vital aspect of this step is synthesis. Synthesis is the process of taking elements of the concept and arranging them in the proper order, sized and dimensioned in the proper way. Outline solutions are prepared for various possible models which are worked out in sufficient details to indicate the means of obtaining each of the required functions.

Stage 5: *Selection of suitable solution or Evaluation*

This stage involves a thorough analysis of the design. The evaluation stage involves detailed calculation, often computer calculation of the performance of the design by using an analytical model. The various solutions obtained in stage 4 are analysed and the most suitable one is selected.

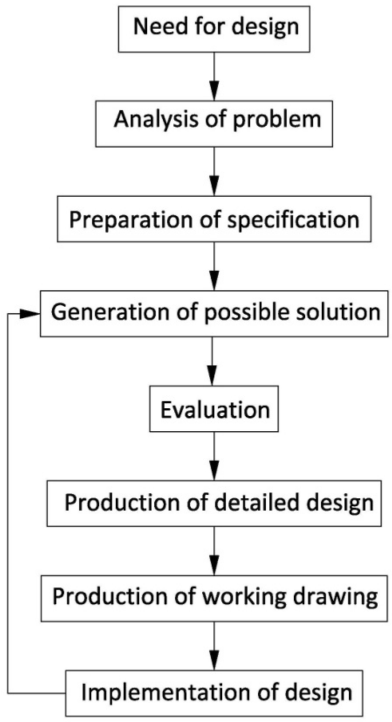


Figure 8. *Stages in designing mechatronics systems*

Stage 6: *Production of detailed design*

The detail of selected design has to be worked out. It might have required at extensive simulated service testing of an experimental model or a full size prototype in order to determine the optimum details of design.

Stage 7*: Production of working drawing*

The finalised drawing must be properly communicated to the person who is going to manufacture. The communication may be oral presentation or a design report. Detailed engineering drawings of each components and the assembly of the machine with complete specification for the manufacturing process are written in the design report.

Stage 8: *Implementation of design*

The components as per the drawings are manufactured and assembled as a whole system.

Case Studies

Mpcnatronics systems are widely used now a day in many industries. Some of the examples are explained here.

1. Case Study 1: Pick and Place Robot

The basic form of a pick and place robot is shown in Figure 9. The robot has three axes about which motion can occur. The following movements are required for this robot.

1. Clockwise and anticlockwise rotation of the robot unit on its base.
2. Linear movement of the arm horizontally i.e., extension or contraction of arm.

3. Up and down movement of the arm and
4. Open and close movement of the gripper.

The foresaid movements can be obtained by pneumatic cylinders which are operated by solenoid valves with limit switches. Limit switches are used to indicate when a motion is completed.

The clockwise, rotation of the robot unit on its base can be obtained from a piston and cylinder arrangement during pistons forward movement. Similarly counter clockwise rotation can be obtained during backward movement of the piston in cylinder. Linear movement of the arm can result during forward and backward movement of the piston in a cylinder.

The upward movement of the arm can result from forward movement of the piston in a cylinder whereas downward movement from its retardation. The griper can also be operated in a similar way as explained above i.e., gripper is opened during forward movement of the piston and closed during backward movement of the piston in the cylinder. Figure 10 shows a mechanism used for this purpose.

A microcontroller used to control the solenoid valves of various cylinders is shown in Figure 11. The micro controller used of this purpose is M68HC11 type. A software program is used to control the robot.

TRIAC optoisolator consists of LED and TRIAC. If the input of the LED is 1, it glows and activates the TRIAC to conduct the current to the solenoid valve. Otherwise TRIAC will not conduct die current to the solenoid valve.

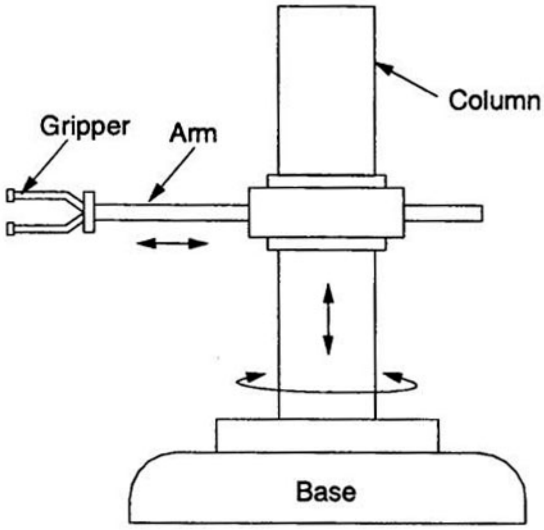


Figure 9. Basic from of a pick and place robot

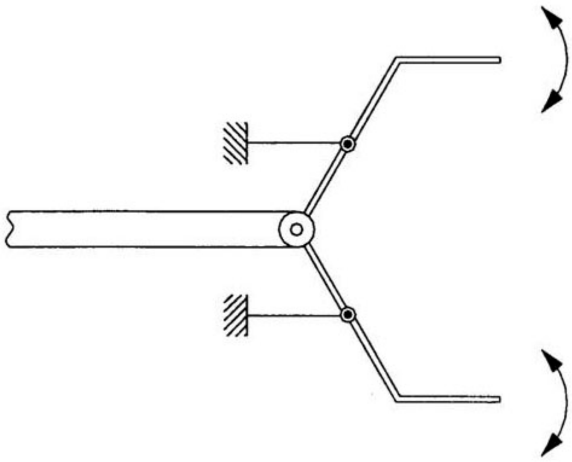


Figure 10. *Gripper mechanism of a robot*

2. Case Study 2: Automatic Car Park System

Consider an automatic car park system with barriers operated by coin inserts. The system uses a PLC for its operation. There are two barriers used namely in barrier and out barrier. In barrier is used to open when the correct money is inserted while out barrier opens when a car is detected in front of it. Figure 5.18 shows a schematic arrangement of an automatic car park barrier. It consists of a barrier which is pivoted at one end, two solenoid valves A and B and a piston cylinder arrangement.

A connecting rod connects piston and barrier as shown in Figure 12. Solenoid valves are used to control the movement of the piston. Solenoid A is used to move the piston upward in turn barrier whereas solenoid B is used to move the piston downward. Limit switches are used to detect me foremost position of the barrier. When current flows through solenoid A, the piston in the cylinder moves upward and causes die barrier to rotate about its pivot and raises to let a car through.

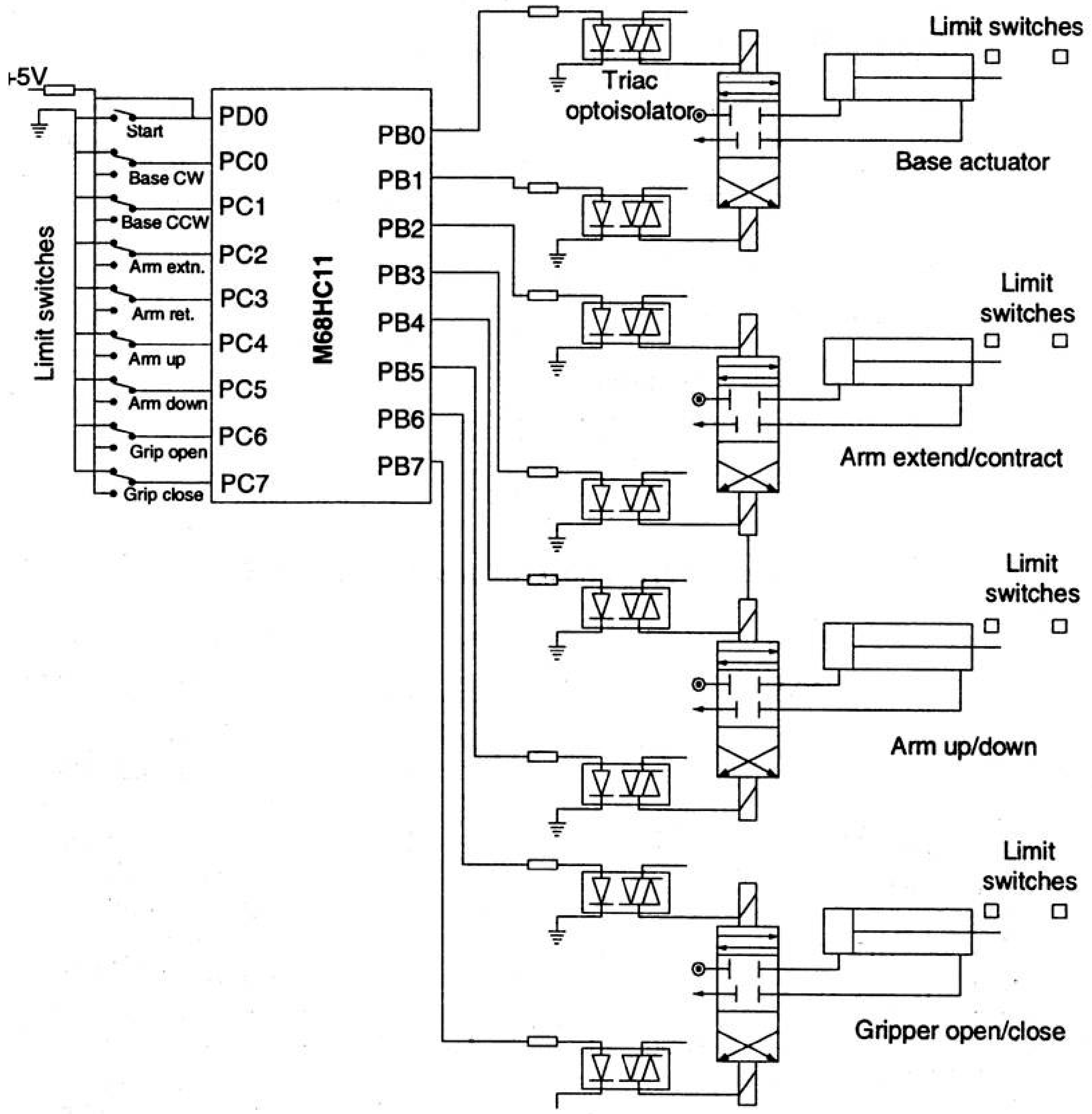


Figure 11 *Microcontroller circuit for pick and place robot*

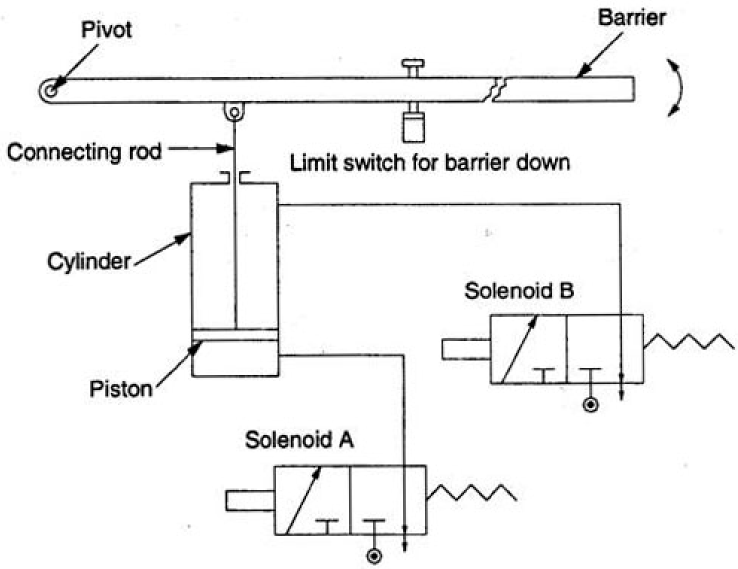


Figure 12 *Microcontroller circuit for pick and place robot*

When the barrier hits the limit switch, it will turns on the timer to give a required time delay. After that time delay, the solenoid B is activated which brings the barrier downward by an operating piston in the cylinder. This principle is used for both the barriers.

3. Case study 3: Engine Management System

An electronic engine management system is made up of sensors, actuators, and related wiring that is tied into a central processor called *microprocessor or microcomputer* (a smaller version of a computer).

Electronic management systems monitor and gather data from a number of sensors in the engine and continuously adjust the fuel supply and injection timing. This minimizes emissions and maximizes fuel efficiency and engine output at any given workload. The electronic engine management generally consists of the following basic components: An electronic control unit (ECU), a fuel delivery system (typically fuel injection), an ignition system and a number of sensors. Figure 13 shows the various components in the typical engine management system.

Electronics control unit (ECU):

The sensors provide feedback to the ECU to indicate how the engine is running so that die ECU can make the necessary adjustments to the operation of the fuel delivery and/or ignition system.

Fuel delivery system:

This system consist high pressure fuel pump which is mounted in or near the tank. The fuel line from the pump passes through a filter before it runs forward to the engine bay. The fuel line connects to a fuel rail that feeds each of the injectors. At the end of the rail is a fuel pressure regulator, with surplus fuel heading back to the tank in the return line.

Ignition system:

Ignition system consists of ignition coil, distributor and spark plug. These components are connected with the ECU to receive the signal for proper timed operation.

Various sensors:

Engine sensors fall into five broad categories: Throttle-Position Sensors, Exhaust Gas Oxygen Sensors, Manifold Absolute Pressure Sensors, Temperature Sensors and Speed/Timing Sensors. All these sensor functions are centrally controlled by microcontroller as shown in Figure 14.

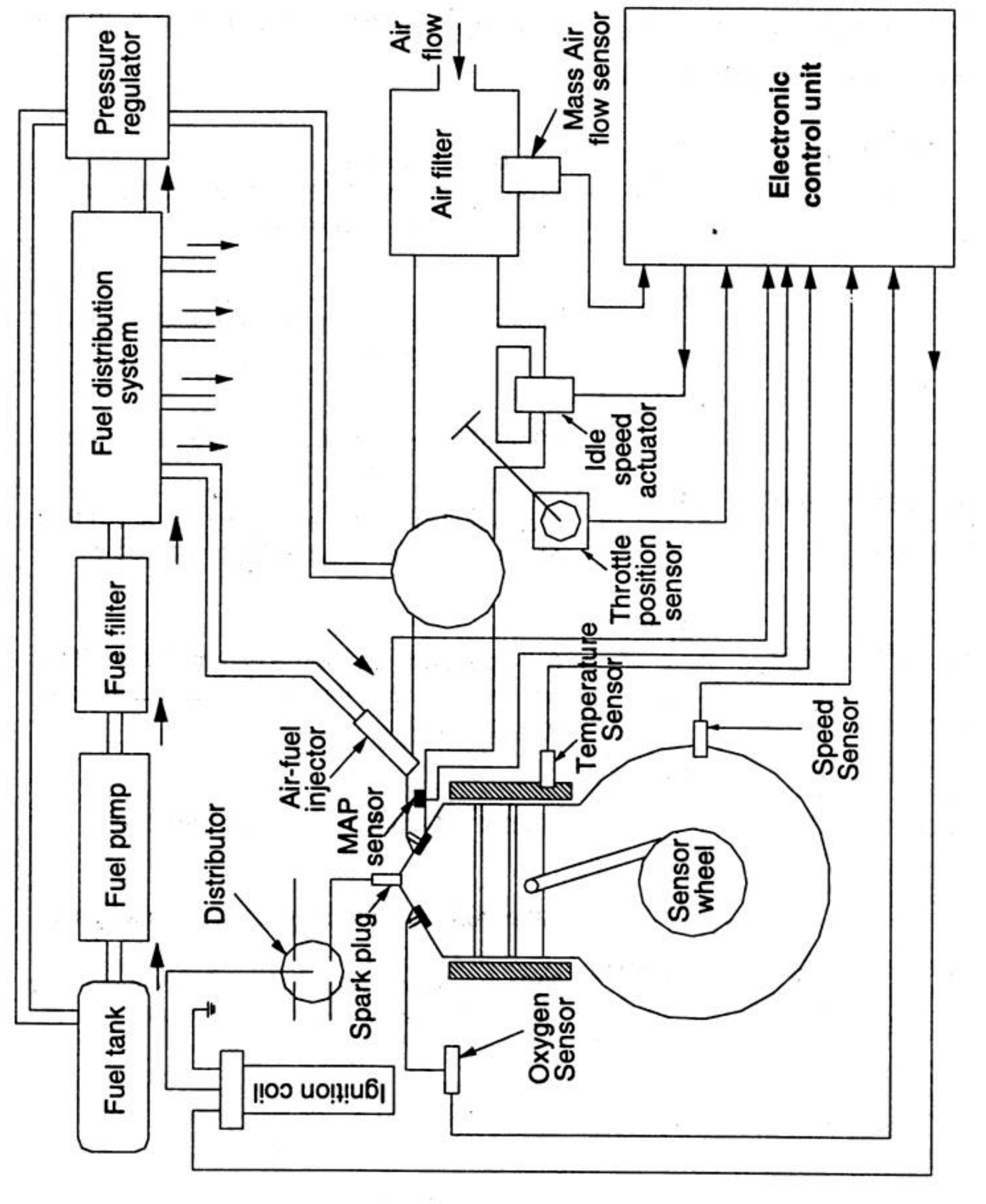


Figure 13 components of engine management system

a. Throttle-Position sensors:

A throttle-position sensor sends the signal to ECU about the throttle opening and the force applied by the driver. Then the ECU controls the fuel delivery and spark timing based on the throttle position. Two common throttle-position sensors are potentiometric and Hall-effect sensors.

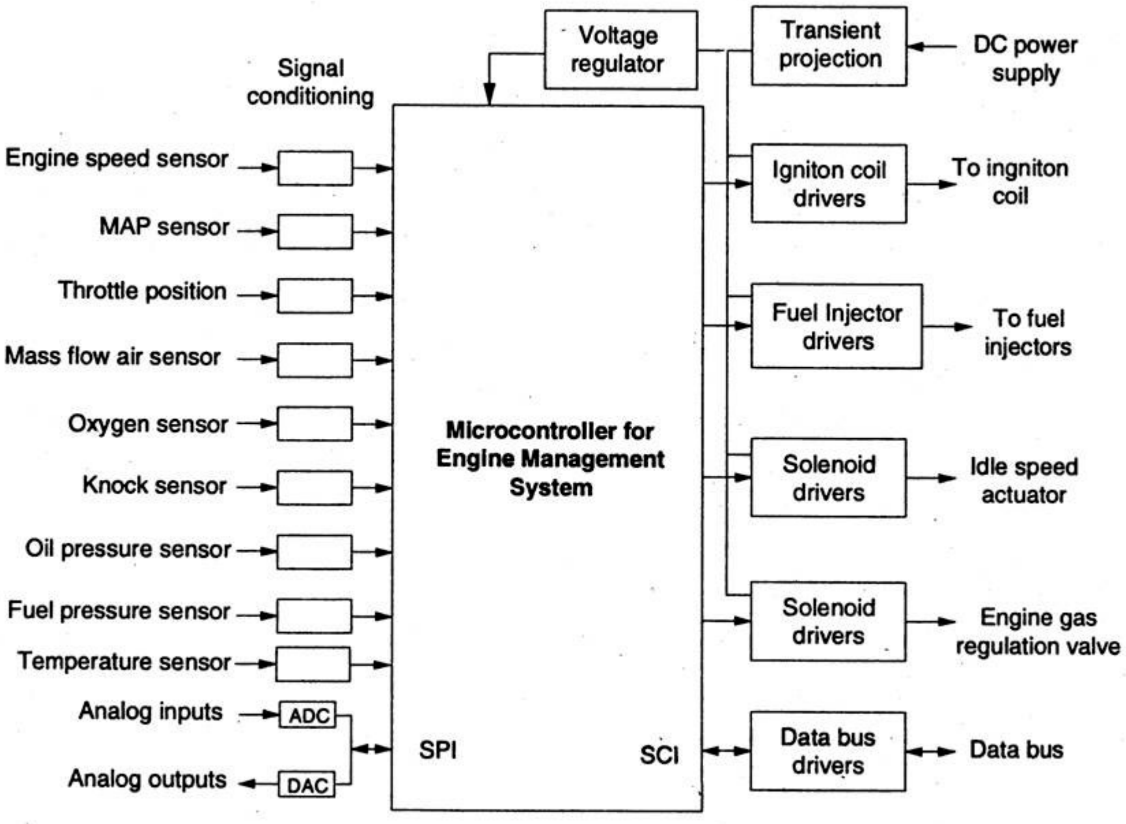


Figure 14 Interfacing of sensor with controller in engine management system

b. Exhaust Gas Oxygen (EGO) Sensors:

Exhaust gas oxygen (EGO) sensors are placed within the engine's exhaust system. The amount of oxygen in the exhaust gas indicates whether or not the ECU has directed the fuel delivery system to provide the proper' air-to-fuel ratio. If the relative amount of air is too high or too low, engine power, smoothness, fuel efficiency and emissions will all suffer.

c. Manifold Absolute Pressure (MAP) Sensors:

Manifold Absolute Pressure (MAP) Sensors measure the degree of vacuum in the engine's intake manifold. The amount of vacuum depends on engine rpm and throttle opening. The most common MAP sensors are piezoresistive .and variable capacitor sensors.

d. Temperature Sensors:

Temperature sensors are used to report engine temperature to the driver/operator via dash panel-mounted temperature gauge, report engine temperatures to the ECU to activate/de-activate cooling fans in water-cooled engines, to richen fuel mixtures for easier starting in cold weather and to lean-out mixtures for maximum fuel economy. Two common temperature sensors are thermistors or thermodiodes.

e. Engine Speed/Timing Sensors:

Speed/timing sensors provide information to the ECU regarding engine speed and the crank position. This information is used by the ECU to control fuel and ignition, as well as to make sure that engine speed does not exceed safe operating limits. It is also used to control the fuel injectors and spark plugs. Most common speed/timing sensors are variable reluctance, optical crankshaft position and Hall-effect sensors.

f. Exhaust gas regulation (EGR) Valve Position Sensor:

The signal from EGR valve position sensor is used to adjust the air fuel mixture. The exhaust gases introduced by the EGR valve into the intake manifold reduce the available oxygen and thus less fuel is needed in order to maintain low hydro carbon level in the exhaust.

g. Mass Air flow (MAF) sensor:

MAF sensor is used to measure engine load to squirt in the right amount of petrol, and fire the spark at just the right moment. The amount of power being" developed depends on how much air the engine is breathing. Most common airflow sensors are Hot Wire Airflow sensor and Vane Airflow Meter.

h. Knock Sensor:

The knock sensor is used to identify the sounds of knocking and sends signal to ECU to avoid knocking. It is screwed into the engine block and is designed to separate out the special noise which means that knocking is occurring. Many Electronic Fuel Injection (EFI) engines run ignition timing very close to knocking.